Symmetry model E9CS

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Motivation 1:

Why do we consider the E9 group (more specifically the Coxeter element of this group)?
1) E9 is an affine group and thus has something to do with extension.
2) The extension is flat as the universe.
3) The key Coxeter element of the group produces symmetries involving our current standard model.

The fundamentals here:
https://en.wikipedia.org/wiki/Coxeter_group
https://de.wikipedia.org/wiki/Wurzelsystem
http://home.mathematik.uni-freiburg.de/soergel/Skripten/XXSPIEG.pdf

Symmetries which arise from the Coxeter element of the E9.

E9CS = SU(5) x SU(3) x SU(2) x U(1) x U(1)  \quad (SU(n) = \text{Special unitary group, } U(1) \text{ unitary group})

Pronounced E9Coxeter-Symmetry
evidently SU(5) x SU(3) x SU(2) x U(1) x U(1) \supset SU(3)c x SU(2)c x U(1)c x U(1)c \quad (\text{Color charge, isospin, Hyper charge})

Write the symmetry in order to:

E9CS = SU(5)s x U(1)y2 x U(1)y1 x SU(2)c x SU(3)c  \quad (=\text{Expansion x actual Standard Model})

Dynkin Diagram E9 (affine one point extension of group E8):

Derivative of the symmetries of E9CS from the invariants of the Coxeter elements E9:

The Coxeter element is the product of the generating reflections of E9.

Coxeterelement = e1.e2.e3.e4.e5.e6.e7.e8.e9

The Coxeterpolynom is the characteristic polynomial of Coxeter elements and has the form:

\[ E_9(x) = \prod_{i=1}^{9} \frac{x^2 - 1}{x - 1} \]

E9CS = SU(5)x SU(3) x SU(2) x U(1)x U(1)

E9(x) ... characteristic polynom of the coxeterelement of E9

E(x) is a polynom with terms of cyclotomic factors \[ E_n(x) = \frac{x^n - 1}{x - 1} \] for n\geq1 and (x-1) for n= 1.

The cyclotomic factors are the characteristic polynom of the An-1 (which is the Dynkin diagram for the SU(n) Liegroup.See more here: https://en.wikipedia.org/wiki/Special_unitary_group).

So finally the symmetry space of the Coxeterelement is SU(5) x SU(3) x SU(2) x U(1) x U(1)

Eigenvalues of the Coxetepolynomial

\[ \zeta_2 = e^{\frac{2\pi i}{3}} \]
\[ \zeta_3 = e^{\frac{2\pi i}{3}} \]
\[ \zeta_4 = e^{\frac{2\pi i}{3}} \]
\[ \zeta_5 = e^{\frac{2\pi i}{3}} \]
\[ \zeta_6 = e^{\frac{2\pi i}{3}} \]
\[ \zeta_7 = e^{\frac{2\pi i}{3}} \]
\[ \zeta_8 = e^{\frac{2\pi i}{3}} \]
\[ \zeta_9 = e^{\frac{2\pi i}{3}} \]

Eigenvalue of the Coxetepolynomial

\[ \mathbb{C}^4 \times \mathbb{C}^3 \times \mathbb{C}^1 \times \mathbb{C}^2 \]
Motivation 2:

What bring us the additional symmetries?

1. These have the potential to describe new particles.
2. These have the potential to describe the space and time.
3. These have the potential to describe gravity.

Wish to analogously represent Graviton to the photon as a blend (Weinberg angle see <8>).

<1> The Idea

Light and gravitation just like photon and graviton have something in common.
Both are massless and propagate with the speed of light.

We know that light by the symmetry breaking 1: SU(2)xU(1)→ U(1) is described as a mixture.
So light is a part of the electro-weak interactions.

we consider analog gravity as a result of a further symmetry breaking
Symmetry breaking 2: SU(5) x U(1) x U(1)→ U(1)
Our extended standard model allows us this.

We will now like to assign our relevant SU(n)'s to algebras division (real numbers, complex numbers, ...).

This 4 division algebras (real numbers, complex numbers, quaternions and octonions) develop through the doubling process
see more at https://de.wikipedia.org/wiki/Verdopplungsverfahren
Considering the dimensions of the SU(2) = 1, SU(3)= 2, SU(5) = 4 then this is double as well.

There appears to be a connection between the division algebras and the SU(n)'s (n = 2,3,5) which I hope is known in analytic geometry or another area.
I assume this connection warrants as simply as given.

