

The Black Hole with A Finite-Size Nucleus Based on The Conservation of Energy and Asymptotic Freedom

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Abstract From analysis of light geodesic along the radial direction, an infinite time for light passing through the event horizon of the black hole seems to be an unreasonable physical solution. The same situation is also for the massive particle. To compress all particles to a singularity also causes another energy problem because the black hole is evolutionary from a star which only has finite mass and stored energy. From the viewpoint of the gravitation self-energy, we deduct that the black hole should have a finite-size nucleus. Then the black hole with finite-size nucleus is proposed and satisfies the gravitational criteria of the black hole. According to the successful theorem of the asymptotic freedom in the strong interaction, several possible structure models are considered in the high-density quark matter phases. Next, using the Kerr-Newman metric, light propagating along the radial direction demonstrates finite speed forwardly and backwardly at any position larger than the Schwarzschild radius and no mathematical singularity at $r=0$ and $\theta=\pi/2$. This very important fact shows that light can propagate from the outer space into a black hole and vice versa. The proposed structure model for the black hole is either rotational or charged and its nucleus can have strong magnetic dipole that causes the relativistically charged mass ejection from the black hole as happened in GRS 1915+105 and Galaxy M87. In thermal equilibrium, the nucleus of the black hole should have temperature higher than the cosmic microwave background temperature of 2.726 K. The entropy increases and the second law of thermodynamics should be still useful.

Keywords: Black hole, Schwarzschild metric, Kerr-Newman metric, asymptotic freedom

I. Introduction

Through the astronomical observations, some candidates of the black holes were found and they exist in some space with heavy mass. The mass of the black hole produces the great gravity to affect neighboring stars and planets. We can use our mathematics and physics based on General Relativity to describe the movements of the bodies or particles till to the event horizon of a black hole in the present knowledge. What would be in the inner of the black hole is still unknown although the concept of the singularity has been proposed for a long time. However, shrinking all mass to a singularity gives rise to some questions. As we know, there are four interactions existing in nature: the strong interaction, the electromagnetic interaction, the weak interaction, and the gravitational interaction. When the gravity does the work to shrink all mass to

a small region, it also needs to transfer energy between these interactions. So we might ask: does the gravitational force of a big ended star have enough energy to compress all particles to a point? According to the Einstein's mass-energy equivalence, it seems to tell us the maximum useful energy for Gravity equal to the mass M_{sun} of a star times the square of the speed of light in free space c , that is, $M_{\text{sun}}c^2$. The singularity including all mass and all charges there seems to be an un-physical phenomenon. It makes another scientific question: how much energy can achieve this phenomenon? Does all mass have to gather at this singularity so that a black hole forms? Based on these questions, it causes the curiosity to think about the reasonable structure inside a black hole. We use several viewpoints including the light geodesic inside the black hole and the ability of size reduction of an atomic nucleus in Quantum Chromodynamics (QCD) to discuss this problem and propose three models for the possible inner structure of the black hole. By these structure models, the relativistically massive ejection from the black hole can be explained.

II. Analysis

First of all, the existence of the photon in or out the black hole is a good starting discussion to gradually build our structure model. According to the Generalized Uncertainty Principle (GPU), the photon has possibility to exist in a region larger than a black hole [1]. This kind of photon has very long wavelength and a corresponding much low energy. The position uncertainty of the photon is about $2R_S$ where R_S is the Schwarzschild radius of the Schwarzschild black hole. This photon has possibility inside the black hole or outside it. However, this quantum description is not so satisfied because it lacks the concept of movement for massive or massless particles as the basic description in General Relativity. It might cause confusion whether photon can propagate away from the inner of a black hole or not?

However, theoretically speaking, the reversibility of light should predict that light can enter a black hole and come back along the same geodesic if there is a mirror inside the black hole to normally reflect it back to the original geodesic without considering any change on the event horizon. But this phenomenon seems to be forbidden by the nowadays theory of the black hole. Does the gravity of the black hole really block it returning back to the universe? Furthermore, we might ask why only the gravitational wave can escape [2] but light cannot leave away the black hole? A way to check it is to consider the propagation process of light entering into the black hole at the polar point and forwarding to the singularity, and the light geodesic is on the radial direction where the origin of the coordinate is at the center of the black hole or the singularity. When light propagates toward the singularity without any absorption, light will pass through it and propagate continuously toward the event horizon at the other side. The energy of

light should keep the same value as it enter into the black hole. Will this geodesic stop at the event horizon finally?

