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# ON COMPLEX DYNAMICS OF REGIONAL INTERACTION

**Georges Sarafopoulos**
Democritus University of Thrace, Greece.

**Abtract**

In this paper we consider the interaction between regions during the implementation of a reform on regional development through a discrete – time duopoly game with heterogeneous players. The existence and stability of equilibria of this system are studied. We show that a parameter of the system may change the stability of equilibrium and cause a structure to behave chaotically. For the low values of this parameter the game has a stable Nash equilibrium. Increasing these values, the Nash equilibrium becomes unstable, through period-doubling bifurcation. The complex dynamics, bifurcations and chaos are displayed by computing numerically Lyapunov numbers and sensitive dependence on initial conditions.

**Keywords**: Regional Development, Institutional Reforms of Local Governments, Game Theory, Nash Equilibrium, Discrete Dynamical System; Chaotic Behavior.

**JEL Classification:** C61, C72, R11

## Introduction

Regional economic activity is a complicate process as integrates apart from economic, societal, cultural and institutional elements. Interregional differences in these characteristics create diversified local government outcomes. Recent studies of relational geography shed strong light in the notion of spatial heterogeneity so to comprehend the terms of interregional divergences (Ettlinger, 2001;Deas & Lord, 2003; Harrison, 2013). Effective use of local comparative advantages is one of the explanatory variables, when local cooperative culture and path dependence in compliance to formal and informal institutions have significant impact as well (Knoben & Oerlemans, 2012). On the other hand, the advancement of local governments by new powers alternates the terms that local actors operate. Especially during the implementation of institutional reform new prospects of cooperation and conflict arise into a region. The role of local governments into local milieu is upgraded as they have the potential to schedule and to implement local public policy. Therefore, the ability of local governments to execute effectively the reforms is a crucial variable of local comparative advantages (Kuhlmann & Wollmann, 2011). It should be notified also, that institutional reforms of local governments are directly associated with the broadest tend of local communities to adopt evolvements. Especially, regions with levels at cooperative actions in investment strategies and joint exploitation of comparative advantages have significantly more chances to interact efficiently with local governments. As a sequence, reform's implementation is determined by a twofold framework: in one pillar stands local government's action and in the other social cohesion of local community (Sarafopoulos & Ioannidis, 2014). The competition of local communities is shaped by the strategies of local entrepreneurships, the efficiency of local governments and the activities of societal organizations. Interaction among the abovementioned actors can determine the options of regional development. So it can be stated that the more effective this interaction is, the biggest is the possibility for a region to compete efficiently neighboring regions.

Intraregional competition among regional unities is an important variable of local economic activity. Neighboring regional unities function in order to attract investments and to promote local comparative advantages. The exploitation of local comparative advantages may result by the affiliation of local actors to commonly accepted strategies that upgrade local prosperity. Local governments can adopt crucial role in this process as they hold the monopoly of public service delivery (Citroni et al, 2013; Ioannidis, 2013). Cognitive cooperation strategies generate strong conditions for local development by diminishing transaction cost and by increasing the efficiency of local resources. One of the most important dimensions of regional development stands in the field of local governments institutional reforms. Institutional reforms of local government produce changes in local communities as new condition of public policy are created. Under this perspective the ability of a local community to implement efficiently reforms can be associated by the prospect of local development (Sarafopoulos & Ioannidis, 2014).

Ardinat, 2011, Krugman, 1994 and Krugman, 2011 point to the difficulty of applying concepts of microeconomic theory (competition between products and firms) at the political level (competition between regions). To emphasize this difficulty we present a model of competition between two local governments in order to show complexity and non-predictability. In previous work (Sarafopoulos & Ioannidis, 2013), it was emphasized in the impact of interaction among local governments on reforms' implementation. In this paper we focus on the competition among local governments. Game Theory has been used as a methodological tool that study diverse options of interaction among local actors (Fontitni, 2003; Steinacker, 2004; Feiock & Park, 2005). Complicate options of interaction can be represented either by static or dynamic forms. Dynamic forms have the potential to reveal in a more multidimensional way the multiple outcomes of equilibrium.

The structure of the paper is the following: In Section 2, we study the interaction between two local governments through a discrete dynamical system. The equilibrium points and local stability of the game are investigated. As a parameter of the model is varied, the stability of the Nash equilibrium point is lost and the complex (periodic or chaotic) behavior occurs. In Section 3, numerical simulations are presented to show that the game behave chaotically.

