THE VALUE OF STRATEGIC COMMITMENT IN MIXED DUOPOLY GAMES WITH A FOREIGN LABOR-MANAGED FIRM

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ABSTRACT

This paper investigates an economy consisting of a state-owned firm and a foreign labor-managed firm. First, the paper considers a Cournot game where only the state-owned firm is allowed to offer lifetime employment as a strategic commitment, and demonstrates that there is a stable equilibrium which is profitable only for the state-owned firm. Next, the paper examines a Cournot game where only the foreign labor-managed firm can offer lifetime employment as a strategic commitment, and shows that there are two stable equilibria. The purpose of this study is to show the value of lifetime employment as a strategic commitment.

Keywords: State-owned firm, Labor-managed firm, Two-stage Cournot model, Lifetime employment

JEL Classification: C72, D21, L20
Introduction

It is well known that state-owned public firms exist in many countries of the world such as developing, developed and former communist economies. Since Merrill and Schneider (1966), many researchers have done the theoretical studies on mixed oligopoly markets including public firms. For example, Nett (1994) examines a mixed duopoly market where a state-owned public firm coexists with a private firm, and proves that the private firm produces at a lower variable cost compared to the public firm. He also shows that the private firm sells a greater amount of output than the public firm. Poyago-Theotoky (1998) considers a mixed R&D duopoly where firms compete to introduce a new product or process under uncertainty and easy imitation, and shows that the profit-maximizing private firm invests less in R&D than the public firm. There are also many excellent papers (see, e.g., George and La Manna, 1996; Chang, 2005; Lu and Poddar, 2005, 2007; Beladi and Chao, 2006; Chao and Yu, 2006; Bárcena-Ruiz, 2007; Han and Ogawa, 2008; Ohnishi, 2008; Saha and Sensarma, 2008; Artz, Heywood and McGinty, 2009; Roy chowdhury, 2009; Wang and Wang, 2009; Wang, Wang and Zhao, 2009; Heywood and Ye, 2010; Wang and Lee, 2010; Zhang and Li, 2013; Pal and Saha, 2014). However, these studies do not include any labor-managed firms.

Ward (1958) theoretically examined firm behavior in the former Yugoslavia. Following this seminal paper, numerous researchers have investigated the behavior of labor-managed firms. For example, Neary and Ulph (1997) consider a mixed duopoly model with one labor-managed firm and one profit-maximizing capitalist firm, and with strategic investment. They show that there is no equilibrium at which the firms can simultaneously make zero profits. Lambertini and Rossini (1998) consider a mixed duopoly market where a labor-managed firm coexists with a profit-maximizing firm, and show that the profit-maximizing firm tends to under-invests while the opposite holds for the labor-managed firm regardless of the rental cost of capital. There are also numerous other excellent works (see, e.g., Law and Stewart, 1983; Mai and Hwang, 1989; Horowitz, 1991; Stewart, 1991; Cremer and Cremer, 1992; Askildsen and Ireland, 1993; Okuguchi, 1993; Ireland and Stewart, 1995; Futagami and Okamura, 1996; Lambertini, 1997, 2001; Ireland, 2003; Cuccia and Cellini, 2009; Luo, 2013; Kalashnikov et al., 2015). However, these studies do not include any state-owned firms.

Only a few studies investigate mixed oligopoly markets that consist of labor-managed and state-owned firms. For example, Delbono and Rossini (1992) consider a Cournot mixed duopoly model with one state-owned firm and
one labor-managed firm, and demonstrate that there is a unique Cournot-Nash solution where the state-owned firm produces more than the labor-managed firm. Ohnishi (2009) investigates the behavior of a labor-managed firm and a state-owned firm in a two-stage mixed duopoly game, and shows that if both firms are allowed to install capacity in stage one, then there is a subgame perfect Nash equilibrium in which only the labor-managed firm installs capacity. In addition, Ohnishi (2015) examines a three-stage model where a state-owned firm and a labor-managed firm can sequentially offer a wage-rise contract as a strategic commitment before competing in quantities, and demonstrates that there is an equilibrium solution where neither firm offers a wage-rise contract as a strategic commitment.