Next, let's use the Schwarzschild metric to further discuss the propagation of light in the black hole. The Schwarzschild metric [1] for a black hole of mass M is

$$ds^2 = -c^2 \left(1 - \frac{2GM}{c^2 r}\right) dt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2, \quad (1)$$

where G is the gravity constant, ds is the invariant interval, t is the coordinate time, r is the radius coordinate, θ is the polar angle, and ϕ is the azimuth angle. The coordinate time in a gravitational field is the time read by the clock stationed at infinity because the proper time and coordinate time becomes identical [3]. Considering a case that a particle of mass m starts freely falling at a place r_0 with initial velocity zero, then the spending time t when it reaches the place r is [4]

$$T = \frac{1}{c} \sqrt{1 - \frac{R_S}{r_0}} \int_r^{r_0} \frac{1}{\left(1 - \frac{R_S}{r}\right) \sqrt{\frac{R_S}{r} - \frac{R_S}{r_0}}} dr, \quad (2)$$

Where $R_S = 2GM/c^2$ is the Schwarzschild radius. It shows that massive particles will spend infinite time at $r=R_S$. The radial speed of light also shows similar result. The geodesic of light obeys $ds^2=0$, then we have the speed of light v_r at the black hole [5-8] is

$$v_r^2 = \left(\frac{dr}{dt}\right)^2 = \left(1 - \frac{2GM}{c^2 r}\right)^2 c^2. \quad (3)$$

It is obviously that the radial velocity is zero at $r=R_S$. The Schwarzschild metric indicates that light as well as other massive particles will spend infinite time at $r=R_S$ by an observer in a reference frame far away from the black hole like on Earth. This observer will not see any particle or light really absorbing by the black hole no matter spend how much the universal time, and it results in a phenomenon that the particle never reach the singularity to induce the whole gravitational field of the black hole rearranged. So can the observer see the expansion of the black hole because the absorption never happens observably? If the observer can investigate the expansion of the black hole, the phenomenon will violate causality. Although some reference explains that it is only a finite interval of proper time in a reference frame moving with the particle [4], however, the astronomical observations have never shown an absorption event by a black hole spending infinite time. So this problem causes another curiosity question: is the Schwarzschild metric really suitable for describing the movements of all particles close to or inside the black hole? If everything stops at the

event horizon, how dose the supermassive black hole [9] having mass several billion times as much as our sun come?

Let's think about the spending time for a particle traveling from the event horizon to the singularity. Here just using the logical deduction to discuss this phenomenon. Considering the situation before the formation of a black hole, every particle falling on the surface of the star spends finite time in the Earth's frame. Similarly, the particle falling into the black hole from the outer space to reach the singularity should also spend finite time. An observer in a reference frame far away the black hole like on Earth shouldn't see a lot of particles and bodies resting on the event horizon and more and more accumulation taking place there. On the contrary, the observer finds out the particles and bodies indeed passing through the event horizon into the black hole as the absorption of the accretion cloud around a black hole. Then in the following time, the observer discovers the real change of the Schwarzschild radius or gravitation because of the occurrence of the absorption. It reveals that the Schwarzschild metric giving infinite time for one happening at the event horizon is not appropriate to describe light and the massive particles passing through the horizon event into the black hole. It should satisfy the actually astronomical phenomena.

Actually, the event horizon of the black hole is just a conceptual boundary between the black hole and the outer space. This boundary is determined by the mass, charges, and rotation of the black hole. Furthermore, we check the speed of light calculated in the Schwarzschild metric again. Except for the problem of the radial speed of light $v_r = 0$ at $r = R_s$, Eq. (3) shows that it becomes an imaginary value when r is less than the Schwarzschild radius. Especially at the singularity $r=0$, the absolutely radial velocity of light is infinite. This is an un-physical description for light. Other problems of singularity in the black hole have also been discussed [10]. Should light have infinite speed in the black hole?

