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INVENTORY HOLDING AND INTERNATIONAL MIXED DUOPOLY

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Abstract

This paper considers a two-period international mixed duopoly model in which a domestic state-owned welfare-maximizing public firm and a foreign profit-maximizing private firm are allowed to hold inventories as a strategic device. In the first period, each firm simultaneously and independently chooses how much it sells in the current market and the level of inventory it holds for the second-period market. By holding inventory, a firm can change the competitive environment in the second period. The paper demonstrates that inventories are used by the domestic state-owned public firm to achieve a collusive outcome.

Keywords: International mixed duopoly, Quantity competition, Inventory holding, Domestic public firm, Foreign private firm

JEL Classification: C72, D21, D43, F23, L30

Introduction

The theoretical analysis of mixed oligopoly models has received significant attention in the past few decades and has been extensively studied by many researchers. For example, Cremer et al., (1991) examine a mixed oligopoly in which firms choose product characteristics. Mujumdar and Pal (1998) examine taxation in a mixed duopoly. Delbono and Denicolò (1993) and Poyago-Theotoky (1998) investigate mixed markets with R&D. Willner (1994) and Wen and Sasaki (2001) construct mixed markets in which firms choose capacity. Bárcena-Ruiz and Garzón (2003) consider a mixed duopoly model in which a private firm and a public firm merge or one of them acquires the other. Pal (1998) examines a Stackelberg-type sequential-move mixed oligopoly with a single homogeneous product. White (1996) investigates the welfare effects of domestic production subsidies in a mixed oligopoly regarding privatization and efficiency, and Anderson et al., (1997) consider a mixed oligopoly with product differentiation that privatizes a public firm. In addition, Fershtman (1990), George and La Manna (1996), Matsumura (1998), Fujiwara (2007), and Lu and Poddar (2007) investigate the partial privatization of public firms. However, these studies consider mixed market models with domestic firms and do not include foreign firms.

As is well known, examples of international mixed oligopolies can be found in developed and developing countries as well as in former communist countries. State-owned public firms compete against foreign private firms in many industries, such as the airline, banking, life insurance, shipbuilding, steel, and tobacco.

Some studies consider international mixed market models with foreign private firms. For example, Fjell and Pal (1996) extend the analysis to an international context by considering a mixed market where a state-owned public firm competes with both domestic and foreign private firms and examine the effects of entry by an additional private firm. Pal and White (1998) examine the welfare effects of privatization in the presence of strategic trade policies within an international mixed oligopoly serving a single market. Matsumura (2003) examines an international mixed duopoly model where a domestic public firm and a foreign private firm first choose the timing for choosing their quantities. Ohnishi (2008) investigates the equilibrium of the following three-stage model: First, a social-welfare-maximizing domestic public firm can adopt either a lifetime employment contract or a wage-rise contract as strategic commitments; Second, the foreign private firm decides whether or not to enter the market; Third, if the foreign private firm enters, each firm independently chooses its actual output, while if the foreign private firm does not enter, the public firm acts as a monopolist. There are also other studies, such as Fjell and Heywood (2002), Chang (2005), Chao and Yu (2006), and Han and Ogawa (2008). However, to the best of my knowledge, the analysis of international mixed market models with inventories as a strategic device has been ignored.

Therefore, we study an international mixed market model in which a domestic state-owned public firm and a foreign private firm are allowed to hold inventories as a strategic device.[1] There are two periods in the model. In the first period, each firm simultaneously and independently chooses how much it sells in the current market and the level of inventory it holds for the second-period market. By holding inventory, a firm can change the competitive environment in the second period.

The purpose of this paper is to draw the reaction curves of the domestic state-owned and foreign private firms and to describe the equilibrium in the international mixed duopoly model with inventories as a strategic device.

The Model

There is an industry composed of one foreign private firm (FPF) and one domestic state-owned public firm (DSF), producing perfectly substitutable goods. In the remainder of this paper, subscripts F and D denote the FPF and the DSF, respectively, and superscripts 1 and 2 denote period 1 and period 2, respectively. In addition, when i and j are used to refer to firms in an expression, they should be understood to refer to F and D with $i \neq j$. The demand and cost conditions that firms face remain unchanged over time. The price of each period is determined by $P(S^t)$, where $S^t = s_{\mathrm{F}}^t + s_{\mathrm{D}}^t$ is the aggregate sales in period $t(t=1,2)$. We assume $P' < 0$ and $P'' < 0$.

