Representation of gravity field equation and solutions, Hawking Radiation in Data General Relativity theory

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ABSTRACT

In the general relativity theory, we find the representation of the gravity field equation and solutions. We treat the representation of Schwarzschild solution, Reissner-Nodstrom solution, Kerr-Newman solution, Robertson -Walker solution. Specially, Robertson -Walker solution is an uniqueness. We found new general relativity theory (we call it Data General Relativity Theory;DGRT). We treat the data of Hawking radiation by Data general relativity theory. This theory has to apply black hole (specially, Primordial Massive Black Hole; PMBH) because black hole(PMBH) is an idealistic structure.

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Data General relativity theory,

Hawking radiation

Primordial Massive Black Hole

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1. Introduction

In the general relativity theory, our article's aim is that we find the representation of the gravity field equation and solutions. We found new general relativity theory(we can call it Data General relativity theory). We more obtain data of Hawking radiation by Data general relativity theory.

First, the gravity potential $g_{\mu\upsilon}$ is

$$ds^2 = g_{\mu\upsilon} dx^\mu dx^\upsilon \qquad (1)$$

In gravity potential $g_{\mu\upsilon}$, we introduce tensor $f_{\mu\upsilon}$ and scalar K.

$$f_{\mu\upsilon} = K g_{\mu\upsilon} \;, \quad \frac{\partial K}{\partial x^\lambda} = 0$$

$$ds'^2 = f_{\mu\upsilon} dx^\mu dx^\upsilon = \bar{g}_{\mu\upsilon} d\bar{x}^\mu d\bar{x}^\upsilon = \bar{g}_{\mu\upsilon} \frac{\partial \bar{x}^\mu}{\partial \bar{x}'^\alpha} \frac{\partial \bar{x}^\upsilon}{\partial \bar{x}'^\beta} d\bar{x}'^\alpha \, d\bar{x}'^\beta$$

$$= \bar{g}'_{\alpha\beta} \, d\bar{x}'^\alpha \, d\bar{x}'^\beta = f'_{\alpha\beta} \, dx'^\alpha \, dx'^\beta$$

$$\bar{g}'_{\alpha\beta} = \bar{g}_{\mu\upsilon} \frac{\partial \bar{x}^\mu}{\partial \bar{x}'^\alpha} \frac{\partial \bar{x}^\upsilon}{\partial \bar{x}'^\beta} \;, \quad f'_{\alpha\beta} = f_{\mu\upsilon} \frac{\partial x^\mu}{\partial x'^\alpha} \frac{\partial x^\upsilon}{\partial x'^\beta} \;,$$

$$g_{\mu\upsilon} = \bar{g}_{\mu\upsilon} \qquad (2)$$

In inverse gravity potential $g^{\mu\upsilon}$,

$$f^{\mu\upsilon} f_{\mu\upsilon} = \delta_\mu{}^\upsilon = (\frac{1}{K} g^{\mu\upsilon})(K g_{\mu\upsilon}) \,, \; f^{\mu\upsilon} = \frac{1}{K} g^{\mu\upsilon} \qquad (3)$$

In Christoffel symbol $\Gamma^\rho{}_{\mu\upsilon}$,

$$\Gamma'^\rho{}_{\mu\upsilon} = \frac{1}{2} f^{\rho\lambda} (\frac{\partial f_{\mu\lambda}}{\partial x^\upsilon} + \frac{\partial f_{\upsilon\lambda}}{\partial x^\mu} - \frac{\partial f_{\mu\upsilon}}{\partial x^\lambda})$$

$$= \frac{1}{2} (\frac{1}{K} g^{\rho\lambda}) (K \frac{\partial g_{\mu\lambda}}{\partial x^\upsilon} + K \frac{\partial g_{\upsilon\lambda}}{\partial x^\mu} - K \frac{\partial g_{\mu\upsilon}}{\partial x^\lambda}) = \Gamma^\rho{}_{\mu\upsilon}$$

$$\bar{\Gamma}^\rho{}_{\mu\upsilon} = \frac{1}{2} \bar{g}^{\rho\lambda} (\frac{\partial \bar{g}_{\mu\lambda}}{\partial \bar{x}^\upsilon} + \frac{\partial \bar{g}_{\upsilon\lambda}}{\partial \bar{x}^\mu} - \frac{\partial \bar{g}_{\mu\upsilon}}{\partial \bar{x}^\lambda}) = \frac{1}{\sqrt{K}} \Gamma^\rho{}_{\mu\upsilon} \qquad (4)$$