As discussed in the first paragraph, a star of mass M with the equivalence energy of Mc^2 can offer itself gravitational field using this energy to do work and compress the massive particles in a space smaller than the original star size. As we know, the gravitational potential energy is proportional to $1/r$. For a singularity, the infinite large gravitational field as well as the gravitational potential energy at $r=0$ is unphysical and unreasonable. The formation of a black hole is believed that the heavier star cannot proceed the nuclear fusion so the explosion of a supernova causes its nucleus much denser and shrink to a smaller space. Finally, a case of the non-rotational and uncharged black hole with a Schwarzschild radius is formed. From the viewpoint of the physical mechanism, once the force equilibrium reaches, then it makes the dense body keep at certain size and stop shrinking. During the shrinking process, it needs gravity to do a lot of work against the strong interaction and the electromagnetic interaction, and at

most uses the maximum stored energy. Reasonably speaking, as long as all mass within a region smaller than the Schwarzschild radius for this kind of the black hole, its gravitation out of the event horizon is fairly enough to reach the gravitational criteria of a black hole.

In order to check this possibility, the density of our sun is used as an example to estimate what situation makes its mass gathering within the Schwarzschild radius without becoming a singularity. As we know, the average density of our sun is 1.409 g/cm^3 , and 92.1% of the sun is hydrogen atom and 7.8% is helium atom [11]. Using this information to calculate the average atom weight, it is about 1.233 and there are average 6.94×10^{23} molecules per cm^3 . The average volume for a hydrogen atom or helium atom is 1.44 \AA^3 . The charged radius of the proton is about $0.84 \sim 0.87 \text{ fm}$ [12] and it is almost the same order for a neutron. The average atomic radius is about 8.0×10^4 times larger than that of the nucleus and the most of the space in an atom is empty. Those empty space can fill more nuclei under strong gravity theoretically. It make us think about a situation that all nuclei contact each other very closely similar to the neutron star, and what is the radius of the sun? Here we don't consider any nuclear fusion and just think about the possibility of the useful space. The original radius of the sun is $R_{sun} = 6.96 \times 10^5 \text{ km}$ [12] and that of above crowded situation is about

$$R_{reduced} = 6.96 \times 10^5 / 8.0 \times 10^4 \text{ km} = 8.70 \text{ km}, \quad (4a)$$

when the radius is reduced 8×10^4 times. The Schwarzschild radius of the sun is

$$R_{Schwarzschild} = 2GM_{sun}/c^2 = 2.95 \text{ km}, \quad (4b)$$

where M_{sun} is the mass of the sun, and c is the speed of light in free space. $R_{reduced}$ and $R_{Schwarzschild}$ are at the same order. It implies that it doesn't need to shrink all mass to be a singularity but just make all the nuclei more crowded to reach the gravitational criteria of the black hole. A proton is consisting of three quarks it has some flexibility to change its size by the strong gravity. The pressure produced by the strong gravitational force on a proton can cause three quarks closer, and the work done by the gravity makes more energy store in the gluon field. From the comparison of the value in Eq. (4a) with the value in Eq. (4b), all the atomic nucleus only reduces its radius 3~4 times or increases its density 27~64 times enough to matches the gravitational criteria in the region out of a black hole. Recently, the charge densities of the neutron and proton were proposed [13], and the compression of neutrons and helium atoms under extreme pressure has been studied [14,15]. As we know, both proton and neutron are baryons. The deformations of baryons is possibly to be a much denser package. When considering the possible face-centered cubic package for piling up baryons, it shows that there are about 26% empty space. When all these space is occupied, the minimum value for the

criteria for producing a quasi black hole is no more than 20 times the uncompressed baryon density in our demonstrated case.

We may ask that is it still stable for a proton or a neutron when their sizes as well as the distance between two quarks are reduced 3~4 times? According to the asymptotic freedom in the strong interaction [16-18], the interaction becomes weaker when the distance between two quarks becomes shorter. It shows that the quark matter phase would exist at very high density. The dense quark matter has been studied on the compact star with mass in the range $1.3\sim 1.6 M_{\text{sun}}$ and radii $8\sim 11$ km [19]. The different density of the baryon performs different quark matter phases [20,21]. When the density of a baryon is roughly less than 10^1 times as large as the original one, it would be in the nuclear superfluid phase [20]. When the density is about 10^1 times even more, it would be the quark-gluon-plasma phase, the non-color-flavor-locked (non-CFL) phase, or the color-flavor-locked (CFL) phase, and those phases are also related to temperature [20,21]. During the mass shrinking process, energy is transferred from the gravitational interaction to the strong interaction and the electromagnetic interaction, and this process stops when the force equilibrium reaches. So physically speaking, the reduction of a baryon or a nuclear size needs gravity to do more and more work and to shrink all particles to a singularity needs an infinite energy. We might ask: where to get the infinite energy?