The timing of the game is as follows. In the first period, each firm non-cooperatively chooses its first-period production $q_i^1 \in [0,\infty)$ and its first-period sales $s_i^1 \in [0, q_i^1]$. Therefore, each firm's inventory I_i^1 becomes $q_i^1 - s_i^1$. In the second period, each firm non-cooperatively chooses its second-period production $q_i^2 \in [0,\infty)$. At the end of the second period, each firm sells $s_i^2 = I_i^1 + q_i^2$ and holds no inventory. For notational simplicity, we take into account the game without discounting.

Since $\sum_{t=1}^2 q_i^t = \sum_{t=1}^2 s_i^t$, each firm's profits are

$$\Pi_i = \sum_{t=1}^{2}\left[P(S^t)s_i^t - c_i q_i^t\right] = \sum_{t=1}^{2}\left[P(S^t)s_i^t - c_i s_i^t\right], \tag{1}$$

[1] Matsumura (1999) examines a Cournot mixed duopoly model in which profit-maximizing private firms are allowed to hold inventories as a strategic device.

where c_i denotes the constant cost. We assume that the DSF is less efficient than the FPF, i.e., $c_{\mathrm{D}} > c_{\mathrm{F}}$.[2] We define

$$\pi_i^t \equiv P(S^t)s_i^t - c_i s_i^t . \tag{2}$$

The objective of the FPF is to maximize the sum of undiscounted profits.

Since $\sum_{t=1}^{2} q_i^t = \sum_{t=1}^{2} s_i^t$, social welfares are

$$W = \sum_{t=1}^{2}\left[\int_0^{S^t} P(x)dx - c_{\mathrm{D}}q_{\mathrm{D}}^t - Pq_{\mathrm{F}}^t\right] = \sum_{t=1}^{2}\left[\int_0^{S^t} P(x)dx - c_{\mathrm{D}}s_{\mathrm{D}}^t - Ps_{\mathrm{F}}^t\right] \tag{3}$$

We define

$$w^t \equiv \int_0^{S^t} P(x)dx - c_{\mathrm{D}}s_{\mathrm{D}}^t - Ps_{\mathrm{F}}^t . \tag{4}$$

The objective of the DSF is to maximize the sum of undiscounted social welfares. We use subgame perfection as our equilibrium concept.

Supplementary Explanations

First, we derive the DSF's reaction functions from (4). In the first period, since there is no inventory available, the DSF's reaction function is defined as:

$$R_{\mathrm{D}}^1(s_{\mathrm{F}}^1) = \arg\max_{s_{\mathrm{D}}^1 \geq 0}\left[\int_0^{S^1} P(x)dx - c_{\mathrm{D}}s_{\mathrm{D}}^1 - Ps_{\mathrm{F}}^1\right]. \tag{5}$$

In the second period, the DSF's reaction function without inventory is defined as:

$$R_{\mathrm{D}}^2(s_{\mathrm{F}}^2) = \arg\max_{s_{\mathrm{D}}^2 \geq 0}\left[\int_0^{S^2} P(x)dx - c_{\mathrm{D}}s_{\mathrm{D}}^2 - Ps_{\mathrm{F}}^2\right], \tag{6}$$

and thus its best response is shown as follows:

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[2] This assumption is justified in Gunderson (1979) and Nett (1993, 1994), and is often used in literature studying mixed oligopolies. Let us assume that the DSF is equally or more efficient than the FPF. In this case, since the DSF is interested in domestic social welfare, it chooses q_{D}^t and s_{D}^t such that price equals marginal cost. Therefore, the FPF has no incentive to participate in the market, and the DSF maximizes domestic social welfare by supplying monopolistically.

$$\overline{R}_{\mathrm{D}}^{2}(s_{\mathrm{F}}^{2}) = \begin{cases} R_{\mathrm{D}}^{2}(s_{\mathrm{F}}^{2}) & \text{for } s_{\mathrm{D}}^{2} > I_{\mathrm{D}}^{1}, \\ I_{\mathrm{D}}^{1} & \text{for } s_{\mathrm{D}}^{2} = I_{\mathrm{D}}^{1}. \end{cases} \tag{7}$$

We now present the following lemma:

Lemma 1. In the international mixed duopoly model, $R_{\mathrm{D}}^{t}(s_{\mathrm{F}}^{t})$ is upward sloping.