Notes but no clear allocation can be found in this direction at Corinne A. Manogue and Tevian Dray, John Baez, etc.
Therefore, we rely analogously on the Higgsfield (2 x complex = doublet)

\[ \phi = \begin{pmatrix} \phi^+ \\ \phi^- \end{pmatrix} = \begin{pmatrix} \phi^+_1 + i \phi^+_2 \\ \phi^-_1 + i \phi^-_2 \end{pmatrix} \]

<2> the Oktoquintenfield (5 x Oktonions= Quintett).

\[ \phi = \begin{pmatrix} \phi^G \\ \phi^R \\ \phi^S \\ \phi^O \end{pmatrix} = \begin{pmatrix} \phi^{G0} + i \phi^{G1} + i \phi^{G2} + i \phi^{G3} + i \phi^{G4} + i \phi^{G5} + i \phi^{G6} + i \phi^{G7} \\ \phi^{R0} + i \phi^{R1} + i \phi^{R2} + i \phi^{R3} + i \phi^{R4} + i \phi^{R5} + i \phi^{R6} + i \phi^{R7} \\ \phi^{S0} + i \phi^{S1} + i \phi^{S2} + i \phi^{S3} + i \phi^{S4} + i \phi^{S5} + i \phi^{S6} + i \phi^{S7} \\ \phi^{O0} + i \phi^{O1} + i \phi^{O2} + i \phi^{O3} + i \phi^{O4} + i \phi^{O5} + i \phi^{O6} + i \phi^{O7} \end{pmatrix} \]

or written otherwise so that the equivalence to the Higgs field is clear (where i4 is pulled from)

\[ \phi = \begin{pmatrix} \phi^G \\ \phi^R \\ \phi^S \\ \phi^O \end{pmatrix} = \begin{pmatrix} \phi^{G0} + i \phi^{G1} + i \phi^{G2} + i \phi^{G3} + i \phi^{G4} + i \phi^{G5} + i \phi^{G6} + i \phi^{G7} \\ \phi^{R0} + i \phi^{R1} + i \phi^{R2} + i \phi^{R3} + i \phi^{R4} + i \phi^{R5} + i \phi^{R6} + i \phi^{R7} \\ \phi^{S0} + i \phi^{S1} + i \phi^{S2} + i \phi^{S3} + i \phi^{S4} + i \phi^{S5} + i \phi^{S6} + i \phi^{S7} \\ \phi^{O0} + i \phi^{O1} + i \phi^{O2} + i \phi^{O3} + i \phi^{O4} + i \phi^{O5} + i \phi^{O6} + i \phi^{O7} \end{pmatrix} \]

Comparison

Oktoquintenfield & Higgsfield

\[ \phi = \begin{pmatrix} \phi^+ \\ \phi^- \end{pmatrix} = \begin{pmatrix} \phi^+_1 + i \phi^+_2 \\ \phi^-_1 + i \phi^-_2 \end{pmatrix} \]

This provides 40 degrees of freedom.

24 of which will be “spent” for our SU(5) tensor bosons for the 5th longitudinal spin degree of freedom (24 Goldstone bosons swallowed over gauge transformation)
thus remain 16 left.

The S, F, R, G and H charges are the 5 charges of the SU (5) analogous to the 3 color charges of SU (3) and the 2 charges (+. -) of SU (2).
The letters stand for S = See, F = feeling, R=smelling G = Taste and H = Hear
Calling therefore the charges of the SU (5) sense charges.
Note: These charges have (such as the color charges of quarks with color) nothing to do with the senses, but to give a name to the child for reference only.

We now want to look at the 16 (40-24 = 16) remaining degrees of freedom.

Make the following division for the 40 field components of the Oktoquinten field as a physical approach:
Take care that the division is not unique because for the left half 4 gray fields we can use 4.3.2.1= 24 Permutations of them in the orange area.
And for the left 4 charges we have five over 4 = 5 Permutations.
So at all we have 5 x 24 = 120 possible permutations.
On the Higgsfield we have 2 x 1= 2 permutations.
Analogous to the Higgs potential we declare a Potential on the Oktoquintenfield

\[ V(\phi) = \frac{\gamma^2}{2} |\phi|^2 + \frac{\mu^2}{4} |\phi|^4 + \frac{\lambda^2}{8} |\phi|^6 \quad \text{with} \quad \phi \in \mathbb{D}^5 \]

\[ \gamma, \lambda \in \mathbb{R} \quad \text{(imaginary)} \quad \text{and} \quad \mu \in \mathbb{R} \]

\[ \frac{\gamma^2}{2} \ldots \text{momentum} \quad (\nu, \omega)^2 = \left( \frac{\hbar \gamma}{m \nu} \right)^2 \]

\[ \frac{\mu^2}{4} \ldots \text{mass} \quad (\nu, \omega)^2 = \left( \frac{\hbar \mu}{m \nu} \right)^2 \]

\[ \frac{\lambda^2}{8} \ldots \text{spin} \quad (\nu, \omega)^2 = \left( \frac{\hbar \lambda}{m \nu} \right)^2 \]

