

Shortest refutation of Gödel's completeness theorem

Copyright © 2018 by Colin James III All rights reserved.

We assume the method and apparatus of Meth8/VL4 with \top as the designated *proof* value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). Results are a 16-valued truth table in row-major and horizontal

LET: $p, q, r: x, y, R; \&$ And; $>$ Imply, greater than, \rightarrow ;
 $\%$ possibility, $<$, for one or some, \exists ; $\#$ necessity, $[],$ for every or all, \forall .

From: en.wikipedia.org/wiki/Gödel's_completeness_theorem

By Gödel's completeness result, the formula $(\forall x.R(x,x)) \rightarrow (\forall x \exists y.R(x,y))$ (1.1)
 holds in all structures, and hence must have a natural deduction proof.

$(\#p \& (r \& (p \& p))) > ((\#p \& \%q) \& (r \& (p \& q)))$; TTTT TCTT TTTT TCTT (1.2)

Eq. 1.2 is *not* tautologous, meaning it does not hold in all structures and serves as a contra-example. Hence Gödel's completeness theorem is refuted.

Remark: When reduced to an abstract and atomic state, Eq. 1.1 becomes weakened as $R(x,x) \rightarrow R(x,y)$ or $(r \& (p \& p)) > (r \& (p \& q))$; TTTT TF TT TTTT TF TT.