Lemma 1 means that the DSF treats s_{D}^{t} as strategic complements.[3]

Second, we derive the FPF's reaction functions from (2). In the first period, since there is no inventory available, the FPF's reaction function is defined as:

$$R_{\mathrm{F}}^{1}(s_{\mathrm{D}}^{1}) = \arg\max_{s_{\mathrm{F}}^{1} \geq 0} \left[P(S^{1}) s_{\mathrm{F}}^{1} - c_{\mathrm{F}} s_{\mathrm{F}}^{1} \right]. \tag{8}$$

In the second period, the FPF's reaction function without inventory is defined as:

$$R_{\mathrm{F}}^{2}(s_{\mathrm{D}}^{2}) = \arg\max_{s_{\mathrm{F}}^{2} \geq 0} \left[P(S^{2}) s_{\mathrm{F}}^{2} - c_{\mathrm{F}} s_{\mathrm{F}}^{2} \right], \tag{9}$$

and thus its best response is shown as follows:

$$\overline{R}_{\mathrm{F}}^{2}(s_{\mathrm{D}}^{2}) = \begin{cases} R_{\mathrm{F}}^{2}(s_{\mathrm{D}}^{2}) & \text{for } s_{\mathrm{F}}^{2} > I_{\mathrm{F}}^{1}, \\ I_{\mathrm{F}}^{1} & \text{for } s_{\mathrm{F}}^{2} = I_{\mathrm{F}}^{1}. \end{cases} \tag{10}$$

We now state the following lemma:

Lemma 2. In the international mixed duopoly model, $R_{\mathrm{F}}^{t}(s_{\mathrm{D}}^{t})$ is downward sloping.

Lemma 2 means that the FPF treats s_{F}^{t} as strategic substitutes.

Third, we state the Cournot-Nash equilibrium of the international mixed duopoly model. In each period, each firm selects s_{i}^{t} simultaneously and independently. The DSF maximizes domestic social welfare with respect to s_{D}^{t} given s_{F}^{t}, while the FPF maximizes its profit with respect to s_{F}^{t} given s_{D}^{t}. A

[3] The concepts of strategic complements and substitutes were introduced by Bulow et al., (1985).

Cournot-Nash equilibrium is a pair $(s_{\mathrm{D}}^{t*}, s_{\mathrm{F}}^{t*})$ of sales levels where each firm maximizes its objective given the other firm's sales.

Fourth, we consider Stackelberg games. If the DSF is the Stackelberg leader, then the DSF selects s_{D}^{t}, and the FPF selects s_{F}^{t} after observing s_{D}^{t}. The DSF maximizes domestic social welfare $w^{t}(s_{\mathrm{D}}^{t}, R_{\mathrm{F}}^{t}(s_{\mathrm{D}}^{t}))$ with respect to s_{D}^{t}. On the other hand, if the FPF is the Stackelberg leader, then it maximizes its profit $\pi_{\mathrm{F}}^{t}(s_{\mathrm{F}}^{t}, R_{\mathrm{D}}^{t}(s_{\mathrm{F}}^{t}))$ with respect to s_{F}^{t}. We state the following lemma:

Lemma 3. In each period, each firm's Stackelberg leader sales are lower than its Cournot sales without inventory.

Lemma 3 means that each firm prefers lower sales than its Cournot sales without inventory.

Results

We begin by considering the equilibrium in the first period. There is no inventory available in the first period, and moreover s_i^1 does not affect s_i^2 and s_j^2. Since each firm's payoff decreases by deviating from the Cournot-Nash solution, it has no incentive to do so, and therefore the equilibrium in the first period coincides with the Cournot-Nash solution without inventory.

We now consider the second period. It is thought that the equilibrium of the second period is decided by the level of I_i^1. We discuss the following three cases.

Case 1: Only the DSF can hold inventory.

Case 2: Only the FPF can hold inventory.

Case 3: Each firm can hold inventory.

We discuss these cases in order.