The coefficients of the potential comes from self interactions. Therefore we make the assumption that we have the following relation:

\[ C := \frac{-\mu^2}{\lambda^2} = \left( 1 + \frac{\gamma^2}{\mu^2} \right)^2 \quad \gamma^2, \lambda^2 < 0 \quad \mu^2 > 0 \]

Then it follows by exact calculation that

\[ C = c^2 v^2 \]

\[ c \ldots \text{speed of light} \]
\[ v \ldots \text{golden ratio} = 1.6180... \]

The first mixing angle which comes from the minimum of the Oktoquintenpotential is approx. equal to the WEINBERG – ANGLE \( \approx 28,89^\circ \)

For aesthetic reasons we want keep in mind that for the coming formulas \( \phi = \text{absolut}(\phi) \).
We want that the second part of the Oktoquintenpotential is our quadratic vacuumenergydensity.

\[
\frac{\rho_1^2}{4c^2} = \frac{1}{4} \left( \frac{\Lambda G}{8\pi} \right)^2 = \frac{1}{4} (\phi_{\text{vacuum}} c^2)^2
\]

\[\Lambda...,\text{cosmological constant}\]
\[\phi_{\text{vacuum}}...,\text{vacuum massdensity}\)

then with the relation \[\rho^2 = \frac{\mu^2}{\lambda^2} = (4\frac{\mu^2}{\lambda^2})^2\] we get the potential as

\[
E\text{INSTEIN - FORM}
\]

\[
V(\phi) = \left( \frac{\Lambda G}{8\pi} \right)^2 \left( -\frac{\mu^2}{\lambda^2} \right)^2 \left( \frac{\phi^4}{c^2} + 2\phi^2 \left( \frac{\phi}{c} \right)^4 - \left( \frac{\phi}{c} \right)^8 \right)
\]

**Lagrangedensity of the Oktoquintenfield/Oktoquintenpotential**

Hint:

I do the same steps as shown in this cooking recipe for the Higgsfield.

https://www.lsw.uni-heidelberg.de/users/mcamenzi/HD_Higgs.pdf

Analogous to the electroweak theory, we want to talk about a gravito super weak theory here. The electroweak theory brings together the electrical with the weak interactions. The gravito-sensecharge theory brings together the gravitational with the dark interactions.

**Similar to the SU(3) Vectorbosons which are named Gluons we name our SU(5) Tensorbosons Repletions.**

The force between different charged Repletions is repulsive because they are tensorbosons (2nd - order).

**Similar to the Higgsfield we assign our Repletions to the Oktoquintenfield by the following scheme.**

The numbers are the sense charges (see < 2 >).

5 = See
4 = Feeling
3 = Smelling
2 = Taste
1 = Hear

\[
\begin{pmatrix}
\begin{array}{cccc}
\end{array}
\end{pmatrix}
\]

similar to the the higgsfield where the vacuumexpectation is

\[\phi_{\text{vac}} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \]

the vacuumexpectation of the Oktoquintenfield is (green, yellow, orange)

\[\phi_{\text{vac}} = v. \begin{pmatrix} 0 + i_1 + i_2 + i_3 \\ 1 + 0 + i_2 + i_3 \\ 1 + i_1 + 0 + i_3 \\ 1 + i_1 + i_2 + 0 \\ 1 + i_1 + i_2 + i_3 \end{pmatrix} \]

where \(v = \epsilon_1\) ist the minimum of the Oktoquintenpotential and \(i_1, i_2\) and \(i_3\) the imaginary quaternions.
STEP 1: Lorentz-invariant Lagrangian density for the Oktoquintenfield

$$\mathcal{L}_\phi = (D^\mu \phi)^\dagger (D_\mu \phi) - V(\phi^\dagger \phi)$$

with $\phi \in \mathcal{Q}^5$ Octonions

The potential $V$ is shown in <3>

$$\bar{\mathcal{T}}_{ij}^{\dagger}$$

Generators of the SU(5)

$$\begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 \\
\end{pmatrix} \qquad \begin{pmatrix}
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 \\
\end{pmatrix} \qquad \begin{pmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
\end{pmatrix} \qquad \begin{pmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 \\
\end{pmatrix} \qquad \begin{pmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 \\
\end{pmatrix} \qquad \begin{pmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 \\
\end{pmatrix}
$$

