Refutation of generalized Hardy's paradox

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, `F` as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). Results are non-repeating fragments of 6-valued truth tables in row-major and horizontal

LET: p, q, r, s, t: A1, A2, B1, B2, probability; also,
p, q, r, s, t: a+, b+, c+, c-, probability;
~ Not; & And; + Or; - Not Or; > Imply, greater than; < Not Imply, lesser than;
@ Not Equivalent; % possibility, for one or some; (x@x) ordinal zero, 0.

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> "In any local theory, if the events A2<B1, B1<A1, and A1<B2 never happen, then naturally the event A2<B2 must never happen. According to quantum theory, however, there exist two-particle entangled states and local projective measurements that break down these local conditions; that is, in terms of probabilities, P(A2<B1)=P(B1<A1)=P(A1<B2)=0, and P(A2<B2)>0, where the last condition evidently conflicts with the prediction of local theory, leading to a paradox. In [fn] the author showed that for the n-qubit Greenberger-HorneZeilinger (GHZ) state the maximal success probability (i.e., the last condition above) can reach [1+cos n− π 1]/2n.
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> Moreover, a quantum paradox can be naturally transformed to a corresponding Bell's inequality. For instance, the paradox mentioned above can be associated to the following Hardy's inequality P(A2<B2)−P(A2<B1)−P(B1<A1)−P(A1<B2)≤0, which is equivalent to Zohren and Gill's version [fn] of the Collins-GisinLinden-Massar-Popescu inequalities (i.e., tight Bell's inequalities for two arbitrary d-dimensional systems, and the inequality becomes the CHSH inequality for d=2).[fn] ... "

If the events A2<B1, B1<A1, and A1<B2 never happen, then naturally the event A2<B2 must never happen. (1.1)

~((%(q<r)&%(r<p))&%(p<s))>~(%(q<s)) ;
`TTCC TTCC TTTT TTTT,   TTCC TTCC TTTT TTTT` (1.2)

In terms of probabilities, P(A2<B1)=P(B1<A1)=P(A1<B2)=0, (2.1)
and P(A2<B2)>0, (3.1)
where the last condition evidently conflicts with the prediction of local theory, leading to a paradox. (4.1)

Moreover, a quantum paradox can be naturally transformed to a corresponding Bell's inequality. For instance, the paradox mentioned above can be associated to the following Hardy's inequality P(A2<B2)−P(A2<B1)−P(B1<A1)−P(A1<B2)≤0, (5.1) which is equivalent to Zohren and Gill's version [fn] of the Collins-GisinLinden-Massar-Popescu inequalities (i.e., tight Bell's inequalities for two arbitrary

d-dimensional systems, and the inequality becomes the CHSH inequality for d = 2) [fn]. See also [fn] for a connection between Hardy’s inequality and Wigner’s argument [of joint probabilities as $p(a+;\ b+) - p(a+;\ c+) - p(c+;\ b+) - p(c-;\ c-) \leq 0$]. (6.1)

(((t&(q<r))=(t&(r<p)))=(t&(p<s)))=(t@t) ;
TTTT TTTT TTTT TTTT, TFFT FFFF TTFF FTFT (2.2)

(t&(p<s)) >(t@t) ; TTTT TTTT TTTT TTTT, TTFF TTFF TTTT TTTT (3.2)

((((t&(q<r))=(t&(r<p)))=(t&(p<s)))=(t@t))&((t&(q<s))>(t@t)) ;
TTTT TTTT TTTT TTTT, TFFF FFFF TTFF FTFT (4.2)

~((((t&(q<s))-(t&(q<r)))-((t&(r<p))-(t&(p<s))))>(t@t))=(p=p) ;
FFFF FFFF FFFF FFFF, FFFF FFFF FFFF FFFF (5.2)

LET p, q, r, s, t: a+, b+, c+, c-, probability

~((((t&(p&q))-(t&(p&r)))-((t&(r&q))-(t&(r&r))))>(t@t))=(p=p) ;
FFFF FFFF FFFF FFFF, FFFF FFFF FFFF FFFF (6.2)

Eqs. 2.2, 3.2, 4.2, 5.2, and 6.2 as rendered are *not* tautologous. This means the generalized Hardy's paradox is refuted. Eqs. 5.2 and 6.2 are contradictory. The means Hardy's inequality and Wigner's argument of joint probabilities are refuted, as is a claimed connection.

Remark: The basis of the entire claim is Eq. 1.1: "If the events A2<B1, B1<A1, and A1<B2 never happen, then naturally the event A2<B2 must never happen." As rendered in Eq. 2, this is not tautologous with result values of contingency (falsity). This is a gross example of mathematical logic exposing the mistaken assumptions of quantum field theory.