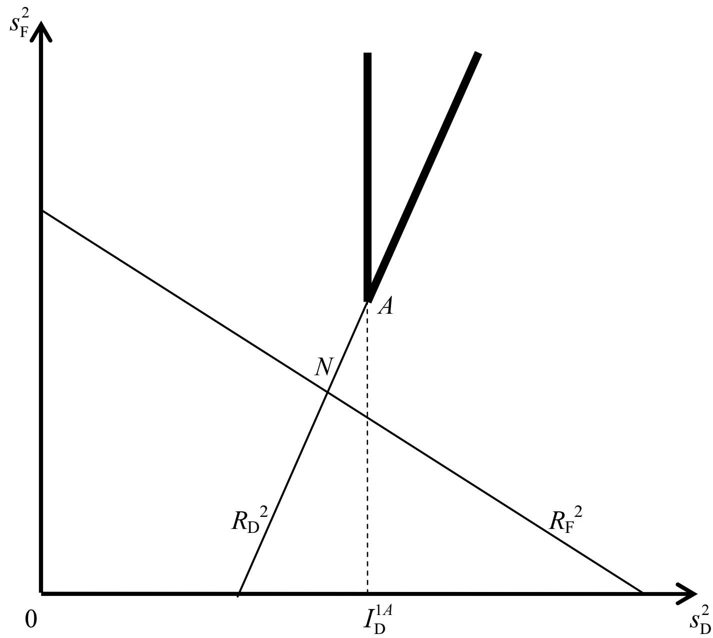

Figure 1: The reaction curves do not cross each other.

Case 1

Case 1 is illustrated in Figures 1 and 2, where R_i^2 denotes the reaction curve without inventory. R_D^2 slopes upward, whereas R_F^2 slopes downward. First, we consider Figure 1. Suppose that the DSF holds I_D^{1A} in the second period. By holding inventory, the DSF's best response changes to (7). The DSF's inventory holding thus creates a kink in its reaction curve at the level of I_D^{1A}. Therefore, the DSF's reaction curve becomes the kinked bold lines. The solution is decided in a Cournot fashion. In other words, the intersection of the reaction curves gives us the equilibrium outcome of the game. In Figure 1, the reaction curves do not cross each other. Therefore, if the DSF maintains the level of I_D^{1A}, then there is no solution.

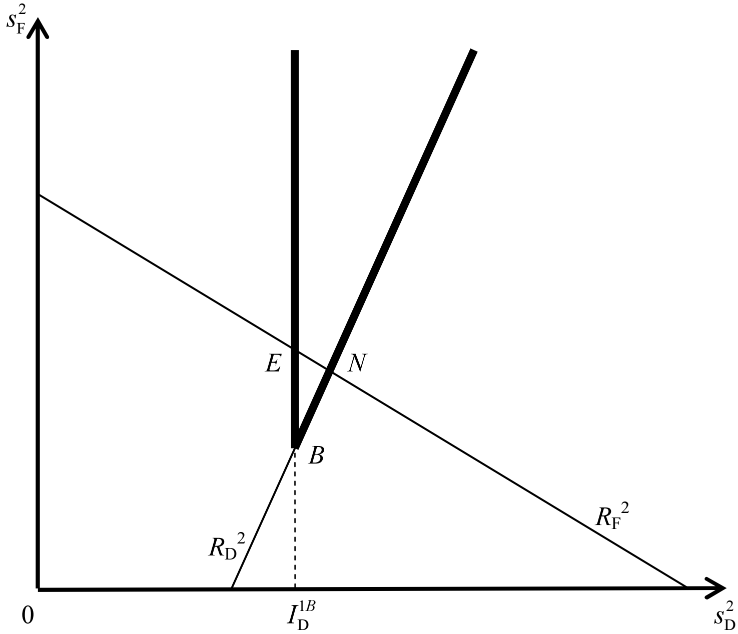

Figure 2: The reaction curves cross at points *E* and *N*.

Next, we consider Figure 2. Suppose that the DSF holds I_D^{1B}. The DSF's reaction curve is the kinked bold lines. The intersection of the reaction curves gives us the equilibrium of the game. The inventory level of I_D^{1B} changes the equilibrium of the game. The intersection of the reaction curves is the equilibrium sales in the second period. The reaction curves cross twice as depicted in Figure 2. We can see easily that both E and N are stable equilibria. That is, there are two stable equilibria. However, we see that if the DSF prefers to hold the level of I_D^{1B}, then both social welfare and the FPF's profit are higher at E than at N.