**The game**

In this section, a dynamic game between two local governments is presented. The game is taking place just after the implementation of a local government institutional reform. The local government is the primary local administrative body. Its basic task is the effective execution of local government institutional reforms.

We consider a simple Cournot-type duopoly game where the players are the local governments. Let $x$ (resp. $y$ ) the variable that measures the degree of adaptation of the reform in the region 1 (resp. in the region 2), $0 \leq x \leq 1,\ 0 \leq y \leq 1$. We assume that the tax per unit reform is linear and decreasing:

$$T = a - b(x + y) \tag{1}$$

and the cost functions are:

$$C_1(x) = cx,\ C_2(y) = cy \tag{2}$$

where $a,b,c > 0$. With these assumptions the tax profits of the local governments are given by

$$\begin{aligned} \Pi_1(x,y) &= x[a - b(x + y)] - cx \\ \Pi_2(x,y) &= y[a - b(x + y)] - cy \end{aligned} \tag{3}$$

Then the marginal profit of the local government at the point $(x,y)$ of the strategy space is given by

$$\begin{aligned} \frac{\partial \Pi_1}{\partial x} &= a - c - 2bx - by \\ \frac{\partial \Pi_2}{\partial y} &= a - c - 2by - bx \end{aligned} \tag{4}$$

and

$$\frac{\partial \Pi_2}{\partial y} = 0 \Rightarrow y = \frac{1}{2b}(a - c - bx) \tag{5}$$

We suppose that the first local government decides to increase its level of adaptation if it has a positive marginal profit, or decreases its level if the marginal profit is negative (boundedly rational player) and the second is a naïve player. Then the dynamical equations of the players are:

$$\begin{aligned} \frac{x(t + 1) - x(t)}{x(t)} &= \frac{\partial \Pi_1}{\partial x} \\ \mathrm{y(t+1)} &= \arg\max_{y} \Pi_2(x(t),y(t)) \end{aligned} \tag{6}$$

The dynamical system of the players is described by

$$\begin{cases} x(t+1) = x(t) + x(t)\left(a - 2bx(t) - by(t) - c\right) \\ y(t+1) = \dfrac{1}{2b}(a - c - bx(t)) \end{cases} \tag{7}$$

We will focus on the dynamics of the system (6) to the parameter $a - c$.

**The equilibria of the game**

The equilibria of the dynamical system Eq.(7) are obtained as nonnegative solutions of the algebraic system

$$\begin{cases} x(t)\left(a - c - 2bx(t) - by(t)\right) = 0 \\ a - c - 2by(t) - bx(t) = 0 \end{cases} \tag{8}$$

which obtained by setting $x(t+1) = x(t)$, $y(t+1) = y(t)$ in Eq. (7) and we can have two equilibriums $E_0 = (0, a - c / 2b)$ and $E^* = (x^*, y^*)$, where

$$x^* = \frac{a-c}{3b}, \quad y^* = \frac{a-c}{3b} \tag{9}$$

The fixed point $E_0$, is the boundary equilibrium. The equilibrium $E^*$ is called Nash equilibrium, provided that $a > c$.

The study of the local stability of equilibrium solutions is based on the localization on the complex plane of the eigenvalues of the Jacobian matrix of the two dimensional map (Eq. (7)). In order study the local stability of equilibrium points of the model Eq. (7), we consider the Jacobian matrix along the variable strategy $(x, y)$.

$$J(x,y) = \begin{bmatrix} 1 + (a - 4bx - by - c) & -bx \\ -0.5 & 0 \end{bmatrix} \qquad (10)$$

The matrix $J(E_0)$ has two eigenvalues $l_1 = 0, l_2 = 1 + 0.5(a - c)$. It follows that $l_2 > 1$, then $E_0$ is unstable fixed point for the system Eq.(7). The Nash equilibrium $E^*$ is locally stable if the following conditions are hold

$$\begin{cases} 1 - T + D > 0 \\ 1 + T + D > 0 \\ 1 - D > 0 \end{cases} \qquad (11)$$

where T is the trace and D is the determinant of the Jacobian matrix

$$J(E^*) = \begin{bmatrix} 1 - 2bx^* & -bx^* \\ -0.5 & 0 \end{bmatrix} \qquad (12)$$