$W$ Boson scheme

$$\begin{pmatrix}
W^{11} & W^{12} & W^{13} & W^{14} & W^{15} \\
W^{21} & W^{22} & W^{23} & W^{24} & W^{25} \\
W^{31} & W^{32} & W^{33} & W^{34} & W^{35} \\
W^{41} & W^{42} & W^{43} & W^{44} & W^{45} \\
W^{51} & W^{52} & W^{53} & W^{54} & W^{55} \\
\end{pmatrix}$$

hint: $W^{ij} = W^{ji}$

We take a look on the symmetry

$$SU(5) \times U(1) \times U(1)$$

$W^{0}$ $B^0$ $B^1$

calculate covariant derivative

$$D_\mu \phi = (\partial_\mu + \frac{i}{2} \tau_{ij} W^0_{ij} + \frac{i}{2} \frac{g_\mu}{\sqrt{2}} - i d^a \Phi^a_{\mu} + \frac{i}{2} g_\mu c_0 H). \phi$$

$$\tau_{ij}.W^0_{ij} = \begin{pmatrix}
W^{11} & W^{12} & W^{13} & W^{14} & W^{15} \\
W^{21} & W^{22} & W^{23} & W^{24} & W^{25} \\
W^{31} & W^{32} & W^{33} & W^{34} & W^{35} \\
W^{41} & W^{42} & W^{43} & W^{44} & W^{45} \\
W^{51} & W^{52} & W^{53} & W^{54} & W^{55} \\
\end{pmatrix}$$

$$\tau_{ij}.W^1_{ij} = \begin{pmatrix}
W^{11} + i W^{12} & W^{11} + i W^{13} & W^{11} + i W^{14} & W^{11} + i W^{15} \\
W^{21} + i W^{22} & W^{21} + i W^{23} & W^{21} + i W^{24} & W^{21} + i W^{25} \\
W^{31} + i W^{32} & W^{31} + i W^{33} & W^{31} + i W^{34} & W^{31} + i W^{35} \\
W^{41} + i W^{42} & W^{41} + i W^{43} & W^{41} + i W^{44} & W^{41} + i W^{45} \\
W^{51} + i W^{52} & W^{51} + i W^{53} & W^{51} + i W^{54} & W^{51} + i W^{55} \\
\end{pmatrix}$$

$$\tau_{ij}.W^2_{ij} = \begin{pmatrix}
W^{11} - i W^{12} & W^{11} - i W^{13} & W^{11} - i W^{14} & W^{11} - i W^{15} \\
W^{21} - i W^{22} & W^{21} - i W^{23} & W^{21} - i W^{24} & W^{21} - i W^{25} \\
W^{31} - i W^{32} & W^{31} - i W^{33} & W^{31} - i W^{34} & W^{31} - i W^{35} \\
W^{41} - i W^{42} & W^{41} - i W^{43} & W^{41} - i W^{44} & W^{41} - i W^{45} \\
W^{51} - i W^{52} & W^{51} - i W^{53} & W^{51} - i W^{54} & W^{51} - i W^{55} \\
\end{pmatrix}$$

$$\tau_{ij}.W^3_{ij} = \begin{pmatrix}
W^{11} + W^{12} & W^{11} + W^{13} & W^{11} + W^{14} & W^{11} + W^{15} \\
W^{21} + W^{22} & W^{21} + W^{23} & W^{21} + W^{24} & W^{21} + W^{25} \\
W^{31} + W^{32} & W^{31} + W^{33} & W^{31} + W^{34} & W^{31} + W^{35} \\
W^{41} + W^{42} & W^{41} + W^{43} & W^{41} + W^{44} & W^{41} + W^{45} \\
W^{51} + W^{52} & W^{51} + W^{53} & W^{51} + W^{54} & W^{51} + W^{55} \\
\end{pmatrix}$$

