Gravity without Newton's Gravitational Constant and No Knowledge of Mass Size

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Abstract: In this paper, we show that the Schwarzschild radius can be extracted easily from any gravitationally-linked phenomena without having knowledge of Newton's gravitational constant or the mass size of the gravitational object. Further, the Schwarzschild radius can be used to predict any gravity phenomena accurately, again without knowledge of Newton's gravitational constant and also without knowledge of the size of the mass, although this may seem surprising at first.

Hidden within the Schwarzschild radius are the mass of the gravitational object, the Planck mass, and the Planck length, which we will assert contain the secret essence related to gravity, in addition to the speed of light, (the speed of gravity). This seems to indicate that gravity is quantized, even at the cosmological scale, and this quantization is directly linked to the Planck units. This also supports our view that Newton's gravitational constant is a universal composite constant of the form $G = \frac{l_p^2 c^3}{\hbar}$, rather than relying on the Planck units as a function of G. This does not mean that Newton's gravitational constant is not a universal constant, but rather that it is a composite universal constant, which depends on the Planck length, the speed of light, and the Planck constant.

This is, to our knowledge, the first paper that shows how a long series of major gravity predictions and measurements can be completed without any knowledge of the mass size of the object, or Newton's gravitational constant. As a minimum, we think it provides an interesting new angle for evaluating existing theories of gravitation, and it may even provide a hint on how to combine quantum gravity with Newton and Einstein gravity.

Key words: Schwarzschild radius, Planck mass, Planck length, measurement, gravitational constant, Heisenberg.

I. INTRODUCTION

Let's examine the following hypothetical formula:

$$r_e = \frac{g^2 R^2}{c^2} \tag{1}$$

Here, g is the gravitational acceleration, R is the radius from the center of the planet (gravitational mass) to the surface, and c is the speed of light. The gravitational acceleration is easy to measure without any knowledge of gravity; it is about 9.8 m/s^2. The radius of the Earth is about 6,371,000 meters. As for the speed of light, we can measure it with a low cost kit and or just take the standard accepted speed, which is defined as exactly 299,792,458 m/s. The main point is that one needs no knowledge of gravity to measure each of these input factors.

Next we plug these values into the formula above and get

$$r_e = \frac{9.8 \times 6371000^2}{299792458^2} \approx 0.00442588 \tag{2}$$

Some will recognize that this is very similar to half of the value of the Schwarzschild radius of the Earth; this is not a coincidence, as that is indeed exactly what it is. Next, we can use this value of r_e and plug it into any of the formulas below to calculate almost any major gravity predictions. We can predict the orbital velocity of a satellite or the Moon, for example, by the formula

$$v_o = c\sqrt{\frac{r_e}{R_o}} \tag{3}$$

where R_o is the radius from the center of the Earth to the object for which we want to predict orbital velocity. Further, the time dilation between two clocks at different altitudes around a planet is given by

$$\frac{T_h}{T_L} = \frac{\sqrt{1 - 2\frac{r_e}{R_L}}}{\sqrt{1 - 2\frac{r_e}{R_h}}} \tag{4}$$

where R_h is the radius further from the center of the Earth than R_L. We can test this by placing one atomic clock at sea level and one at the top of a 2,000-meter mountain. We need to synchronize the clocks before performing this task. The clocks will be consistent with our gravity prediction. Again, all we need is r_e, which we can easily extract from the gravitational acceleration on the surface of the Earth.

Next we can predict the red-shift; it is given by using the following formula

$$\lim_{r \to +\infty} z(r) = \frac{1}{\sqrt{1 - \frac{2r_e}{R}}} - 1 \tag{5}$$

If both the emitter and the receiver are inside the gravitational field and we focus on frequency rather than change in wavelength, we have the well-known formula

$$f_h = f_L \frac{\sqrt{1-\frac{r_e}{R_L}}}{\sqrt{1-\frac{r_e}{R_h}}} \tag{6}$$

Based on this, we can accurately predict the results of experimental set ups equal to the Pound Rebka experiment [1]. This is just one example of how we can perform a series of gravitational predictions that can be confirmed by experiment without any knowledge of Newton's gravitational constant or the mass size of any object. What we have relied on instead is r_e, which can simply be obtained from the gravitational acceleration at the surface of the Earth, the speed of light, and the radius of the Earth.