Physically, the formation of a black hole obeys energy conservation, so it exists the following energy relationship at least

$$\begin{aligned} & U_{g,\text{star}} + U_{e,\text{star}} + U_{\text{rot},\text{star}} + U_{\text{mech},\text{star}} - (U_{g,\text{black}} + U_{e,\text{black}} + U_{\text{rot},\text{black}} + U_{\text{mech},\text{black}}) \\ & = \Delta E + \Delta mc^2. \end{aligned} \quad (5)$$

In Eq. (5), the electric energy, the gravitational energy, the rotational energy and other mechanical energy are $U_{e,\text{star}}$, $U_{g,\text{star}}$, $U_{\text{rot},\text{star}}$ and $U_{\text{mech},\text{star}}$ for a star, respectively, and $U_{e,\text{black}}$, $U_{g,\text{black}}$, $U_{\text{rot},\text{black}}$ and $U_{\text{mech},\text{black}}$ for a black hole, respectively. The terms in the right-hand side of Eq. (5) are the change of energy ΔE in the atomic scales, and the energy loss Δmc^2 in terms of massive or massless particles, or light radiating to the universe. According to it, the singularity with a lot of mass and charges is un-physical solution and all mass as well as energy might be almost lost before compressing it to a singularity.

For example, considering a supernova with average density ρ , radius R , and mass $M=4/3\pi\rho R^3$. Its classically gravitational self-energy is

$$\frac{3GM^2}{5R} = \frac{16}{15} G\pi^2 R^5 \rho^2. \quad (6)$$

Now, suppose the explosion of this supernova losses its mass and the remainder mass is M/α with $\alpha > 1$. Relativistically speaking, the released energy is $(1-1/\alpha)Mc^2$ from

this explosion and the rest mass can obtain this energy from the explosion at most. Actually, there is always some part of energy radiated in electromagnetic wave or massive particles, etc. The obtained energy can do work to compress the rest mass and result in much higher density. Finally, the radius and density become R' and ρ' . However, the strength of the strong interaction is about 10^{39} times as large as the gravitational interaction, and almost all the obtained energy for the rest mass are used to do the work for the strong interaction. The high density is the factor that the particles such as the proton or neutron are compressed and the increase of the gravitational energy is very tiny, so the classically gravitational energy is smaller than that before the explosion or compression. Then we have

$$\frac{16}{15} G\pi^2 \frac{R'^5 \rho'^2}{\alpha^2} < \frac{16}{15} G\pi^2 R^5 \rho^2, \quad (7a)$$

or

$$\rho' < \alpha \left(\frac{R}{R'} \right)^{5/2} \rho. \quad (7b)$$

From Eq. (7b), it shows the upper limit of the final density ρ' dependent on R , ρ , α , and R' . The more mass losses, the higher density could be. However, even $R' = 0$, ρ' is still finite in Eq. (7b).

We further consider the situation after formation that the black hole absorbs a lot of massive things and increases its total mass to βM with $\beta > 0$. The radius and density are R'' and ρ'' . Similarly, as the reason in Eq. 7(a), it gives

$$\frac{16}{15} G\pi^2 R''^5 \rho''^2 < \beta^2 \frac{16}{15} G\pi^2 R^5 \rho^2, \quad (8a)$$

or

$$\rho'' < \beta \left(\frac{R}{R''} \right)^{5/2} \rho. \quad (8b)$$

The choice “<” is the reason that some energy such as electromagnetic wave can radiate to the outer space during the absorption. Both Eqs. (7b) and (8b) have the same form and result in the same conclusion. So $R' > 0$, $R'' > 0$, and both ρ' and ρ'' are finite for the black hole that further supports our model. The black hole should have a finite-size nucleus.