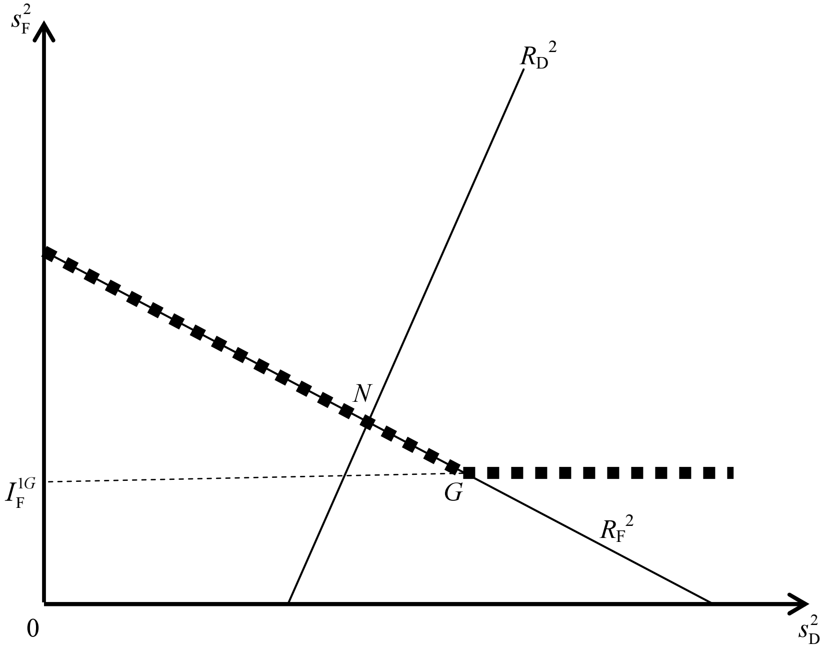

Figure 3: The equilibrium occurs at point *N*.

Case 2

Case 2 is illustrated in Figures 3 and 4. First, we look at Figure 3. Suppose that the FPF holds I_F^{1G} in the second period. By holding inventory, the FPF's best response changes to (10). The FPF's inventory holding thus creates a kink in its reaction curve at the level of I_F^{1G}. That is, the FPF's reaction curve becomes the kinked bold broken lines. The intersection of the reaction curves gives us the equilibrium of the game. Figure 3 shows that the intersection of reaction curves is not affected by the kink. Hence, the equilibrium occurs at N.

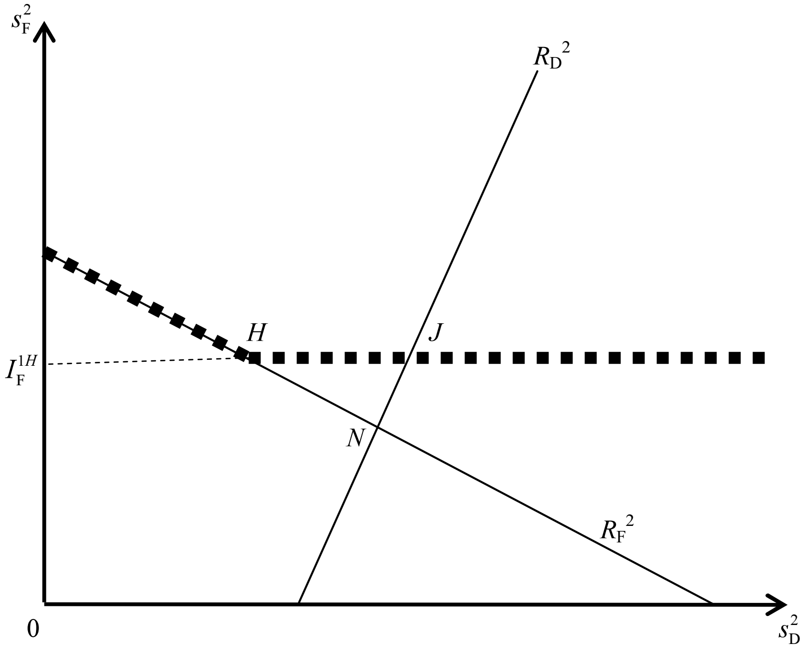

Figure 4: The reaction curves cross at point *J*.