and for example

$$W_{12} = \frac{W^{12} + W^{21}}{\sqrt{2}}$$

The boson which changes the charge from 1 (bear) to 2 (taste).
Then

\[ D_{\mu} \phi_{\text{vac}} = -\frac{\phi_{\text{vac}}}{2} \begin{pmatrix} g^{\text{I}} \begin{pmatrix} B^0_0 + g^0_0 \cdot B^1_0 & 0 & 0 & 0 & 0 \\ 0 & B^0_0 + g^0_0 \cdot B^1_0 & 0 & 0 & 0 \\ 0 & 0 & B^0_0 + g^0_0 \cdot B^1_0 & 0 & 0 \\ 0 & 0 & 0 & B^0_0 + g^0_0 \cdot B^1_0 & 0 \\ 0 & 0 & 0 & 0 & B^0_0 + g^0_0 \cdot B^1_0 \end{pmatrix} \end{pmatrix} \begin{pmatrix} 1 + i_1 + i_2 + i_3 \\ 1 + i_1 + i_2 + i_3 \\ 1 + i_1 + i_2 + i_3 \\ 1 + i_1 + i_2 + i_3 \end{pmatrix} \]

then

\[(D_{\mu} \phi_{\text{vac}})(D_{\nu} \phi_{\text{vac}}) = -\frac{\phi_{\text{vac}}}{4} \begin{pmatrix} 4(1 + g^0_0) & 4(1 + g^0_0) & 4(1 + g^0_0) & 4(1 + g^0_0) \\ 4(1 + g^0_0) & 4(1 + g^0_0) & 4(1 + g^0_0) & 4(1 + g^0_0) \\ 4(1 + g^0_0) & 4(1 + g^0_0) & 4(1 + g^0_0) & 4(1 + g^0_0) \\ 4(1 + g^0_0) & 4(1 + g^0_0) & 4(1 + g^0_0) & 4(1 + g^0_0) \end{pmatrix} + \text{something} + \text{something} + \text{something} + \text{something} \]

\[ W^{ij} = W^i_\mu \text{ and } B^0 = B^0_\mu \text{ and } B^1 = B^1_\mu \]

and the result of the Higgs field we expect something like that :

\[(D_{\mu} \phi_{\text{vac}})(D_{\nu} \phi_{\text{vac}}) = \frac{e^2}{8} \left( g^2 (W^+)^2 + g^3 (W^-)^2 + (g^2 + g^3) Z_\mu Z^\mu + 0.4 \cdots \right) \]

We have a lot of summands so we first want to take a look on the diagonal elements of the covariant derivation.

In the Higgsfieldtheory we get as result the massive Z – Bosons and the Photon as a mixing of neutral W and B bosons.

We calculate the expression which is a symmetric bilinear form:

\[ \begin{pmatrix} W^{01} & W^{02} & W^{03} & W^{04} & B^0 \end{pmatrix} \begin{pmatrix} 8g^2 & 4g^2 & 4g^2 & 4g^2 & 4g^2 \\ 4g^2 & 8g^2 & 4g^2 & 4g^2 & 4g^2 \\ 4g^2 & 4g^2 & 8g^2 & 4g^2 & 4g^2 \\ 4g^2 & 4g^2 & 4g^2 & 8g^2 & 4g^2 \\ 4g^2 & 4g^2 & 4g^2 & 4g^2 & 8g^2 \end{pmatrix} \begin{pmatrix} 0 \\ 20g^2 \end{pmatrix} \text{ linearly independent} \]

and compare it with the red area of the dynamic lagrange part.

Someone can easy proof that is identical.

Then with diagonalizing the Momentundensity – Matrix we get the following result:

\[(D_{\mu} \phi_{\text{vac}})(D_{\nu} \phi_{\text{vac}}) = \phi_{\text{vac}} \begin{pmatrix} 20g^2 + g^2 \end{pmatrix} + \text{something} \]

**Momentundensity Quadrats**
The coupling angles $\alpha_1$ and $\alpha_2$ comes from the extremal values of the Octoquintenpotential.

As calculated in $<8>$ we have:

$$SU(5) \times SU(3) \times SU(2) \times U(1) \times U(1)$$

\[ \theta_W \text{ Light} \]
\[ \theta_K \text{ Gravitation} \]

Mixing angles:

$$\theta_W \approx \alpha_1$$  \hspace{1cm}  ($\theta_W$ = Weinberg angle)

$$\theta_K \approx 29,57^\circ$$

$$\alpha_1 \approx 28,89^\circ$$

$$\alpha_2 \approx 121,6^\circ$$

$$g' = R \sin(\alpha_1)$$

$$g'' = R \sin(\alpha_2)$$

Coupling constants $g'$, $g''$

$\theta_K$ - Weinberg angle

$m$ - mass density

$G$ = U(1)  

Gravitation field.

