Numbers interspersed between two Fibonacci numbers

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Introduction

Generally the Fibonacci sequence has been used to study the properties of Fibonacci numbers, however the study of the internal sets of numbers between two Fibonacci numbers has been ignored.

Fibonacci numbers act as border points of a certain number of natural numbers that have a certain relationship with Phi apart from other properties.

Procedure

It occurred to me to do a summation with all the natural numbers between two Fibonacci numbers including these and another summation with the higher Fibonacci number of the other summation and the next Fibonacci number and I verified that the quotient tended to Phi when we used numbers of Fibonacci larger, in addition fractions became irreducible.

Everyone knows more or less the Fibonacci sequence, and I will not go into this topic, it is enough to say that Kepler in the sixteenth century managed to find a relationship between the quotient of two consecutive Fibonacci numbers and it was precisely the Phi number.

The Phi number was discovered in Ancient Greece and was the ratio of a segment to its greater part as the greater part to the smaller part of the segment.

Kepler referred to him as an amazing jewel

I decided to do the opposite of working with the Fibonacci succession to find new properties of the succession itself.

Results

I dedicated to add the interspersed numbers between two Fibonacci numbers.

Although this sum according to common sense should give a number greater than Phi, surprisingly if we make the sum between the numbers that are interspersed among three Fibonacci numbers, including the respective Fibonacci numbers and we divide them by Phi squared

For example, we have three Fibonacci numbers

144-89-55

The sum of the numbers between 144 and 89 including these same numbers is equal to 6524, and the sum of the numbers between 89 and 55 does not give 2520

If we divide these quantities 6524/2520 we see that it gives us an approximation to Phi squared, in this case 2.58, the higher the Fibonacci numbers the closer they get to Phi square, the Fibonacci numbers act as a kind of border, where the sum of the numbers between them gives Phi squared.

The explicit formula is:

$$\lim_{x \to inf} \frac{\sum_{n=Fibonacci(x)}^{Fibonacci(x+1))} n}{\sum_{n=Fibonacci(x-1))}^{Fibonacci(x))} n} = \phi^2$$

These subsets maintain a structure.

On the other hand it is an alternative method to find Phi decimals.

Other properties

In topology, it is a quotient of closed intervals with a point in common that is precisely the Fibonacci number that is in the middle of the three.

The quotient between the sum of the elements of the two closed sets forms fractions that are irreducible.

These ratios tend relatively slowly to Phi squared.

- As an example, the division between two Fibonacci numbers tends faster to Phi, for example fibonacci (16) / fibonacci (15) = 1.6180327 we see that it has 6 significant figures equal to Phi. However, the quotient of the sum of Fibonacci (15) to Fibonacci (16) is equal to 301833 if we divide it between the sum of Fibonacci (14) and Fibonacci (15) whose sum is 115479
We obtain :

301833/115479 = 2.61374

In this case we obtain only three significant figures of Phi square exactly half.

It would be possible that the two speeds of approach on one hand to Phi and on the other hand to Phi square by this method were related.

Discusión:

Esté método sirve para comprobar si hay algún tipo de desfase entre las cifras decimales de Phi y de Phi cuadrado y para cualquier otro tipo de relación que exista en la naturaleza teniendo en cuenta estos subconjuntos de números naturales.

Bibliography

https://en.wikipedia.org/wiki/Fibonacci_number