Therefore, in the curvature tensor $R^\rho{}_{\mu\upsilon\lambda}$,

$$R'^\rho{}_{\mu\upsilon\lambda} = \frac{\partial \Gamma'^\rho{}_{\mu\upsilon}}{\partial x^\lambda} - \frac{\partial \Gamma'^\rho{}_{\mu\lambda}}{\partial x^\upsilon} + \Gamma'^\sigma{}_{\mu\upsilon} \, \Gamma'^\rho{}_{\sigma\lambda} - \Gamma'^\sigma{}_{\mu\lambda} \, \Gamma'^\rho{}_{\sigma\upsilon}$$

$$= \frac{\partial \Gamma^{\rho}{}_{\mu\upsilon}}{\partial x^{\lambda}} - \frac{\partial \Gamma^{\rho}{}_{\mu\lambda}}{\partial x^{\upsilon}} + \Gamma^{\sigma}{}_{\mu\upsilon}\Gamma^{\rho}{}_{\sigma\lambda} - \Gamma^{\sigma}{}_{\mu\lambda}\Gamma^{\rho}{}_{\sigma\upsilon} = R^{\rho}{}_{\mu\upsilon\lambda}$$

$$\overline{R}^{\rho}{}_{\mu\upsilon\lambda} = \frac{\partial \overline{\Gamma}^{\rho}{}_{\mu\upsilon}}{\partial \overline{x}^{\lambda}} - \frac{\partial \overline{\Gamma}^{\rho}{}_{\mu\lambda}}{\partial \overline{x}^{\upsilon}} + \overline{\Gamma}^{\sigma}{}_{\mu\upsilon}\overline{\Gamma}^{\rho}{}_{\sigma\lambda} - \overline{\Gamma}^{\sigma}{}_{\mu\lambda}\overline{\Gamma}^{\rho}{}_{\sigma\upsilon}$$

$$= \frac{1}{K}\left(\frac{\partial \Gamma^{\rho}{}_{\mu\upsilon}}{\partial x^{\lambda}} - \frac{\partial \Gamma^{\rho}{}_{\mu\lambda}}{\partial x^{\upsilon}} + \Gamma^{\sigma}{}_{\mu\upsilon}\Gamma^{\rho}{}_{\sigma\lambda} - \Gamma^{\sigma}{}_{\mu\lambda}\Gamma^{\rho}{}_{\sigma\upsilon}\right) = \frac{1}{K} R^{\rho}{}_{\mu\upsilon\lambda} \quad (5)$$

In Ricci tensor $R_{\mu\upsilon}$,

$$R'_{\mu\upsilon} = R'^{\rho}{}_{\mu\rho\upsilon} = R^{\rho}{}_{\mu\rho\upsilon} = R_{\mu\upsilon} \quad , \overline{R}_{\mu\upsilon} = \overline{R}^{\rho}{}_{\mu\rho\upsilon} = \frac{1}{K} R^{\rho}{}_{\mu\rho\upsilon} = \frac{1}{K} R_{\mu\upsilon}$$

(6)

In curvature scalar R

$$R' = f^{\mu\upsilon} R'_{\mu\upsilon} = \frac{1}{K} g^{\mu\upsilon} R_{\mu\upsilon} = \frac{1}{K} R$$

$$\overline{R} = \overline{g}^{\mu\upsilon} \overline{R}_{\mu\upsilon} = \frac{1}{K} g^{\mu\upsilon} R_{\mu\upsilon} = \frac{1}{K} R \quad (7)$$

Hence, in the gravity field equation of Einstein,

$$R'_{\mu\upsilon} - \frac{1}{2} f_{\mu\upsilon} R' = R_{\mu\upsilon} - \frac{1}{2} K g_{\mu\upsilon} \left(\frac{1}{K} R\right)$$

$$= R_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} R = -\frac{8\pi G}{c^4} T_{\mu\upsilon}$$

$$\overline{R}_{\mu\upsilon} - \frac{1}{2} \overline{g}_{\mu\upsilon} \overline{R} = \frac{1}{K}\left(R_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} R\right)$$

$$= -\frac{8\pi G}{c^4} \frac{1}{K} T_{\mu\upsilon} \quad (8)$$

In Newtonian approximation, Energy-momentum tensor $T_{\mu\upsilon}$ is

$$\nabla^2 f_{00} = \nabla^2 K g_{00} \approx -\frac{8\pi G}{c^4} K T_{00} = -\frac{8\pi G}{c^4} T'_{00}$$

$$\rho c^2 = T_{00}, \quad K\rho c^2 = T'_{00} \quad (9)$$

$$\overline{\nabla}^2 \overline{g}_{00} = \frac{1}{K} \nabla^2 g_{00} \approx -\frac{8\pi G}{c^4} \frac{1}{K} T_{00} = -\frac{8\pi G}{c^4} \overline{T}_{00}$$

$$\rho c^2 = T_{00}, \quad \frac{1}{K} \rho c^2 = \overline{T}_{00} \quad (10)$$

Hence,

$$T'_{\mu\upsilon} = KT_{\mu\upsilon}, \quad \frac{1}{K}T_{\mu\upsilon} = \overline{T}_{\mu\upsilon} \tag{11}$$