III. The Proposed Possible Structure Models

According to Eq. (5), the black hole reasonably has a nucleus with finite size, not a singularity. In the following, three possible structures of the black hole with a nucleus

are boldly proposed. As we know, the surface temperature of a star is usually several thousand K and its core temperature is at least several million K. When ^{56}Fe is produced in the core region, so high temperature has very possibly made ^{56}Fe ionize and gradually a lot of ^{56}Fe nuclei gather in the core region. Above the core region, there should be other materials such as neutrons, hydrogen, helium ...etc., to cover it and also protect the ^{56}Fe nuclei from the interaction with negatively charged particles like electrons. Because proton can quickly transfer to neutrons due to the interaction with electrons, there should be some layer like neutron for protecting ^{56}Fe nuclei. Due to this protection, a lot of ^{56}Fe in the core region can sustain for a very long time. Or there is the other possibility that almost all protons in ^{56}Fe nuclei transfer to neutrons because of interactions.

After formation of a black hole, its deeper region of the nucleus originally comes from a lot of ^{56}Fe nuclei in a fairly high density, and those protons and neutrons in ^{56}Fe are in the ultra-high density quark matter phases. Even all ^{56}Fe nuclei can possibly mix each other to become some special or unknown macro matter. Those quark matter phases start from the superfluid or the quark liquid phase to the non-CFL phase [20,21]. Furthermore, as the pressure continuously increases from the surface to the core, the density of these baryons can exceed the critical value and they would become the CFL phase [21] in the much deeper region till to the core region of the nucleus of the black hole. Simply speaking, it is from the high-density quark matter phase to the super ultra-high-density quark matter one. According to the early research of the compact stars [19], it has mentioned the inner structure of the CFL phase for this kind of star. Usually, the phase diagram of quantum chromodynamics (QCD) is shown in terms of the baryon chemical potential μ , where μ is proportional to the cubic root of the baryon density d . The range of the CFL phase [20,21] can cover the above mentioned value 27~64, so it is reasonable to propose the finite-size nucleus model for the black hole with the CFL phase in the deeper inner region of the nucleus of the black hole with total mass roughly equal to our sun.

When we focus our attention from the inner region of the nucleus to the place above this nucleus, it possibly exists the quantum phenomena for small charged or uncharged particles. We don't have to think all particles falling down on the nucleus of the black hole like the binding electrons around an atom. There is an advanced possible structure model that a lot of negatively charged particles moving around the nucleus on orbitals and existing mainly close to the surface of the nucleus. Except for the negatively charged particle, the positively charged particles also have their orbitals. The positively charged particles are repulsed by the electromagnetic force but attracted by the gravitational force due to the strong gravity. Their orbitals mainly exist a little far away from the surface of the nucleus. Except for these two kinds of charged-particle orbitals,

the charge-free particles can also have the third orbitals purely attracted by the gravitational force and these orbitals may spread broadly. Recently, a concept of the Bohr-like black holes has been proposed for considering the orbitals of the particles like neutrons using gravity as an attractive force [22].

To sum up, the above descriptions make the whole black hole like a very big atom with a nucleus consisting of matters from high-density quark matter phase to super ultra-high-density quark matter one as shown in Fig. 1(a). These negatively charged particles form a very dense cloud and positively charged ones form another one with a larger radius. These two dense clouds may have some overlaps that might be unstable and easily causes the positively and negatively charged particles disappear or become neutral. Then all the charge-free cloud distributes mainly between these two charged clouds. In this structure model, the charged particles from the outer space can also continuously occupy some empty orbitals of the black hole. The total charges Q in the black hole can be fluctuated in time.

The second possible structure is shown in Fig. 1(b), where the main constituents in the core region as Fig. 1(a) with a nucleus from high-density quark matter phase to super ultra-high-density quark matter one. The difference between the first and the second structures is the negatively and positively charged clouds disappeared. There might exist a thin charged-deposition region in the buffer layer close to the surface. They can absorb the charged particles from the outer space, so this buffer layer have unstable charges. Furthermore, this kind of buffer layer might be eventually divided into several layers and alternatively appears with different electricity. The total charges Q of this black hole can be varied in time.

The third possible structure is shown in Fig. 1(c) where the main constituents are originally from neutrons but here it has much higher density than the neutron star. It has been pointed out the range of the quark matter phase for the neutron star on the phase diagram of QCD [21] at low temperature. So this kind of the black hole is similar to the neutron star and becomes denser as the CFL phase [20,21]. This structure can absorb positively or negatively charged particles resulting in average charge fluctuated in time.