From Eq.(11) it follows that the Nash equilibrium is locally stable if

$$0 < a - c < 2.4 \qquad (13)$$

**Numerical simulations**

To provide some numerical evidence for the chaotic behavior of the system Eq.(7), as a consequence of change in the parameters $a - c$, we present various numerical results here to show the chaoticity, including its bifurcations diagrams, strange attractor, Lyapunov numbers and sensitive dependence on initial conditions (Kulenovic, M., Merino, O., 2002). In order to study the local stability properties of the equilibrium points, it is convenient to take $b = 1$. Numerical experiments are computed to show the bifurcation diagram with respect to $a - c$, strange attractor of the system (7) in the phase plane $(x,y)$, and the Lyapunov numbers . Fig. 1 show the bifurcation diagrams with respect to the parameter $a - c$ for $b = 1$. In this figure one observes complex dynamic behavior such as cycles of higher order and chaos. Fig. 2 show the graph of strange attractor and Lyapunov numbers of the orbit of (0.1, 0.1) for $a - c = 3.7$ and $b = 1$. From these results when all parameters are fixed and only a-c is varied the structure of the game becomes complicated through period doubling bifurcations, more complex bounded attractors are created which are aperiodic cycles of higher order or chaotic attractors.

To demonstrate the sensitivity to initial conditions of the system (7), we compute two orbits with initial points (0.1, 0.1) and (0.101, 0.1), respectively. Fig. 3 shows sensitive dependence on initial conditions for x-coordinate of the two orbits, for the system (6), plotted against the time with the parameters values b=1, a-c = 3.7. At the beginning the time series are indistinguishable; but after a number of iterations, the difference between them builds up rapidly. From Fig. 3 we show that the time series of the system Eq. (7) is sensitive dependence to initial conditions, i.e. complex dynamics behaviors occur in this model.

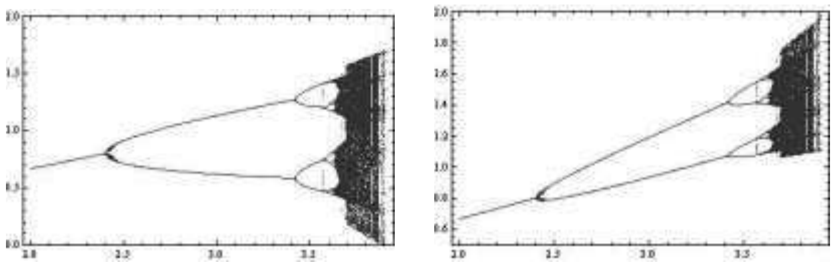

Fig.1. Bifurcation diagrams with respect to the parameter a-c against variable x (left) and against y (right) for b=1, with 550 iterations of the map Eq.(7).

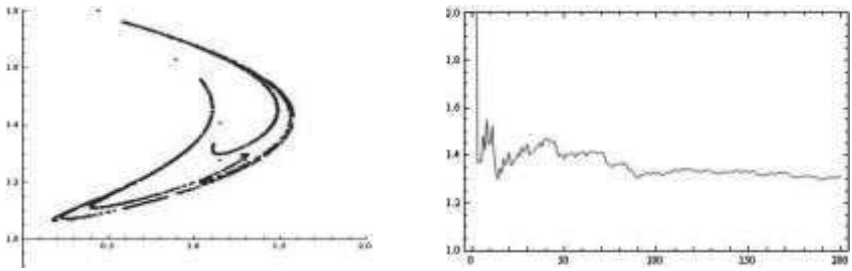

Fig.2. Strange attractor with 2000 iterations (left) and Lyapunov numbers versus the number of iterations (right), of the orbit orb. (0.1, 0.1), for b=1, a-c = 3.7

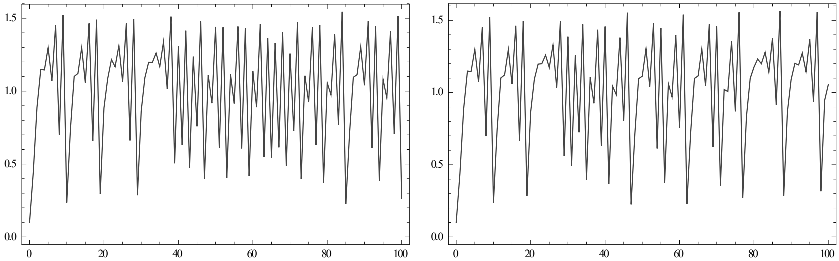


Fig.4. Sensitive dependence on initial conditions, for x–coordinate plotted against the time: The two orbits orb.(0.1, 0.1) (left) and orb.(0.101, 0.1) (right), for the system (6), with the parameters values b=1, a-c = 3.7