Then the Gravitation and the $\Gamma$ - Boson is a mixing:

$$\begin{pmatrix} G_H \\ G_V \end{pmatrix} = \begin{pmatrix} \cos(\theta_K) & -\sin(\theta_K) \\ \sin(\theta_K) & \cos(\theta_K) \end{pmatrix} \cdot \begin{pmatrix} B^0 \\ B_\mu \end{pmatrix}$$

The Impulse density and therefore the mass density and therefore the mass of the $W$ and $Z$ Bosons are equal because they have the same coupling constant $g$.

The relation of the $\Gamma$ particle mass to the $W$ - Boson mass is:

$$\frac{M_\Gamma}{M_W} = \frac{20(g^2 + g''^2)}{8g^2} = \frac{5(g^2 + g''^2)}{2g^2}$$

We develop around the vacuum expectation $\phi_{vac}$ to see the interaction terms.

$$\phi = \frac{\phi_{vac}}{c} (c + H)$$

$$\begin{pmatrix} 1 + i_1 + i_2 + i_3 \\ 1 + i_1 + i_2 + i_3 \\ 1 + i_1 + i_2 + i_3 \\ 1 + i_1 + i_2 + i_3 \end{pmatrix}$$
then with \( \frac{\gamma \omega}{c} = \beta \) and \( c = \text{speed of light} \)

\[
(D^a \phi)(D_a \phi) = 2 \beta^2 (\partial^a (\partial_a H)) (\partial_a H) + \\
2 \beta^2 \gamma^2 (|W_{12}|^2 + |W_{21}|^2 + ... ) + \quad W - \text{Boson Momentumdensity} \\
2 \beta^2 \gamma^2 (|Z_0|^2 + |Z_1|^2 + |Z_2|^2 + |Z_3|^2) + \quad Z - \text{Boson Momentumdensity} \\
2 \beta^2 \gamma^2 (2 \gamma H + H^2) (W_{12} W_{21} + ...) + \quad H - W \text{ Interaction} \\
2 \beta^2 \gamma^2 (2 \gamma H + H^2) (Z_0 + Z_1 Z_1 + Z_2 Z_2 + Z_3 Z_3) + \quad H - Z \text{ Interaction} \\
5 \beta^2 (g^2 + \gamma^2) (e^2 + 2 \gamma H + H^2) |\Gamma|^2 \\
\Gamma - \text{Boson Momentumdensity} \quad H - \Gamma \text{ Interaction}
\]

In our Lagrange we have massdensities instead of mass!
So how can we talk about particles with mass?

One possibility is to take a look on Planckunits.
More concrete the W–Bosons could be Planckparticles with some special properties.

- Planckparticles have Planckmass
- Planckparticles are something like the smallest Blackhole

What can we say about the geometrical shape of such a particle?
We assume that a Clifford–Torus is the right object and will proof this.
Then it follows that a Planckparticle is an object with dimension = 2.
This explains the holographical principle because a Planckparticle then is a two – dimensional object.

The Octogonpotential has two (classes of) nontrivial zero points on \(|\phi| = c\) and \(|\phi| = c \sqrt{\phi}\) with \(\phi\). ... golden ratio and \(c\) speed of light.
This means the Energydensity there is zero!
Let us assume that nature prefers this two states.
The unit of \(\phi\) is speed.

Multiplying it with \(\frac{c}{G \hbar}\) results a curvature \(\phi \sqrt{\frac{c}{G \hbar}} = \frac{1}{r_0}\)
Then for \(\phi = c\) we get as curvature

\[
\sqrt{\frac{c^2}{G \hbar}} = \frac{1}{r_0} \text{ and for } \phi = c \sqrt{\phi}
\]

With this two radii \(r_p\) and \(r'_p\) we want to define a special
Clifford – Torus.

A Clifford – Torus is defined as \(\mathbb{T}^2 = S^1_a \times S^1_b \subset S^3_{\sqrt{a^2 + b^2}} \subset \mathbb{R}^4\) where
\(S^1 = 1 – \text{Sphere and } a, b \text{ are the radii of the spheres}.\)
Then we write our special Clifford – Torus as

\(\mathbb{T}^2_p = S^1_a \times S^1_b_{r_p}\)

and name it Planck – Torus
Then the Planck – Torus \(\mathbb{T}^2_p\) lays in \(S^3_{\sqrt{a^2 + b^2}} = S^1_a \times S^1_b_{r_p}\)
This is the 3 – Sphere with radius \(r_p \sqrt{\phi}\)
Motivated by our results in \(<6\) we are thinking about two such Planck – Tori

\(T^2_{\text{ps}} \text{ and } T^2_{\text{pd}}\)

Visual

\[
\begin{align*}
\begin{array}{c}
\text{Time} \rightarrow \\
\text{Spin} \\
\text{UP} \\
\text{DOWN}
\end{array}
\end{align*}
\]

To generate an 2 – dimensional object with spin = 2 we have to connect the coordinate cycles like in \(<6.4\)>.