In the above structure model, the black hole can possess intrinsic magnetic dipole from its nucleus and exist the North and South poles as the most planets and stars. This magnetic field from the nucleus of the black hole can cause the high speed plasma near the poles outside the black hole give rise to the phenomenon similar to the coronal mass ejection (CME) [23]. The observations of the relativistic jet from high-speed rotational GRS 1915+105 [24,25] or Galaxy M87 [26-29] radiated the polarized electromagnetic waves [30] that can be explained from the accelerated electrons in an axial magnetic field where the motions of electrons performed helical trajectories with a gradually

increasing rotational radius along the axis within a small conical angle [31]. The total fields combine the magnetic field of the black hole and that induced by the relativistically charged particles. This mass ejection extends five thousand light years at least and the small polar angle 6~7 degrees is observed at a distance of 37.5 light years (12 pc) from the source [28]. This phenomenon means that the axial magnetic field is very strong for the relativistically moving particles. This strong magnetic field is reasonable from the supermassive black holes in the centers of GRS 1915+105 and Galaxy M87, respectively.

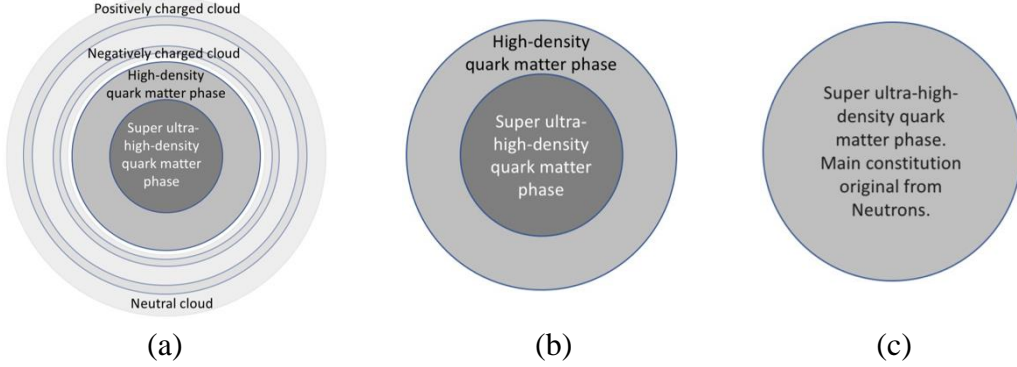


Fig. 1 (a) The nucleus of the black hole is mainly consisting of materials the from high-density the quark matter phase to super ultra-high-density quark matter phase. It is surrounded by small negatively and positively charged particles as well as neutral ones to form a structure like a very big atom. The totally charges Q are fluctuated in time. (b) The nucleus of the black hole still contains materials from high-density quark matter phase to super ultra-high-density quark matter phase. Its total charges Q is fluctuation in time. (c) The nucleus of the black hole is mainly consisting of neutrons from high-density quark matter phase to super ultra-high-density quark matter phase to form a basic structure. The total charges Q are fluctuated in time.

IV. Discussion of The Speed of Light In The Black Hole

Next, we discuss the propagation of light at the black hole. To avoid the un-physical problems in Sec. II, the Kerr-Newman metric is considered here for light propagating in the black hole. The Kerr-Newman metric [32] in the spherical polar coordinate (r, θ, ϕ) with the coordinate time t is

$$\begin{aligned}
 ds^2 = & -c^2(\Delta - a^2 \sin^2 \theta) \frac{1}{\rho^2} dt^2 + \frac{\rho^2}{\Delta} dr^2 + \rho^2 d\theta^2 \\
 & - (\Delta a^2 \sin^2 \theta - (r^2 + a^2)^2) \frac{\sin^2 \theta}{\rho^2} d\phi^2 \\
 & - 2ac(-\Delta + (r^2 + a^2)) \frac{\sin^2 \theta}{\rho^2} dt d\phi,
 \end{aligned} \tag{9}$$

where $a=J/Mc$ with J the angular momentum of the black hole, $\rho^2 = r^2 + a^2 \cos^2 \theta$,