## Conclusions

In this paper, we analyzed through a discrete dynamical system the intraregional competition during the implementation of local government institutional reforms. The stability of equilibria, bifurcation and chaotic behavior are investigated. We show that a parameter may change the stability of equilibrium and cause a structure to behave chaotically. For low values of this parameter there is a stable Nash equilibrium. Increasing these values, the equilibrium becomes unstable, through period-doubling bifurcation.

## References

Ardinat, G., *Géographie de la compétitivité*, (2011), Thèse de doctorat, Université Paul- Valery, Montpellier.

Bischi, G.I., Naimzada, A., (1999), Global analysis of a dynamic duopoly game with bounded rationality. In: Filar JA, Gaitsgory V, Mizukami K, (eds). (2000). *Advances in dynamic games and applications,* vol. 5. Basel: Birkhauser: 361–85.

Citroni, G., Lippi, A. and Profeti, S., (2013), 'Remapping the State: Inter-Municipal Cooperation through Corporatization and Public-Private Governance Structures', Local Government Studies, 39,2: 208-234.

Deas, I., Lord, A., (2003), From a New Regionalism to an Unusual Regionalism? The Emergence of Non-standard Regional Spaces and Lessons for the Territorial Reorganisation of the State, Urban Studies, 43, 10 1847–1877.

Ettlinger, N, (2001), A Relational Perspective in Economic Geography: Connecting Competitiveness with Diversity and Difference. Antipode 33, (2), 216-227.

Feiock, R., Park, H.J., (2005), Bargaining, Networks and Institutional Collective Action in Local Economic Development, Paper presented at the Annual Meeting of the American Society for Public Administration, 2-5/04/2005.

Fontini, F. (2003), "A Firms-Government Coordination Game under Strategic Uncertainty", Studi Economici, 80:2, 73-84

Harrison J., (2013), Configuring the New 'Regional World': On being Caught between Territory and Networks, Regional Studies, 47,(1), 55-74.

Ioannidis, P.G., (2013), Local Government Institutional Reforms and Cooperation among Local Actors: The Case of Eastern Macedonia and Thrace ', Paper presented at the 6th Biennial Hellenic Observatory PhD Symposium on Contemporary Greece and Cyprus 6-7 June 2013, Hellenic Observatory, London School of Economics.

Knoben, J., Oerlemans, L. A. G., (2012), Configurations of Inter-organizational Knowledge Links: Does Spatial Embeddedness Still Matter?, Regional Studies, 46, (8), 1005-1021.

Krugman, P., (1994), Competitiveness: A dangerous obsession, *Foreign Affairs*, New York, vol. 73, no 2, March- April;

Krugman, P., (2011), *The competition myth*, The New York Times, January 23, 2011.

Kuhlmann S., Wollmann H., (2011) The Evaluation of Institutional Reforms at Sub-national Government Levels: A Still Neglected Research Agenda, Local Government Studies, 37:5, 479-494

Kulenovic, M., Merino, O., (2002), Discrete Dynamical Systems and Difference Equations with Mathematica. Chapman & Hall/Crc.

Sarafopoulos G., Ioannidis P.G., (2014), Local Agents' Cooperation as a Signal Game: Firms, Local Governments and Investment Strategies, Paper Presented at the 5th International Conference The Economies of Balkan and Eastern Europe Countries in the changed world (EBEEC 2013) Istanbul, Turkey, May 9-12 2013 (accepted for publication at Procedia Economics and Finance-Elsevier).

Sarafopoulos, G., Ioannidis, P.G., (2013), Institutional Reforms and Interaction of Local Governments: A Dynamic Approach, Paper Presented at the 2nd International Conderence on Econophysics, 13-14 September, 2013, Kavala, Greece.

Steinacker, A.,(2004), Game Theoretic Models of Metropolitan Cooperation in Metropolitan Governance: Conflict, Competition and Cooperation edited by Richard C. Feiock. Washington D.C.: Georgetown University Press.