We study an economy in which a state-owned firm and a foreign labor-managed firm coexist. First, we consider a Cournot game in which only the state-owned firm is allowed to offer lifetime employment as a strategic commitment. Second, we examine a Cournot game in which only the foreign labor-managed firm can offer lifetime employment as a strategic commitment. We present the equilibrium outcomes of these two games. The main purpose of this study is to show the value of lifetime employment as a strategic commitment in international mixed duopoly competition.

**The Basic Setting**

Let us consider mixed duopoly competition with one domestic state-owned firm (firm S) and one foreign labor-managed firm (firm L). In the remainder of this paper, subscripts S and L denote firm S and firm L, respectively. In addition, when $i$ and $j$ are used, they should be understood to refer to S and L with $i \neq j$. There is no possibility of entry or exit. The market price is determined by the inverse demand function $P(Q)$, where $Q = q_S + q_L$ denotes the aggregate quantity. We assume that $P' < 0$ and $P'' < 0$.

The market is modeled by means of the following two-stage competition. In the first stage, firm $i$ is allowed to offer lifetime employment. At the end of the first stage, firm $j$ observes firm $i$’s action. In the second stage, both firms simultaneously and independently choose actual outputs $q_S > 0$ and $q_L > 0$. If firm $i$ offers lifetime employment, then it chooses an output level $q_i^* > 0$ and enters into a lifetime employment contract with the number of employees necessary to achieve $q_i^*$. Ohnishi (2001) gives a detailed explanation of lifetime employment as a strategic commitment.
Therefore, domestic social welfare, which is the sum of firm S’s profit and consumers’ surplus, is given by

\[
W = \begin{cases} 
\int_0^Q P(x)dx - rq_S - wq_S - Pq_L - 2f & \text{if } q_s > q_s^*, \\
\int_0^Q P(x)dx - rq_S - wq_s^* - Pq_L - 2f & \text{if } q_s \leq q_s^*, 
\end{cases}
\] (1)

where \( r > 0 \) denotes the capital cost for each unit of output, \( w > 0 \) is the labor cost for each unit of output, and \( f > 0 \) is the fixed cost. Let “\( Pq_L + f \)” be the sum of firm L’s cost and profit. Since firm L’s profit is channeled out of firm S’s home country, it is not included in \( W \).

Firm L’s income per worker is given by

\[
V_L = \begin{cases} 
\frac{P(Q)q_L - rq_L - f}{l(q_L)} & \text{if } q_L > q_L^*, \\
\frac{P(Q)q_L - rq_L - f}{l(q_L^*)} & \text{if } q_L \leq q_L^*, 
\end{cases}
\] (2)

where \( l \) is the labor input function. We assume that \( l' > 0 \) and \( l'' > 0 \). This assumption means that the marginal labor input is increasing. Throughout this paper, we adopt subgame perfection as our solution concept.

Supplementary Explanations

First, we derive firm S’s best response from (1). If firm S’s marginal cost is \( r + w \), then its reaction function is defined by

\[
R_s^\sigma(q_L) = \arg \max_{q_s} \left[ \int_0^Q P(x)dx - rq_S - wq_S - Pq_L \right],
\] (3)

and if firm S adopts lifetime employment and reduces its marginal cost to \( r \), then its reaction function is defined by

\[
R_s^l(q_L) = \arg \max_{q_s} \left[ \int_0^Q P(x)dx - rq_S - Pq_L \right].
\] (4)

Therefore, firm S’s best response is shown as follows:
Firm S aims to maximize domestic social welfare with respect to its own output, given firm L’s output. The first-order condition for (3) is

\[ P - r - w - P'q_L = 0, \] (6)

and the second-order condition is

\[ P' - P''q_L < 0. \] (7)

Moreover, the first-order condition for (4) is

\[ P - r - P'q_L = 0, \] (8)

and the second-order condition is

\[ P' - P''q_L < 0. \] (9)

Therefore, we have

\[ R_S''(q_L) = R_S'(q_L) = -\frac{-P''q_L}{P' - P''q_L}. \] (10)

Since it is assumed that \( P'' \) is negative, \( -P''q_L \) is positive.