Therefore, revised Einstein's gravity field equation is

$$R'_{\mu\upsilon} - \frac{1}{2}f_{\mu\upsilon}R' = R_{\mu\upsilon} - \frac{1}{2}g_{\mu\upsilon}R = -\frac{8\pi G}{c^4}T_{\mu\upsilon} = -\frac{8\pi G}{c^4}\frac{T'_{\mu\upsilon}}{K}$$

$$\overline{R}_{\mu\upsilon} - \frac{1}{2}\overline{g}_{\mu\upsilon}\overline{R} = \frac{1}{K}(R_{\mu\upsilon} - \frac{1}{2}g_{\mu\upsilon}R) = -\frac{1}{K}\frac{8\pi G}{c^4}T_{\mu\upsilon} = -\frac{8\pi G}{c^4}\overline{T}_{\mu\upsilon}$$

(12)

Therefore, revised gravity field equation of tensor $\overline{g}_{\mu\upsilon}$ is able to reduce Einstein's gravity field equation.

Therefore,

$$\overline{g}^{\mu\upsilon}[\overline{R}_{\mu\upsilon} - \frac{1}{2}\overline{g}_{\mu\upsilon}\overline{R}] = \frac{1}{K}g^{\mu\upsilon}[R_{\mu\upsilon} - \frac{1}{2}g_{\mu\upsilon}R] = -\frac{8\pi G}{c^4}\frac{1}{K}g^{\mu\upsilon}T_{\mu\upsilon} = -\frac{8\pi G}{c^4}\frac{1}{K}T^{\lambda}{}_{\lambda}$$

$$= -\frac{8\pi G}{c^4}\overline{g}^{\mu\upsilon}\overline{T}_{\mu\upsilon} = -\frac{8\pi G}{c^4}\overline{T}^{\lambda}{}_{\lambda}$$

$$\rightarrow -\overline{R} = -\frac{1}{K}R = -\frac{8\pi G}{c^4}\frac{1}{K}T^{\lambda}{}_{\lambda} = -\frac{8\pi G}{c^4}\overline{T}^{\lambda}{}_{\lambda} \tag{13}$$

Hence,

$$\frac{1}{K}T^{\lambda}{}_{\lambda} = \overline{T}^{\lambda}{}_{\lambda} \tag{14}$$

Ricci tensor is

$$\overline{R}_{\mu\upsilon} = \frac{1}{K}R_{\mu\upsilon} = -\frac{8\pi G}{c^4}\frac{1}{K}(T_{\mu\upsilon} - \frac{1}{2}g_{\mu\upsilon}T^{\lambda}{}_{\lambda}) = -\frac{8\pi G}{c^4}(\overline{T}_{\mu\upsilon} - \frac{1}{2}\overline{g}_{\mu\upsilon}\overline{T}^{\lambda}{}_{\lambda}) \tag{15}$$

The proper distance is

$$ds^2 = g_{\mu\upsilon}dx^{\mu}dx^{\upsilon}, \; ds'^2 = f_{\mu\upsilon}dx^{\mu}dx^{\upsilon} = \overline{g}_{\mu\upsilon}d\overline{x}^{\mu}d\overline{x}^{\upsilon} \tag{16}$$

2. Weak gravity field approximation.

Weak gravity field approximation is

$$g_{\mu\upsilon} = \eta_{\mu\upsilon} + h_{\mu\upsilon}, \quad |h_{\mu\upsilon}| << 1$$

$$R_{\mu\upsilon} = -\frac{8\pi G}{c^4}(T_{\mu\upsilon} - \frac{1}{2}g_{\mu\upsilon}T^{\lambda}{}_{\lambda}) \tag{17}$$