Next, we consider Figure 4. Suppose that the FPF holds I_F^{1H}. The FPF's reaction curve becomes the kinked bold broken lines. From Figure 4, we see that the inventory level of I_F^{1H} changes the equilibrium of the game. The intersection of the reaction curves is the equilibrium sales in the second period. That is, if the FPF holds I_F^{1H}, then the solution occurs at J. However, we see that the FPF's profit is higher at N than at J.

Case 3

This case is illustrated in Figures 5 and 6. First, we look at Figure 5. Suppose that the DSF and the FPF hold I_D^{1M} and I_F^{1K}, respectively. The DSF's reaction curve is the kinked bold lines, and the FPF's reaction curve is the kinked bold broken lines. In Figure 5, the new reaction curves do not cross each other. Therefore, if the DSF and the FPF maintain I_D^{1M} and I_F^{1K}, respectively, then there is no solution.

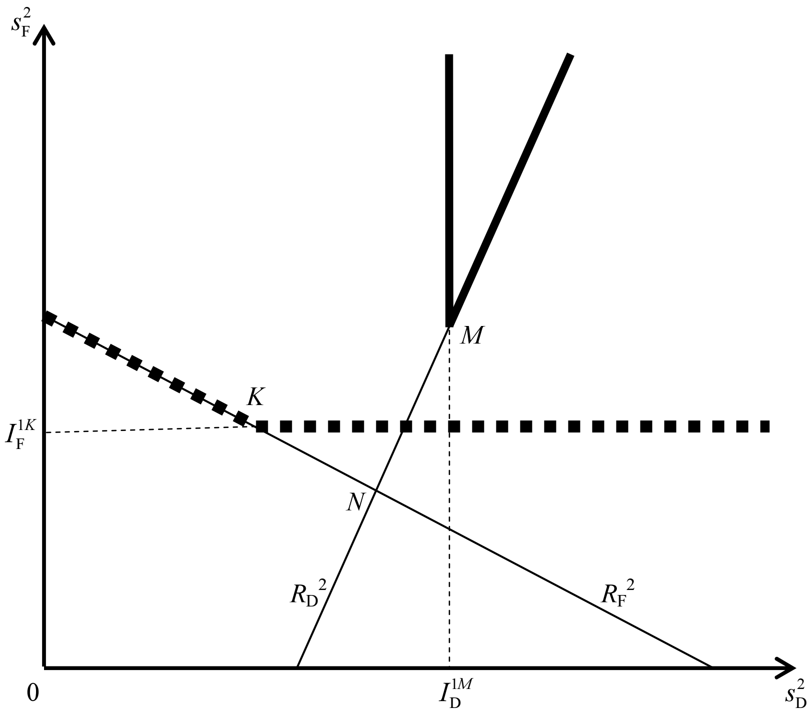

Figure 5: The new reaction curves do not cross each other.

Next, we suppose that the DSF holds I_D^{1W} and the FPF holds I_F^{1T}. In Figure 6, the new reaction curves cross twice. We see easily that both U and V are stable solutions. However, the FPF's profit is higher at U than at V, and therefore the FPF prefers U to V. In addition, the FPF's profit is higher at Y than at U. The FPF can increase its profit by reducing I_F^1.

We can now state the following proposition:

Proposition 1. In the second period of the international mixed duopoly model, there is an equilibrium that coincides with the Stackelberg solution where the DSF is the leader. At equilibrium, both domestic social welfare and the foreign private firm's profit are higher than in the game without inventory.