We can now state the following lemma:

**Lemma 1.** Under Cournot competition, both \( R_S''(q_L) \) and \( R_S'(q_L) \) are upward sloping.

Second, we derive firm L’s best response from (2). If firm L does not offer lifetime employment, then its reaction function is defined by
\[ R_L^*(q_S) = \arg \max_{q_L} \left[ \frac{P(Q)q_L - rq_L - f}{l(q_L)} \right], \quad (11) \]

and if firm L adopts lifetime employment and produces \( q_L \leq q_L^* \), then its reaction function is defined by

\[ R_L^i(q_S) = \arg \max_{q_L} \left[ \frac{P(Q)q_L - rq_L - f}{l(q_L^*)} \right]. \quad (12) \]

Therefore, firm L’s best response is as follows:

\[
R_L(q_S) = \begin{cases} 
R_L^*(q_S) & \text{if } q_L > q_L^*, \\
q_L^* & \text{if } q_L = q_L^*, \\
R_L^i(q_S) & \text{if } q_L < q_L^*. 
\end{cases} \quad (13)
\]

Firm L seeks to maximize income per worker with respect to its own output, given firm S’s output. The first-order condition for (11) is

\[ (P'q_L + P - r)l - (Pq_L - rq_L - f')l' = 0, \quad (14) \]

and the second-order condition is

\[ (P''q_L + 2P')l - (Pq_L - rq_L - f)l'' < 0. \quad (15) \]

Moreover, the first-order condition for (12) is

\[ P'q_L + P - r = 0, \quad (16) \]

and the second-order condition is

\[ P''q_L + 2P' < 0. \quad (17) \]

Therefore, we obtain

\[
R_L'(q_J) = -\frac{P''q_Jl + P'(l - q_Jl')}{(P''q_J + 2P')l - (Pq_J - rq_J - f)l''} \quad (18)
\]
and

\[ R'_L (q_S) = -\frac{P''q_L + P'}{P''q_L + 2P'} \]  \hspace{1cm} (19)

Since \( l'' > 0 \), \( l - q_L l' \) is negative, so that \( P''q_L + P'(l - q_L l') \) is positive. On the other hand, since \( P' < 0 \) and \( P'' < 0 \), \( P''q_L + P' \) is negative.

We now state the following lemma:

**Lemma 2.** Under Cournot competition, \( R^n_L (q_S) \) slopes upward, whereas \( R'_L (q_S) \) slopes downward.

**Equilibrium Outcomes**

In this section, we analyze the equilibrium outcomes of two games: Game 1 and Game 2.

**Game 1**

In this game, only firm S is allowed to offer lifetime employment. That is, the two stages of the game run as follows. In stage one, firm S decides whether to offer lifetime employment. In stage two, the firms simultaneously and independently choose quantities, and both domestic social welfare and firm L’s income per worker are decided.

Firm S aims to maximize domestic social welfare. Therefore, we can see easily that firm S will offer lifetime employment if domestic social welfare increases by doing so, and firm S will not offer lifetime employment if domestic social welfare decreases by doing so.

We use Figure 1 to discuss the equilibrium outcome of this game. The horizontal axis is firm S’s output level, the vertical axis is firm L’s output level, \( R^n_i \) is firm \( i \)’s reaction curve without lifetime employment, and \( R^n_S \) is firm S’s reaction function with zero marginal labor costs.