Schematic of our second curvature tensor for the double Clifford torus

Shear – Components

\[
W_{\text{Boson}} = \begin{pmatrix}
\begin{array}{cccc}
\bullet & \bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet & \bullet \\
\end{array}
\end{pmatrix} \subset S^3_{r,\sqrt{5}}
\]

This construction has 3 different Tori \(\subset S^3_{r,\sqrt{5}}\)

<table>
<thead>
<tr>
<th>(T^2_{\text{ps}})</th>
<th>(T^2_{\text{pd}})</th>
<th>(T^2_{\text{ps}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(r = t_p)</td>
<td>(r = \frac{t_p}{\sqrt{5}})</td>
<td>(r = t_p\sqrt{\frac{5}{2}})</td>
</tr>
<tr>
<td>(r = \frac{t_p}{\sqrt{5}})</td>
<td>(r = \frac{t_p}{\sqrt{5}})</td>
<td>(r = \frac{t_p}{\sqrt{5}})</td>
</tr>
</tbody>
</table>

The \(W_{\text{Boson}}\) has Planck mass then it follows that the frequency is:

\[
\omega_p = \frac{1}{t_p} = \sqrt{\frac{\hbar c}{G}}
\]

Resting \(W_{\text{Boson}}\):

\[
W_{\text{Boson}} = \begin{pmatrix}
\begin{array}{cccc}
\frac{t_p}{t_p}e^{-it_p} & 0 & 0 & \frac{t_p}{t_p}e^{-it_p} \\
0 & \frac{t_p}{\sqrt{5}}e^{it_p} & \frac{t_p}{t_p}e^{-it_p} & 0 \\
0 & \frac{t_p}{t_p}e^{it_p} & \frac{t_p}{\sqrt{5}}e^{-it_p} & 0 \\
\frac{t_p}{\sqrt{5}}e^{-it_p} & 0 & 0 & \frac{t_p}{t_p}e^{-it_p}
\end{array}
\end{pmatrix}
\]

Properties of \(W_{\text{Boson}}\):

1) is a Tensor boson
2) lies in a 3 – Sphere
3) is build by 3 Clifford Tori
4) Is a flat two dimensional object
We set $l_p = 1$ for easier handling.

The class of all Clifford – Tori in $S^3_{\sqrt{\varphi}}$ then is

$$C_a := \{z_1, z_2 \in \mathbb{C}^2 : |z_1| = \sqrt{\frac{1 + a}{2}}, |z_2| = \sqrt{\frac{1 - a}{2}}\}$$

for a real parameter $a \in (-1, 1)$. This is an embedded surface in $S^3_{\sqrt{\varphi}}$ with mean curvature constant equal to

$$H_a := \frac{2a}{\sqrt{\varphi} \cdot \sqrt{1 - a^2}}$$

Then our three Clifford – Tori $T^2_{pu}$, $T^2_{pa}$ and $T^2_{ps}$ can be written as

$$T^2_{pu} = C_a \text{ with } a = \frac{2 - \sqrt{\varphi}}{\varphi} \text{ and }$$

$$T^2_{pa} = C_{-a} \text{ and }$$

$$T^2_{ps} = C_0$$

Then the mean curvature of $T^2_{pu}$ is

$$H_a = H_{\frac{2 - \sqrt{\varphi}}{\varphi}}$$

and the mean curvature of $T^2_{pa}$ is

$$H_{-a} = H_{\frac{2 - \sqrt{\varphi}}{-\varphi}} = -H_a$$

and the mean curvature of $T^2_{ps}$ is

$$H_0 = 0$$

The shape of the W – Boson then is:

| topology $W_{Boson} = \{C_{\frac{2 - \sqrt{\varphi}}{\varphi}}, C_0, C_{\frac{\sqrt{\varphi} - 2}{\varphi}}\} \subset S^3_{\sqrt{\varphi}}$ |

<5> Curvature tensors by the Oktoquintenfield

The construction comes from multiplications (symmetric to the diagonal) by 2 degrees of freedom (complex subspaces). With this construction the tensor is symmetric in the diagonal.
10 independent fields.