$\Delta = r^2 - rR_S + a^2 + R_Q^2$, and $R_Q^2 = KQ^2G/c^4$ with Coulomb's constant K . As mentioned before, the coordinate time in a gravitational field is the time read by the clock stationed at infinity because the proper time and coordinate time becomes identical [3]. When we only consider the situation that light is normally incident on the black hole, the geodesic can be straightly toward the center of the black hole. Similarly, the geodesic of light is $ds^2=0$ and it has been used to deduce the velocity of light in the Schwarzschild metric by an observer at infinity [5-8]. Supposing light only along the radial direction, the expression of the radial speed $v_r = dr/dt$ of light deduced from Eq. (9) is

$$v_r^2 = \left(\frac{dr}{dt}\right)^2 = c^2 \left(1 + \frac{-rR_S + R_Q^2}{\rho^2}\right) \left(1 + \frac{a^2 \sin^2 \theta - rR_S + R_Q^2}{\rho^2}\right). \quad (10)$$

In Eq. (10), it is clear that when $r > R_S$, the radial velocity is real and non-imaginary everywhere no matter a and R_Q are, that is

$$v_r = \frac{dr}{dt} = \pm c \sqrt{\left(1 + \frac{-rR_S + R_Q^2}{\rho^2}\right) \left(1 + \frac{a^2 \sin^2 \theta - rR_S + R_Q^2}{\rho^2}\right)}. \quad (11)$$

The speed of light is finite when $r > R_S$ and the two signs of the radial velocity v_r mean that light can propagate into or away from the black hole even the event horizon is at $r > R_S$. Then the ratio of the speed at the pole to the equator at $r=R_S$ is calculated as

$$\text{Speed ratio} = \left(1 + \frac{a^2}{R_Q^2}\right)^{1/2} \left(\frac{1}{1 + \frac{a^2}{R_S^2}}\right). \quad (12)$$

The ratio of the absolute velocity of light at the poles to the equator varied with R_Q/a and R_S/a is shown in Fig. 2. The ranges of both R_Q/a and R_S/a are between 0.01 and 1.00 and the interval is 0.01. Because the time dilation exists in Eq. (9), only $R_S > R_Q$ is reasonably considered. The results reveal that the speed of light at the equator is about 25 times as large as that at two poles when $a \gg R_Q$ and $a \gg R_S$ in our calculations.

In addition, there is a singularity for the Kerr-Newman metric at $r=0$ and $\theta=\pi/2$ which is needed to check in this finite-size nucleus model. From Eq. (10) or (11), the denominator shows a singularity at $r=0$ no matter what the value a is at $\theta=\pi/2$. When we look at the numerator, a and R_Q terms are kept. In this finite-size nucleus model, light experiences $a=0$ and $Q=0$ even no gravitational force at $r=0$ so the speed of light at this point is $dr/dt=c$. It is independent of the propagation direction at this point and more important, the spacetime structure is flat at this point. All those are the advantage of the proposed finite-size nucleus model. Furthermore, the black hole usually inherits

the part of the angular momentum from the previous star so $|a| > 0$ should be a common case in the universe.

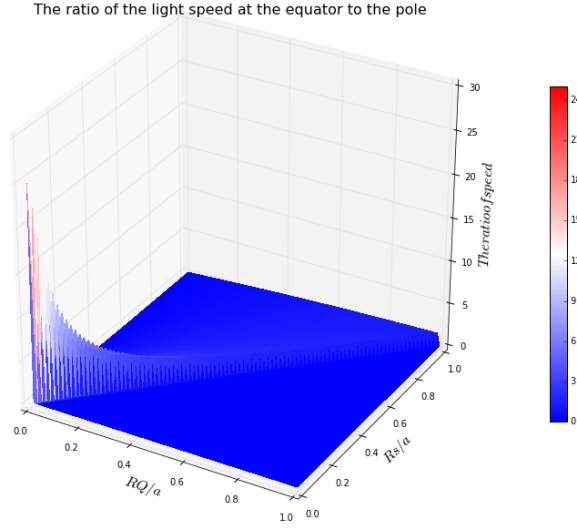


Fig. 2 The ration of the absolute speed of light at the equator to the pole at $r=R_S$. It is the function of R_Q/a and R_S/a and both ranges are between 0.01 to 1.00 with interval 0.01. The ratio of the light speed is as large as 25 times when $a \gg R_Q$ and $a \gg R_S$ in our calculations.