According to Eq(15),

$$\bar{g}_{\mu\upsilon} = \bar{\eta}_{\mu\upsilon} + \bar{h}_{\mu\upsilon}, \ \left|\bar{h}_{\mu\upsilon}\right| << 1$$

$$\bar{R}_{\mu\upsilon} = -\frac{8\pi G}{c^4}(\bar{T}_{\mu\upsilon} - \frac{1}{2}\bar{g}_{\mu\upsilon}\bar{T}^{\lambda}{}_{\lambda}) \tag{18}$$

The tensor of weak gravity field is

$$R_{\mu\upsilon} \approx -\frac{8\pi G}{c^4}S_{\mu\upsilon}, \ \bar{R}_{\mu\upsilon} \approx -\frac{8\pi G}{c^4}\bar{S}_{\mu\upsilon}$$

$$S_{\mu\upsilon} = T_{\mu\upsilon} - \frac{1}{2}\eta_{\mu\upsilon}T^{\lambda}{}_{\lambda},$$

$$\bar{S}_{\mu\upsilon} = \bar{T}_{\mu\upsilon} - \frac{1}{2}\bar{\eta}_{\mu\upsilon}\bar{T}^{\lambda}{}_{\lambda} \tag{19}$$

The solution is

$$h_{\mu\upsilon}(t,\vec{x}) = \frac{4G}{c^2}\int d^4x' \frac{S_{\mu\upsilon}(t - |\vec{x} - \vec{x}'|, \vec{x}')}{|\vec{x} - \vec{x}'|},$$

$$\int d^3x T_{00} = M \tag{20}$$

$$\bar{h}_{\mu\upsilon}(\bar{t},\vec{\bar{x}}) = \frac{4G}{c^2}\int d^4\bar{x}' \frac{\bar{S}_{\mu\upsilon}(\bar{t} - |\vec{\bar{x}} - \vec{\bar{x}}'|, \vec{\bar{x}}')}{|\vec{\bar{x}} - \vec{\bar{x}}'|},$$

$$\int d^3\bar{x}\bar{T}_{00} = \int K\sqrt{K}d^3x\frac{1}{K}T_{00} = \sqrt{K}M = \bar{M} \quad , \ T_{00} = K\bar{T}_{00} \tag{21}$$

As

$$\bar{h}_{00}(\vec{\bar{x}}) \approx \frac{4G}{\bar{r}c^2}\int d^3\bar{x}'[\bar{T}_{00} - \frac{1}{2}\bar{T}_{00}] = \frac{2\sqrt{K}GM}{\bar{r}c^2},$$

$$\bar{h}_{ij}(\vec{\bar{x}}) \approx \frac{4G}{\bar{r}c^2}\int d^3\bar{x}'[\frac{1}{2}\delta_{ij}\bar{T}_{00}] = \frac{2\sqrt{K}GM}{\bar{r}c^2}\delta_{ij}, \tag{22}$$

The proper distance is

$$-ds^2 = c^2d\tau^2 = -g_{\mu\upsilon}dx^{\mu}dx^{\upsilon} \approx (1 - \frac{2GM}{rc^2})c^2dt^2 - (1 + \frac{2GM}{rc^2})\delta_{ij}dx^i dx^j \tag{23}$$

The proper distance is in this theory

$$-ds'^2 = -Kds^2 = Kc^2d\tau^2 = -Kg_{\mu\upsilon}dx^{\mu}dx^{\upsilon}$$

$$\approx K(1 - \frac{2GM}{rc^2})c^2dt^2 - K(1 + \frac{2GM}{rc^2})\delta_{ij}dx^i dx^j$$

$$= (1 - \frac{2\sqrt{K}GM}{\bar{r}c^2})c^2 d\bar{t}^2 - (1 + \frac{2\sqrt{K}GM}{\bar{r}c^2})\delta_{ij} d\bar{x}^i d\bar{x}^j$$

$$= (1 - \frac{2G\overline{M}}{\bar{r}c^2})c^2 d\bar{t}^2 - (1 + \frac{2G\overline{M}}{\bar{r}c^2})\delta_{ij} d\bar{x}^i d\bar{x}^j$$

$$\approx \bar{g}_{\mu\upsilon} d\bar{x}^\mu d\bar{x}^\upsilon$$

$$\sqrt{K}t = \bar{t}, \sqrt{K}x^i = \bar{x}^i, \sqrt{K}x^j = \bar{x}^j, \sqrt{K}r = \bar{r}, \sqrt{K}M = \overline{M} \qquad (24)$$

3. The other representation in Schwarzschild solution, Reissner-Nodstrom solution, Kerr-Newman solution and Robertson-Walker solution