At stage one, firm S is allowed to offer lifetime employment. If firm S offers lifetime employment, then its marginal cost decreases and thus it increases its output. By strategic choice of lifetime employment, firm S’s best response becomes (5). The offer of lifetime employment by firm S thus creates kinks in
the reaction curve at the level of $q_S^*$. That is, if firm S chooses $q_S^*$ and offers lifetime employment, then its reaction curve shifts down for $q_S < q_S^*$ and becomes the bold lines as drawn in Figure 1.

Figure 1: Only firm S offers lifetime employment.
In stage two, each firm independently chooses its actual output. The equilibrium solution is decided in a Cournot fashion. Hence, if firm S does not offer lifetime employment, then the equilibrium occurs at $N$.

Firm L’s reaction curve is $R_L^n$, and hence firm S can select any point on the segment $AN$ of $R_L^n$. If firm S chooses $q_S^*$ corresponding to $B$ and offers lifetime employment, then the reaction curves cross at $B$ as drawn in Figure 1. At stage two, firm S chooses $q_S^*$ and firm L chooses $q_L = R_L^n(q_S^*)$. Hence, the solution occurs at $B$. From Figure 1, we can see straightforwardly that domestic social welfare is higher at $B$ than at $N$ whereas firm L’s income per worker is lower at $B$ than at $N$.

The equilibrium outcome can be stated as follows.

**Proposition 1:** There exists a stable equilibrium where firm S offers lifetime employment. At equilibrium, domestic social welfare is higher than in the Cournot game without lifetime employment, whereas firm L’s income per worker is lower than in the Cournot game without lifetime employment.

Proposition 1 means that lifetime employment can be an effective strategy for firm S.

**Game 2**

In this game, only firm L can offer lifetime employment. The timing of the game run as follows. At stage one, firm L decides whether to offer lifetime employment. At stage two, the firms simultaneously and independently choose quantities, and both firm L’s income per worker and domestic social welfare are decided.

Firm L seeks to maximize income per worker. Hence, firm L will offer lifetime employment if its income per worker increases by doing so, and firm L will not offer lifetime employment if its income per worker decreases by doing so.

Game 2 is depicted in Figure 2, where the horizontal axis is firm L’s output level, the vertical axis is firm S’s output level, $R_i^n$ is firm i’s reaction curve without lifetime employment, and $R_L^I$ is firm L’s reaction function with zero marginal labor costs. $R_L^n$ slopes upward, whereas $R_L^I$ is downward sloping.

At the first stage, firm L is allowed to offer lifetime employment. Therefore, if
firm L chooses $q_L^*$ and offers lifetime employment, then its best response becomes the bold lines as drawn in Figure 2. At the second stage, each firm noncooperatively decides its actual output. The solution is decided in a Cournot fashion.

If firm L chooses $q_L^*$ corresponding to $C$ and offers lifetime employment, then the firms’ reaction curves cross at two points $N$ and $C$ in Figure 2. Both $N$ and $C$ are stable equilibria. We see that firm L’s income per worker is higher at $C$ than at $N$ while domestic social welfare is lower at $C$ than at $N$.

Figure 2: Only firm L offers Lifetime employment.
The equilibrium outcome can be stated as follows.

**Proposition 2:** There exist two stable equilibria where firm L offers lifetime employment. One is the Cournot-Nash solution without lifetime employment, and the other is more profitable only for firm L than the Cournot-Nash solution without lifetime employment.

Proposition 2 indicates that lifetime employment may be an effective strategy for firm L.

**Conclusion**

We have considered two Cournot games in which a state-owned firm and a foreign labor-managed firm coexist. First, we have examined a Cournot game in which only the state-owned firm can offer lifetime employment, and have demonstrated that there is a stable equilibrium which is profitable only for the state-owned firm. Next, we have analyzed a Cournot game in which only the foreign labor-managed firm is allowed to offer lifetime employment, and have shown that there are two stable equilibria. As a result of these analyses, we have showed the value of lifetime employment as a strategic commitment in international mixed duopoly competition.

**References**


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