The black hole can have no rotation, however, it is not easy to sustain because of the impacts by particles in the black hole and the inheritance from the part of the angular momentum of the previous star. The rotational nucleus of a black hole with charges can produce additional magnetic field parallel or antiparallel to its intrinsic magnetic field. So adding negative charged particles may change the rotational speed as well as the magnetism of the nucleus of the black hole. It possibly causes the rotational speed slow down. Eq. (10) or (11) also tell us a very important fact that light can have finite speed at $r > R_S$ even passing through the event horizon from the inside of a black hole to the outer space. So the Kerr-Newman metric is an appropriate one to describe the black hole in our structure model without any correctness and avoids the singularity at $r=0$. In addition, the electromagnetic energy can be shared in the interior as well as the exterior of the Kerr-Newman black hole [33] and seems to tell us that electromagnetic energy can flow between the interior as well as the exterior. We use the radial speed of light to clearly explain why electromagnetic energy is shared because light can propagate from the outer space to the inner of the black hole and vice versa.

Next, the temperature of the black hole as well as the total change of the entropy are discussed. Because light can propagate from the inner of the black hole to the universe, it means the thermal equilibrium with the universe can be reached. The temperature of the cosmic microwave background (CMB) is 2.726 K [34] and it is also the environmental temperature for the black hole where K is the Kelvin degree. After

the formation of the black hole, its nucleus cools down gradually and the temperature changes from high to low. Finally, the temperature T of a black hole measured by an observer in a reference frame far away from it is close to $T_{CMB}=2.726$ K in thermal equilibrium. The radiation spectrum of the black hole is mostly close to the universe and is not easily distinguished from the universe by investigating the radiation spectrum. The black hole at different temperature T releases the heat $\delta Q(T)$ to the environment from the initial temperature T_0 to the final temperature $T=T_{CMB}$. The environment is the universe at an average constant temperature T_{CMB} and the total change of the entropy Δs is

$$\Delta s = \frac{1}{2.726} \int_{T_0}^{2.726} \delta Q(T) dT - \int_{T_0}^{2.726} \frac{\delta Q(T)}{T} dT \geq 0. \quad (13)$$

The second law of classical thermodynamics should be still useful in our structure model. However, due to the strong gravity, the radiation encounters gravitational redshift and the nucleus of the black hole should have temperature higher than 2.726 K actually. The nuclear temperature of the black hole can be approximately deduced from its mass and the Schwarzschild radius.

After the nucleus of black hole forming in thermal equilibrium, the freedom is almost frozen macroscopically but the phenomena due to the strong interaction as well as others continue taking place because of the very large density. It would have some different micro-states and the entropy is non-zero in statistical mechanics. This structure model avoids the black hole existing a singularity such as a non-physical description. After all, the strong interaction is about 10^{39} times as large as the gravitational interaction that the gluon field has much ability to store the energy transferred from the gravitational energy.

V. Conclusion

In summary, a finite-size nucleus model for describing the inner structure of the black hole is proposed. This model describes that a black hole has a nucleus in a finite space. The asymptotic freedom permits the baryon compressible to increase the density to super ultra-high. Furthermore, using the viewpoint of the gravitational self-energy and the conservation of energy, we theoretically explain that the black hole should have a finite-size nucleus because its average density has to be finite. The nucleus of the black hole with finite volume avoids the concept of the singularity in the black hole and satisfies reasonably useful energy. After all, the star has finite energy for gravity compressing the mass and charge. This nucleus of the black hole can have charges and rotate around a certain axis and more important, it can have the magnetic dipole which causes the relativistically charged mass ejection at the poles outer the black hole like

the phenomena observed in GRS 1915+105 and Galaxy M87.

The other important thing is pointed out by the radial speed of light in the Kerr-Newman metric that there are two real solutions existing no matter a and R_Q are when $r > R_S$. It means that light can leave away from the inner of a black hole even the event horizon is at $r > R_S$. According to our analysis, the Schwarzschild metric is inappropriate to describe the movements of particles and the propagation of light in the black hole, and the speed of light with imaginary value or its infinite value at $r=0$ shows the unphysical phenomenon. Furthermore, the most common black holes are rotational and their charges are fluctuated in time. The temperature of the nucleus of the black hole in thermal equilibrium is higher than 2.726 K and the total change of the entropy is positive that the second law of classical thermodynamics should be still useful.

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