Schwarzschild solution (vacuum solution) is

$$R_{\mu\upsilon} = 0$$

$$ds^2 = -c^2(1 - \frac{2GM}{rc^2})dt^2 + \frac{dr^2}{1 - \frac{2GM}{rc^2}} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \qquad (25)$$

The other representation of Schwarzschild solution is

$$ds'^2 = f_{\mu\upsilon} dx^\mu dx^\upsilon = K g_{\mu\upsilon} dx^\mu dx^\upsilon = K \cdot ds^2$$

$$= -c^2 K(1 - \frac{2GM}{rc^2})dt^2 + \frac{Kdr^2}{1 - \frac{2GM}{rc^2}} + Kr^2 d\theta^2 + Kr^2 \sin^2\theta d\phi^2$$

$$= -c^2(1 - \frac{2\sqrt{K}GM}{\bar{r}c^2})d\bar{t}^2 + \frac{d\bar{r}^2}{1 - \frac{2\sqrt{K}GM}{\bar{r}c^2}} + \bar{r}^2 d\bar{\theta}^2 + \bar{r}^2 \sin^2\bar{\theta} d\bar{\phi}^2$$

$$= -c^2(1 - \frac{2G\overline{M}}{\bar{r}c^2})d\bar{t}^2 + \frac{d\bar{r}^2}{1 - \frac{2G\overline{M}}{\bar{r}c^2}} + \bar{r}^2 d\bar{\theta}^2 + \bar{r}^2 \sin^2\bar{\theta} d\bar{\phi}^2 = \bar{g}_{\mu\upsilon} d\bar{x}^\mu d\bar{x}^\upsilon$$

$$\sqrt{K}t = \bar{t}, \sqrt{K}r = \bar{r}, \theta = \bar{\theta}, \phi = \bar{\phi}, \sqrt{K}M = \overline{M} \qquad (26)$$

Reissner-Nodstrom solution is

$$ds^2 = g_{\mu\upsilon} dx^\mu dx^\upsilon$$

$$= -c^2(1 - \frac{2GM}{rc^2} + \frac{kGQ^2}{r^2c^4})dt^2 + \frac{dr^2}{1 - \frac{2GM}{rc^2} + \frac{kGQ^2}{r^2c^4}} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \qquad (27)$$

The other representation of Reissner-Nodstrom solution is

$$ds'^2 = f_{\mu\upsilon} dx^\mu dx^\upsilon = K g_{\mu\upsilon} dx^\mu dx^\upsilon = K \cdot ds^2$$

$$= -Kc^2 (1 - \frac{2GM}{rc^2} + \frac{kGQ^2}{r^2 c^4}) dt^2 + \frac{K dr^2}{1 - \frac{2GM}{rc^2} + \frac{kGQ^2}{r^2 c^4}} + K r^2 d\theta^2 + K r^2 \sin^2 \theta d\phi^2$$

$$= -c^2 (1 - \frac{2\sqrt{K} GM}{\bar{r} c^2} + \frac{KkGQ^2}{\bar{r}^2 c^4}) d\bar{t}^2 + \frac{d\bar{r}^2}{1 - \frac{2\sqrt{K} GM}{\bar{r} c^2} + \frac{KkGQ^2}{\bar{r}^2 c^4}} + \bar{r}^2 d\bar{\theta}^2 + \bar{r}^2 \sin^2 \bar{\theta} d\bar{\phi}^2$$

$$= -c^2 (1 - \frac{2G\overline{M}}{\bar{r} c^2} + \frac{kG\overline{Q}^2}{\bar{r}^2 c^4}) d\bar{t}^2 + \frac{d\bar{r}^2}{1 - \frac{2G\overline{M}}{\bar{r} c^2} + \frac{kG\overline{Q}^2}{\bar{r}^2 c^4}} + \bar{r}^2 d\bar{\theta}^2 + \bar{r}^2 \sin^2 \bar{\theta} d\bar{\phi}^2$$

$$= \bar{g}_{\mu\upsilon} d\bar{x}^\mu d\bar{x}^\upsilon$$

$$\sqrt{K} t = \bar{t}\,,\ \sqrt{K} r = \bar{r}\ \ , \theta = \bar{\theta}, \phi = \bar{\phi}\,, \sqrt{K} M = \overline{M}\,, K Q^2 = \overline{Q}^2 \qquad (28)$$