II. THE SCHWARZSCHILD RADIUS IN A NEW PERSPECTIVE

The Schwarzschild radius comes out from the Schwarzschild metric [2, 3] solution of the Einstein field equation [4] and is given by

$$r_s = \frac{2GM}{c^2} \tag{7}$$

where G is Newton's gravitational constant, M is the mass of the object, and c is the speed of light. In other words, we need to know the mass of the object of interest and Newton's gravitational constant in order to find its Schwarzschild radius. The escape velocity [5] from a mass M at the radius R from the center of the mass is given by

$$v_e = \sqrt{\frac{2GM}{R}} \tag{8}$$

When we replace the radius in the escape velocity with the Schwarzschild radius $r = r_s = \frac{2GM}{c^2}$, we get

$$v_e = \sqrt{\frac{2GM}{\frac{2GM}{c^2}}} = c \tag{9}$$

Thus, if an object with mass M is packed inside the Schwarzschild radius, then we will have a mass where even light cannot escape from inside the radius. This phenomenon is commonly known as a black hole, and the Schwarzschild radius is often linked to black holes.

It is important to be aware of the history of science on this topic. In 1783, the geologist John Michell noted that when the escape velocity calculated from Newton's gravitational theory was equal to the speed of light, the star would be dark [6]. That is, not even light could escape such a gravity-dense object. Since the escape velocities one gets from Newtonian theory and from general relativity theory are the same [5], this means that one can easily calculate a radius identical to the Schwarzschild radius from Newtonian theory. One could even argue that it should be called the "Michell radius," as he was the first to point out the possibility of an escape velocity equal to the speed of light and noted how this would lead to something special, dark stars.

However, the two theories differ strongly in their interpretation of what this special "dark" object is. In general, both theories agree that the gravitational force would be so strong that either only light or perhaps not even light could escape, while GR has a large theory around black holes. In this paper, we will not address the interpretation of black holes, but the background information is important because when we discuss the Schwarzschild radius, we are speaking about it in broad terms – there is a special radius that can be found from Newtonian-type gravity theory.

Any object we have observed directly in the sky or on Earth has mass where the radius is extending outside the Schwarzschild radius. In other words, no mass has directly been detected that has all of its mass inside the Schwarzschild radius, even though recent gravitational wave detections may have detected collisions of black holes.

What is important here is that Newton's gravitational constant, and not the mass of the cosmological or smaller-sized objects, must be known to find the Schwarzschild radius. The Schwarzschild radius can be found directly as described above from the gravitational acceleration of the Earth, or directly from the measured orbital velocity of a satellite such as the Moon by simply using the formula

$$r_s = 2r_e = 2\frac{v_o^2 R}{c^2} \tag{10}$$

where R is now the radius from the center of the Earth to the orbital object of interest. Further, v_o is the "easily" observed orbital velocity of the Moon, for example. Alternatively, we could use two atomic clocks, measure the time dilation between them, and then plug the values into this formula to find the Schwarzschild radius

$$r_e = R\sqrt{1-\frac{t_0^2}{T_f^2}} \tag{11}$$

where T_f is a clock far distant from the gravity field and T_0 is a clock placed at radius R relative to the gravitational object. Naturally, we do not have access to a far-away clock T_f from Earth, but we can certainly have two clocks on Earth at altitude R_L and R_h, and from this we can calculate r_e by solving the following equation with respect to r_e

$$\frac{T_h}{T_L} = \frac{\sqrt{1-2\frac{r_e}{R_L}}}{\sqrt{1-2\frac{r_e}{R_h}}} \tag{12}$$

this gives

$$r_e = \frac{R_h R_L (T_h^2 - T_L^2)}{2(R_L T_h^2 - R_h T_L^2)} \quad (13)$$

In 2016, Haug [7] suggested that the gravitational constant is likely a universal composite constant of the form

$$G = \frac{l_p^2 c^3}{\hbar} \quad (14)$$

Which is basically identical to a similar composite constant suggested by McCulloch [8]. This leads to an evaluation of the Schwarzschild radius at a deeper level by

$$r_s = \frac{GM}{c^2} = \frac{\frac{l_p^2 c^3}{\hbar} M}{c^2} = 2N l_p \quad (15)$$

where N is the number of Planck masses in the mass M. This is not new in itself, but the idea that we can find the Schwarzschild radius with no knowledge off G or even the mass and use this to predict "all" known gravity phenomena is quite new. For gravity phenomena the combined measurement of N and l_p is important; in this case, we do not need to know N (the number of Planck masses) or l_p separately, but the combination of the two, Nl_p, will be sufficient.