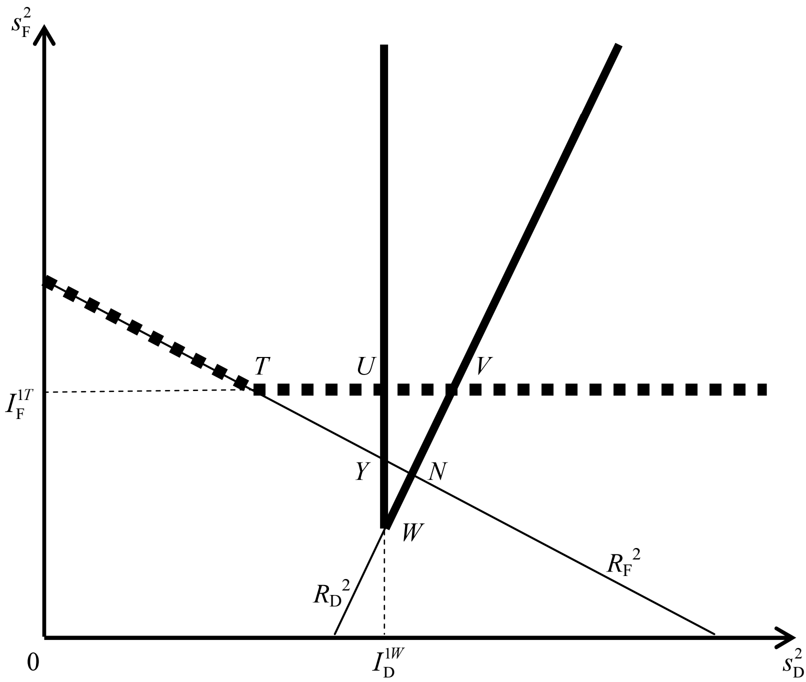

Figure 6: The new reaction curves cross at points *U* and *V*.

Concluding Remarks

We have considered a two-period model in which a domestic state-owned public firm and a foreign private firm are allowed to hold inventories as a strategic device. We have then demonstrated that the equilibrium in the second period coincides with the Stackelberg solution where the domestic state-owned public firm is the leader, and at equilibrium, domestic social welfare and the foreign private firm's profit both are higher than in the game without inventory.

The introduction of inventories into the analysis of international mixed market competition between public and foreign private firms is profitable for the firms. That is, inventory holding enables both firms to get more in a non-cooperative setting. Therefore, we find that inventory holding facilitates tacit collusion. Furthermore, inventory holding by the public firm decreases its sales while increases the foreign private firm's sales, thereby improving domestic social welfare. As a result, we see that governments that wish to increase domestic social welfare should adopt an industrial policy that behaves less aggressively toward foreign private firms; that is, inventory holding might be viewed as just one way to achieve this.

Appendix

Proof of Lemma 1

The DSF aims to maximize domestic social welfare with respect to s_D^t, given s_F^t. The optimal sulution must satisfy the following conditions: The first-order condition for the DSF is

$$P - c_D - P' s_F^t = 0, \tag{11}$$

and the second-order condition is

$$P' - P'' s_F^t < 0. \tag{12}$$

Furthermore, we have

$$R_D^t{}'(s_F^t) = \frac{P'' s_F^t}{P' - P'' s_F^t}. \tag{13}$$

Thus, Lemma 1 follows from $p'' < 0$. Q.E.D.

Proof of Lemma 2

The FPF aims to maximize its profit with respect to s_F^t, given s_D^t. The optimal solution must satisfy the following conditions: The first-order condition for the FPF is

$$P' s_F^t + P - c_F = 0, \tag{14}$$

and the second-order condition is

$$2P' + P'' s_F^t < 0. \tag{15}$$

Furthermore, we have

$$R_F^t{}'(s_D^t) = -\frac{P' + P'' s_F^t}{2P' + P'' s_F^t}. \tag{16}$$

Thus, Lemma 2 follows from $p' < 0$ and $p'' < 0$. Q.E.D.

Proof of Lemma 3

First, we prove that the DSF's Stackelberg leader sales are lower than its

Cournot sales without inventory. If the DSF is the Stackelberg leader, then it maximizes social welfare $\overline{w}^t(s_{\mathrm{D}}^t, R_{\mathrm{F}}^t(s_{\mathrm{D}}^t))$ with respect to s_{D}^t. Therefore, in each period, the DSF's Stackelberg leader sales must satisfy the first-order condition:

$$\frac{\partial w^t}{\partial s_{\mathrm{D}}^t} + \frac{\partial w^t}{\partial s_{\mathrm{F}}^t}\frac{\partial R_{\mathrm{F}}^t}{\partial s_{\mathrm{D}}^t} = 0, \tag{17}$$

where $\partial w^t / \partial s_{\mathrm{F}}^t = -P' s_{\mathrm{F}}^t$ is positive from $P' < 0$, while $\partial R_{\mathrm{F}}^t / \partial s_{\mathrm{D}}^t$ is negative from (16). To satisfy (17), $\partial w^t / \partial s_{\mathrm{D}}^t$ must be positive.