Kerr-Newman solution is

$$ds^2 = g_{\mu\upsilon} dx^\mu dx^\upsilon$$

$$= -c^2 (1 - \frac{2c^2 GMr - kGQ^2}{c^4 \Sigma}) dt^2 + 2(2c^2 MGr - kGQ^2) \frac{a \sin^2 \theta}{c^4 \Sigma} c dt d\phi$$

$$- \frac{c^4 \Sigma}{r^2 - c^2 2GMr + a^2 + kGQ^2} dr^2 - \Sigma d\theta^2$$

$$- \sin^2 \theta [r^2 + a^2 + (2c^2 GMr - kGQ^2) \frac{a^2 \sin^2 \theta}{c^4 \Sigma}] d\phi^2$$

$$\Sigma = r^2 + a^2 \cos^2 \theta \qquad (29)$$

The other representation of Kerr-Newman solution is

$$ds'^2 = f_{\mu\upsilon} dx^\mu dx^\upsilon = K g_{\mu\upsilon} dx^\mu dx^\upsilon = K \cdot ds^2$$

$$= -Kc^2 (1 - \frac{2c^2 GMr - kGQ^2}{c^4 \Sigma}) dt^2 + 2K(2c^2 MGr - kGQ^2) \frac{a \sin^2 \theta}{c^4 \Sigma} c dt d\phi$$

$$- \frac{K \Sigma c^4}{r^2 - 2c^2 GMr + a^2 + kGQ^2} dr^2 - K \Sigma d\theta^2$$

$$- K \sin^2 \theta [r^2 + a^2 + (2c^2 GMr - kGQ^2) \frac{a^2 \sin^2 \theta}{c^4 \Sigma}] d\phi^2$$

$$= -c^2(1-\frac{2c^2G\sqrt{K}M\sqrt{K}r - kGKQ^2}{K\Sigma c^4})d\bar{t}^2 + 2(2c^2\sqrt{K}MG\sqrt{K}r - kGKQ^2)\frac{\sqrt{K}a\sin^2\theta}{K\Sigma c^4}cd\sqrt{K}td\phi$$

$$-\frac{K\Sigma c^4}{Kr^2 - 2c^2G\sqrt{K}M\sqrt{K}r + Ka^2 + kGKQ^2}d(\sqrt{K}r)^2 - K\Sigma d\theta^2$$

$$-\sin^2\theta[Kr^2 + Ka^2 + (2c^2G\sqrt{K}M\sqrt{K}r - kGKQ^2)\frac{Ka^2\sin^2\theta}{K\Sigma c^4}]d\phi^2$$

$$= -c^2(1-\frac{2c^2G\overline{M}\bar{r} - kG\overline{Q}^2}{\overline{\Sigma}c^4})d\bar{t}^2 + 2(2c^2\overline{M}G\bar{r} - kG\overline{Q}^2)\frac{\bar{a}\sin^2\bar{\theta}}{\overline{\Sigma}c^4}cd\bar{t}d\bar{\phi}$$

$$-\frac{\overline{\Sigma}c^4}{\bar{r}^2 - 2c^2G\overline{M}\bar{r} + \bar{a}^2 + kG\overline{Q}^2}d\bar{r}^2 - \overline{\Sigma}d\bar{\theta}^2$$

$$-\sin^2\bar{\theta}[\bar{r}^2 + \bar{a}^2 + (2c^2G\overline{M}\bar{r} - kG\overline{Q}^2)\frac{\bar{a}^2\sin^2\bar{\theta}}{c^4\overline{\Sigma}}]d\bar{\phi}^2$$

$$= \bar{g}_{\mu\upsilon}d\bar{x}^\mu d\bar{x}^\upsilon$$

$$\overline{\Sigma} = K\Sigma = Kr^2 + Ka^2\cos^2\theta = \bar{r}^2 + \bar{a}^2\cos^2\bar{\theta}$$

$$\sqrt{K}t = \bar{t},\ \sqrt{K}r = \bar{r}\ ,\theta = \bar{\theta}, \phi = \bar{\phi}, \sqrt{K}M = \overline{M}, KQ^2 = \overline{Q}^2, \sqrt{K}a = \bar{a}\ ,$$

$$\bar{J} = c\overline{M}\bar{a} = KcMa = KJ \tag{30}$$

In this time, we obtain the data of the time t, the distance r, the mass M, the charge Q and the angular momentum J.