III. IS THE NEWTON GRAVITATIONAL CONSTANT A COMPOSITE?

There are several reason to question whether or not Newton's gravitational constant is a composite constant, including the following points:

1. If we "never" need Newton's gravitational constant for any gravitation observations, not even in calibrating a model, does this imply that it is not central for gravity either? See Table 1 for a series of calculations and observations that can be completed without any knowledge of Newton's gravitational constant. Further, if we want to separate the Planck units, we need to know the size of the mass of the gravitational object, otherwise that is not necessary. For all gravity predictions and phenomena in the table, we need no knowledge of the gravitational constant, or the size of the mass.

2. The output units of Newton's gravitational constant are given by $m^3 \cdot kg^{-1} \cdot s^{-2}$. It would seem strange if something fundamental existed at the deepest level that is meters cubed, divided by kg and seconds squared. It cannot be excluded, but one should first attempt to find a simpler explanation. We will claim this strongly indicates that Newton's gravitational constant must be a composite universal constant consisting of more fundamental constants.

3. By reformulating G as a composite of the form $G = \frac{l_p^2 c^3}{\hbar}$, a series of Planck units are simplified and become more logical. Take, for example, the Planck time, which is described as $t_p = \sqrt{\frac{G\hbar}{c^5}}$, such formulas give minimal intuition. We may ask, what is the meaning of c^5 and what is the deeper logic behind the gravitational constant? When replacing G with its composite form, we simply show the Planck time as $t_p = \frac{l_p}{c}$, so the time it takes for light to travel the Planck length, which is naturally well-known.

4. The Planck mass and the Planck length can be measured totally independent of any knowledge of Newton's gravitational constant, as recently shown by Haug [9**?** , 10]. This means the elements of a composite Newtonian gravitational constant all are known. At a fundamental level, it seems more logical that there exists a unique and likely shortest possible length, namely the Planck length, as well as the speed of light. In addition, we have the Planck constant, which is more complex, but in all observable gravity phenomena, the gravitational constant cancels out. Then we are left with the Schwarzschild radius (or half of this radius in many cases) as the essential thing to know and that can be measured easily. Again, this consists of the number of Planck masses times the Planck length in the gravitational object of interest.

IV. THE UNIQUNESS OF THE SCHWARZSCHILD RADIUS

To find Newton's gravitational constant in a Cavendish apparatus, we need to know the mass of the large lead balls first, in relation to their weight.

$$G = \frac{2L\pi^2 r^2 \theta}{MT^2} \quad (16)$$

where θ is the angle and L is the distance between the two small lead balls hanging in a wire, T is the oscillation time period, M is the mass of one of the two identical, largelead balls, and r is the radius from center to center between a small lead ball and a large lead ball. Next, in order to measure the Schwarzschild radius of the large lead ball, we only need the following formula

$$r_s = \frac{4L\pi^2 r^2 \theta}{c^2 T^2} \quad (17)$$

In this formula, there is no mass, but instead we have the speed of light. Also, remember $r_s =$

$2\frac{M}{m_p}l_p$. So embedded in the Schwarzschild radius is a mass ratio consisting of the mass of the gravitational object divided by the Planck mass multiplied by the Planck length.

V. COMPARING THE SCHWARZSCHILD RADIUS WITH THE GRAVITATIONAL CONSTANT

To find Newton's gravitational constant in a Cavendish apparatus, we need to know the mass of the large lead balls first, in relation to their weight. In our view, the reason for this is that the embedded gravitational constant also contains the reduced Planck constant, $G = \frac{l_p^2 c^3}{\hbar}$. Mass is clearly related to the Planck constant. The Schwarzschild radius, on the other hand, is just a length of the form $\frac{1}{2}r_s = Nl_p$, where N is a mass ratio, namely the mass of the gravitational object divided by the Planck mass. The Schwarzschild contains essential information about mass, while Newton's gravitational constant actually contains unnecessary information.