Next, we prove that the FPF's Stackelberg leader sales are lower than its Cournot sales without inventory. If the FPF is the Stackelberg leader, then it maximizes its profit $\overline{\pi}_{\mathrm{F}}^t(s_{\mathrm{F}}^t, R_{\mathrm{D}}^t(s_{\mathrm{F}}^t))$ with respect to s_{F}^t. Therefore, the FPF's Stackelberg leader sales must satisfy the first-order condition:

$$\frac{\partial \pi_{\mathrm{F}}^t}{\partial s_{\mathrm{F}}^t} + \frac{\partial \pi_{\mathrm{F}}^t}{\partial s_{\mathrm{D}}^t}\frac{\partial R_{\mathrm{D}}^t}{\partial s_{\mathrm{F}}^t} = 0, \tag{18}$$

where $\partial \pi_{\mathrm{F}}^t / \partial s_{\mathrm{D}}^t = P' s_{\mathrm{F}}^t$ is negative from $P' < 0$, and $\partial R_{\mathrm{D}}^t / \partial s_{\mathrm{F}}^t$ is positive from (13). To satisfy (18), $\partial \pi_{\mathrm{F}}^t / \partial s_{\mathrm{F}}^t$ must be positive. Hence, Lemma 3 follows. Q.E.D.

Proof of Proposition 1

First, consider the possibility that the DSF holds inventory as a strategic device. Lemmas 1 and 2 state that R_{D}^2 is upward sloping, and R_{F}^2 is downward sloping, respectively. Lemma 3 states that the DSF's Stackelberg leader sales are lower than its Cournot sales without inventory. From (7) and (10), we see that the equilibrium in the second period is decided by the value of I_{D}^1. Let I_{D}^1 can take values of zero and above. Hence, the DSF can choose $\overline{I}_{\mathrm{D}}^1 \left(= \overline{q}_{\mathrm{D}}^1 - N_{\mathrm{D}}\right)$ associated with its second-period Stackelberg leader solution.

Next, consider the possibility that the FPF holds inventory as a strategic device. If the DSF chooses $\overline{I}_{\mathrm{D}}^1$, then its reaction function will have a flat segment at the level of $\overline{I}_{\mathrm{D}}^1$. Let π_{F}^2 be assumed to be continuous and concave in s_{F}^2. A little change in the FPF's sales does not change the DSF's sales and reduces the FPF's profit. That is, inventory holding by the FPF decreases its profit. Hence, the FPF does not hold inventory as a strategic device.

Our solution concept is the subgame perfect equilibrium and all information in the model is common knowledge. The FPF knows that the DSF holds inventory as a strategic device. Hence, the FPF does not hold inventory as a strategic device. The DSF knows that the FPF does not hold inventory as a strategic device. Lemma 3 states that the DSF's Stackelberg leader sales are lower its Cournot sales without inventory. Let I_{D}^{1} can take values of zero and above. In the first period, the DSF chooses $\overline{I}_{\mathrm{D}}^{1}\left(=\overline{q}_{\mathrm{D}}^{1}-N_{\mathrm{D}}\right)$ associated with its second-period Stackelberg leader solution. In the second period, the DSF sells $\overline{s}_{\mathrm{D}}^{2}=\overline{I}_{\mathrm{D}}^{1}$. Thus, the equilibrium coincides with the Stackelberg solution where the DSF is the leader.

From Lemma 3, we see that the DSF decreases s_{D}^{2} by holding $\overline{I}_{\mathrm{D}}^{1}\left(=\overline{q}_{\mathrm{D}}^{1}-N_{\mathrm{D}}\right)$ associated with its second-period Stackelberg leader solution. Since $\partial\pi_{\mathrm{F}}^{2}/\partial s_{\mathrm{D}}^{2}=P's_{\mathrm{F}}^{2}<0$, decreasing s_{D}^{2} increases π_{F}^{2} given s_{F}^{2}, and thus the proposition follows. Q.E.D.

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