**Refutation of the exclusivity rule (as extended basis of the Born rule and free will theorem)**



We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, `F` as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal.

LET: p, q, r, s: *p*, *i*, *j*, *k*; ~ Not; + Or; & And; > Imply, greater than;
= Equivalent; % possibility, for one or some; # necessity, for all or every;
(%x>#x) ordinal 1; (x=x) `T`, proof.

From: Cabello, A. (2018). A simple explanation of Born's rule. arxiv.org/abs/1801.06347.

> "*The exclusivity (E) principle*. If every two events in a set are exclusive (i.e., if they are exclusive pairwise), then all the events in the set are mutually exclusive (i.e., they are exclusive globally).
>
> For example, if every two events in the set {*i, j, k*} are pairwise exclusive, then their graph of exclusivity is a triangle. If the E principle holds, then the possible probability assignments $\{p_i, p_j, p_k\}$ do not only have to satisfy

$p_i + p_j \leq 1$, (1.1)
$p_i + p_k \leq 1$, and (2.1)
$p_j + p_k \leq 1$, but also (3.1)
$p_i + p_j + p_k \leq 1$." (4.1)
"$p_i + p_j \leq 1$, $p_i + p_k \leq 1$, and $p_j + p_k \leq 1$, but also $p_i + p_j + p_k \leq 1$" (5.1)

~(((p&q)+(p&r))>(%s>#s)) = (s=s); `FFFC FCFC FFFC FCFC` (1.2)
~(((p&q)+(p&s))>(%r>#r)) = (r=r) ; `FFFC FFFC FCFC FCFC` (2.2)
~(((p&r)+(p&s))>(%q>#q)) = (q=q) ; `FFFF FCFC FCFC FCFC` (3.2)
~((((p&q)+(p&r))+(p&s))>(%p>#p)) = (p=p); `FFFC FCFC FCFC FCFC` (4.2)
(~(((p&q)+(p&r))>(%s>#s))&~(((p&q)+(p&s))>(%r>#r)))&
(~(((p&r)+(p&s))>(%q>#q))&~((((p&q)+(p&r))+(p&s))>(%p>#p))) ;
`FFFF FFFC FFFC FCFC` (5.2)

Eq. 5.2 as rendered is *not* tautologous. This means the exclusivity (E) principle is not a theorem. What is proved is something closer to a contradiction. For example, the author(s) could write "$p_i + p_j$ = proof, $p_i + p_k$ = proof, and $p_j + p_k$ = proof, but also $p_i + p_j + p_k$ = proof" in which case the table result of `FFFF FFFF FFFF FFFF` is forced into a contradiction. However, such liberties fly in the face of the intention of the rule which was to map probability as a theorem.

What follows is that if the exclusivity principle is refuted, then so are refuted the extended chain of subsequent assertions in the order of Born's rule and the free will thereon.

**Remark:** This is an example of the faulty mathematical logic which unfortunately peppers the quantum hypothesis field, beginning from about Gödel.