VI. CONCLUSION

We have shown how a long series of gravity predictions and measurements are totally independent of knowledge of the Newton gravitational constant, or the size of the mass in question. One component is (half) of the Schwarzschild radius, which is the number of Planck masses in the gravitational object multiplied by the Planck length. However, for most gravitational observations and predictions we do not need to reduce the Schwarzschild radius into these fundamental components.

We also show that we do not need any knowledge of the mass of the gravitational object or Newton's gravitational constant to find the Schwarzschild radius of a cosmological object, or even a small clump of matter on Earth. This strongly supports our view that Newton's gravitational constant is a composite constant of the form $G = \frac{l_p^2 c^3}{\hbar}$. In understanding this, we may gain a deeper understanding the link between the quantum world and the macroscopic world in terms of gravity. If our theory is correct, then we are able to perform a series of accurate gravity predictions based on measuring the gravitational acceleration on Earth alone, without any knowledge of G or the mass of the object.

[1] Needs further investigation and confirmation; see [11] for more details.

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TABLE I. The table shows that the most common gravitational measurements and predictions can be done without any knowledge of Newton's gravitational constant. Only when we want to separate out the Planck units or the gravitational constant do we need to know the mass size of the gravitational object.

What to measure/predict	**Formula**	**How**	**Is it easy to do**	**Knowledge of mass size**
Half Schwarzschild radius	$r_e = \frac{g^2R^2}{c^2}$	From g (9.8 m/s^2 Earth)	Yes	No
Half Schwarzschild radius	$r_e = \frac{v_o^2 R_o}{c^2}$	From orbital velocity	Yes	No
Half Schwarzschild radius	$r_e = R\sqrt{1 - \frac{T_0^2}{T_f^2}}$	From time dilation needs high precision clocks	Difficult need far away clock	No
Half Schwarzschild radius	$r_e = \frac{R_h R_L (T_h^2 - T_L^2)}{2(R_L T_h^2 - R_h T_L^2)}$	From time dilation needs high precision clocks	Yes	No
Half Schwarzschild radius	$r_e = \frac{R_h R_L (f_h^2 - f_L^2)}{2(R_L f_h^2 - R_h f_L^2)}$	From red-shift	Yes	No
Half Schwarzschild radius	$r_e = \frac{\delta R}{4}$	From light-bending	less so "need" eclipse	No
Gravitational acceleration field	$g = \frac{r_e}{R^2}c^2$	Find r_e first	Yes	No
Orbital velocity	$v_o = c\sqrt{\frac{r_e}{R}}$	Find r_e first	Yes	No
Escape velocity	$v_e = c\sqrt{2\frac{r_e}{R}}$	Find r_e first	Yes	No
Time dilation	$t_2 = t_1\sqrt{1 - 2\frac{r_e}{R}}$	Find r_e first	Yes	No
GR bending of light	$\delta = 4\frac{r_e}{R}$	Find r_e first	Yes	No
Gravitational red-shift	$\lim_{R\to+\infty} z(R) = \frac{r_e}{R}$	Find r_e first	Yes	No
Bekenstein-Hawking luminosity	$P = \frac{1}{15360\pi}\frac{\hbar c^2}{r_e^2}$	Find r_e first	Yes	No
Schwarzschild radius off the Cavendish sphere	$r_s = \frac{4L\pi^2R^2\theta}{c^2T^2}$	Cavendish apparatus	Yes	No
Planck mass	$m_p = \sqrt{\frac{\hbar c M T^2}{L2\pi^2R^2\theta}}$	Cavendish apparatus	Yes	Yes[a]
Planck length	$l_p = \sqrt{\frac{\hbar L2\pi^2R^2\theta}{MT^2c^3}}$	Cavendish apparatus	Yes	Yes[b]
Planck time	$t_p = \sqrt{\frac{\hbar L2\pi^2R^2\theta}{MT^2c^5}}$	Cavendish apparatus	Yes	Yes[c]
Gravitational constant	$G = \frac{\hbar c}{m_p^2} = \frac{l_p^2c^3}{\hbar} \approx 6.67 \times 10^{-11}$ $G = \frac{L2\pi^2R^2\theta}{MT^2}$	Cavendish apparatus	Yes	Yes[d]

[a] Needs to know the mass of the large lead ball in the Cavendish apparatus, can be done by simply weighing it.
[b] Same as footnote above.
[c] Same as footnote above.
[d] Same as footnote above.

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