Robertson-Walker solution is

$$ds^2 = g_{\mu\upsilon}dx^\mu dx^\upsilon$$

$$= -c^2dt^2 + \Omega^2(t)[\frac{dr^2}{1-kr^2} + r^2d\theta^2 + r^2\sin^2\theta d\phi^2] \tag{31}$$

The other representation of Robertson-Walker solution is by the other scalar K',

$$ds'^2 = f_{\mu\upsilon}dx^\mu dx^\upsilon = K' g_{\mu\upsilon}dx^\mu dx^\upsilon = K'\cdot ds^2$$

$$= -K'c^2dt^2 + \Omega^2(t)[\frac{K'dr^2}{1-k\frac{K'r^2}{K'}} + K'r^2d\theta^2 + K'r^2\sin^2\theta d\phi^2]$$

$$= -c^2d\bar{t}^2 + \overline{\Omega}^2(\bar{t})[\frac{d\bar{r}^2}{1-\frac{k\bar{r}^2}{K'}} + \bar{r}^2d\bar{\theta}^2 + \bar{r}^2\sin^2\bar{\theta}d\bar{\phi}^2]$$

$$= -c^2 d\bar{t}^2 + \bar{\Omega}^2(\bar{t})[\frac{d\bar{r}^2}{1-k'\bar{r}^2} + \bar{r}^2 d\bar{\theta}^2 + \bar{r}^2 \sin^2\bar{\theta} d\bar{\phi}^2] = \bar{g}_{\mu\upsilon} d\bar{x}^{\mu} d\bar{x}^{\upsilon}$$

$$\sqrt{K'}t = \bar{t}\ , \Omega(t) = \bar{\Omega}(\bar{t}),$$

$$\sqrt{K'}r = \bar{r}\ , \theta = \bar{\theta}, \phi = \bar{\phi}$$

$$k = (0,1,-1),\ k' = \frac{k}{K'} = (0, \frac{1}{K'}, -\frac{1}{K'}) \tag{32}$$

Hence, $K' = 1$, In this time, ds^2 is an uniqueness. This theory didn't apply the cosmology.

4. Obtaining process information of Hawking radiation

Stephen Hawking found black –hole's thermodynamics. By Hawking Radiation, we obtain the new data from formulas of black-hole's thermodynamics. We start the obtaining process informations of Hawking Radiation. In Wikipedia (Hawking Radiation), we know formulas of Hawking Radiation.

The radiation temperature T of Schwarzschild black hole (In this theory, PMBH)

$$T = \frac{\hbar c^3}{8\pi G M k_B} \tag{33}$$

The radiation temperature $\bar{T}$ is in Data General Relativity theory.

$$\bar{T} == \frac{\hbar c^3}{8\pi G \bar{M} k_B} = \frac{\hbar c^3}{8\pi G \sqrt{K} M k_B} = \frac{T}{\sqrt{K}} \tag{34}$$

The black hole (PMBH)'s entropy

$$dS = 8\pi G k_B M dM / \hbar c = d(4\pi M^2) G k_B / \hbar c = \frac{dQ}{T} \tag{35}$$

The black-hole (PMBH)'s entropy $\bar{S}$ is in Data General Relativity theory.

$$d\bar{S} = 8\pi \bar{M} d\bar{M} G k_B / \hbar c = d(4\pi \bar{M}^2) G k_B / \hbar c = d(4\pi K M^2) G k_B / \hbar c$$

$$= K dS = \frac{d\bar{Q}}{\bar{T}} = \frac{d(\sqrt{K}Q)}{\frac{T}{\sqrt{K}}}$$

$$\bar{S} = KS \quad , \quad \bar{Q} = \sqrt{K}Q \tag{36}$$

Black-hole (PMBH) radiation's power P_{ev} is

$$P_{ev} = A_S \varepsilon \sigma T_H{}^4,\ A_s = 4\pi r_S{}^2,\ r_S = \frac{2GM}{c^2}, \varepsilon, \sigma \text{ is constant} \tag{37}$$

Black-hole (PMBH) radiation's power $\bar{P}_{ev}$ is in Data General Relativity theory.

$$\bar{A}_s = 4\pi \bar{r}_S{}^2 = K4\pi r_S{}^2 = KA_s, \qquad \bar{r}_S = \frac{2G\bar{M}}{c^2} = \frac{2G\sqrt{K}M}{c^2} = \sqrt{K} r_S$$

$$\bar{P}_{ev} = \bar{A}_S \varepsilon \sigma \bar{T}_H{}^4 = KA_s \varepsilon \sigma \frac{T^4}{K^2} = \frac{P_{ev}}{K}$$

Stefan-Boltzmann constant $\sigma = \dfrac{\pi^2 k_B{}^4}{60\hbar^3 c^2}$,

Black-hole (PMBH) is a perfect black body($\varepsilon = 1$)

(38)

The evaporation time t_{ev} of a black hole (PMBH) is

$$t_{ev} = \frac{5120\ \pi G^2 M^3}{\hbar c^4} \tag{39}$$

The evaporation time $\bar{t}_{ev}$ of a black hole (PMBH) is in Data General Relativity theory.

$$\bar{t}_{ev} = \frac{5120\ \pi G^2 \bar{M}^3}{\hbar c^4} = \frac{5120\ \pi G^2 K\sqrt{K} M^3}{\hbar c^4} = K\sqrt{K} t_{ev} \tag{40}$$

The power of evaporation energy of the black-hole (PMBH) is

$$P_{ev} = -\frac{dE_{ev}}{dt_{ev}} \tag{41}$$

The power of evaporation energy of the black-hole (PMBH) is in Data General Relativity theory.

$$\bar{P}_{ev} = -\frac{d\bar{E}_{ev}}{d\bar{t}_{ev}} = -\frac{d(E_{ev}\sqrt{K})}{K\sqrt{K}dt_{ev}} = \frac{P_{ev}}{K},$$

$$\bar{M}_{ev}c^2 = \bar{E}_{ev} = \sqrt{K}E_{ev} = \sqrt{K}M_{ev}c^2 \tag{42}$$

5. Conclusion

We find the other representation of solutions in the General relativity theory. In this time, Robertson-Walker solution is an uniqueness. We more obtain the information of black-hole thermodynamics in Data General Relativity theory.

If we use variable $\bar{A}$ instead of A, Data General Relativity theory is reduced to normal general relativity theory. This theory's remarkable thing is if $\sqrt{K} = 2$ and black hole (PMBH)'s mass M is $M\sqrt{K} = 2M$, black hole (PMBH)'s distant of gravitation r is $r\sqrt{K} = 2r$, black hole (PMBH)'s proper time τ is $\tau\sqrt{K} = 2\tau$.If rotating black hole (PMBH)'s mass M is to be $M\sqrt{K} = 2M$, we predict the angular momentum J of the black-hole (PMBH) is to be $KJ = 4J$.

In this time, we have to apply only black-holes (PMBHs) because black hole (PMBH) is an idealistic structure. BH is Black hole.

Appendix A

In DGRT, we have to apply only black-holes (PMBHs).

According to [27]Paul H. Frampton, Physical Letter B(2017), if the mass of sun is M_{Θ}, data is

Astrophysical obect	Mass solar masses	Period τ seconds	Angular momentum kg㎞²/s
PIMBH1	20M⊝	0.013s	3.0×10^{37}
PIMBH2	100M⊝	0.063s	7.2×10^{38}
PIMBH3	1000M⊝	0.63s	7.2×10^{40}
PIMBH4	10^4 M⊝	6.3s	7.2×10^{42}
PIMBH5	10^5M⊝	63s	7.2×10^{44}
PSMBH6(M87)	6×10^9M⊝	3.8×10^6s	2.6×10^{54}

According to DGRT, BH (PMBH)'s mass M is to be $\sqrt{K}M$, time τ is to be $\tau\sqrt{K}$,Angular momentum J is to be KJ.Hence, PIMBH2's (from PIMBH1) $\sqrt{K}$ is 5, PIMBH3's (from PIMBH 2) $\sqrt{K}$ is 10,PIMBH4's(from PIMBH3) $\sqrt{K}$ is 10, PIMBH5's (from PIMBH4) $\sqrt{K}$ is 10, PSMBH6's (from PIMBH5) $\sqrt{K}$ is 6×10^{4}.

In this time, PIMBH is Primordial Intermediate Massive Black hole, PSMBH is Primordial Super Massive Black Hole. The hypothesis of the constituents of dark matter in the galactic halo are PIMBHs is his theory. According to this theory [27]"Angular momentum of dark matter black holes"

$$20M_{\Theta} \le M_{PIMBH} \le 100{,}000\, M_{\Theta}$$

$$10^{5} M_{\Theta} \le M_{PSMBH} \le 10^{17} M_{\Theta}$$

Therefore, calculated data is in DGRT,

Astrophysical obect	Mass solar masses	Period τ seconds	Angular momentum kg㎞²/s
PIMBH1	20M⊝	0.013s	3.0×10^{37}
PIMBH2	100M⊝	0.065s	7.5×10^{38}
PIMBH3	1000M⊝	0.65s	7.5×10^{40}
PIMBH4	10^4 M⊝	6.5s	7.5×10^{42}
PIMBH5	10^5M⊝	65s	7.5×10^{44}

PSMBH6(M87)	6 x 10^9M⊙	3.9x10^6s	2.7x10^{54}

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