Remark:
for $\phi = c$ we get as curvature the plank curvature
which is the reciprocal of the plank area.
The value of the curvature is:

$0.34 \times 10^{-70} \frac{1}{m^2}$

Second curvature tensor from the oktoquintenfield (generates a spinpotential).
The construction comes from multiplications by 4 degrees of freedome (quatemionic subspaces).
With this construction the tensor is symmetric in both diagonals.

$C_{em} = \frac{c}{G \hbar c}$

$5$ independent fields $A, B, C$ and two in the diagonal (blue and yellow).
So finally we get three derivation- or curvature tensors of the Oktoquintenpotential for twisted spacetime excitation.

The Oktoquintenpotential has two symmetric curvature tensors. This motivates us to extend the Einstein Equation.

I think this shows that the GR (General Relativity) has to be extended by an imaginary part (spin part) to be a consistent quantum theory.

So finally we expect something like $\text{GR} + i \text{GR}^\phi$ where $\text{GR}^\phi$ is the spin part.

with the two curvature tensors $C_{em}$ and $C_{spin}$ we can define following equation:

$$\text{Real}(C_{em}) + i \frac{\psi \pi G}{c^4} (T_{\mu \nu} + \frac{i}{\psi} S_{\mu \nu})$$

where the real part is the GR and the imaginary part is $\text{GR}^\phi$.

GR...General Relativity
$\text{GR}^\phi$...Spin extension of GR
$\psi$...golden ratio

<6> Extension of the ART by the second curvature tensor
the operator Real(A) is defined by

\[
\text{Real}\begin{pmatrix}
 a_{0,0} & t_1 a_{0,1} & t_2 a_{0,2} & t_3 a_{0,3} \\
 t_1 a_{1,0} & a_{1,1} & t_2 a_{1,2} & t_3 a_{1,3} \\
 t_2 a_{2,0} & t_3 a_{2,1} & a_{2,2} & t_3 a_{2,3} \\
 t_3 a_{3,0} & t_2 a_{3,1} & t_1 a_{3,2} & a_{3,3}
\end{pmatrix} = \begin{pmatrix}
 a_{0,0} & a_{0,1} & a_{0,2} & a_{0,3} \\
 a_{1,0} & a_{1,1} & a_{1,2} & a_{1,3} \\
 a_{2,0} & a_{2,1} & a_{2,2} & a_{2,3} \\
 a_{3,0} & a_{3,1} & a_{3,2} & a_{3,3}
\end{pmatrix}
\]

the reversing Real^{-1} is:

\[
\text{Real}^{-1}\begin{pmatrix}
 a_{0,0} & a_{0,1} & a_{0,2} & a_{0,3} \\
 a_{1,0} & a_{1,1} & a_{1,2} & a_{1,3} \\
 a_{2,0} & a_{2,1} & a_{2,2} & a_{2,3} \\
 a_{3,0} & a_{3,1} & a_{3,2} & a_{3,3}
\end{pmatrix} = \begin{pmatrix}
 a_{0,0} & t_1 a_{0,1} & t_2 a_{0,2} & t_3 a_{0,3} \\
 t_1 a_{1,0} & a_{1,1} & t_2 a_{1,2} & t_3 a_{1,3} \\
 t_2 a_{2,0} & t_3 a_{2,1} & a_{2,2} & t_3 a_{2,3} \\
 t_3 a_{3,0} & t_2 a_{3,1} & t_1 a_{3,2} & a_{3,3}
\end{pmatrix}
\]

\(i_1, i_2, i_3, \ldots \text{imaginary quaternions}\)

more detailed with the two curvature tensors of the oktoquintenfield:

\[
\frac{8 \pi G}{c^2} T_{\mu \nu} + \frac{i}{\varphi} C_{\mu \nu} = \text{Real}(C_{em}) + \frac{i}{\varphi} C_{spin} - \frac{c}{G \hbar} \left[ \begin{array}{cccc}
\gamma^0 \delta^1 & \gamma^0 \delta^2 & \gamma^0 \delta^3 & \gamma^0 \\
\gamma^1 \delta^0 & \gamma^2 \delta^0 & \gamma^3 \delta^0 & \gamma^4 \\
\gamma^2 \delta^1 & \gamma^3 \delta^1 & \gamma^4 \delta^1 & \gamma^5 \\
\gamma^3 \delta^2 & \gamma^4 \delta^2 & \gamma^5 \delta^2 & \gamma^6
\end{array} \right] + \frac{i}{e^2 \varphi} \left[ \begin{array}{cccc}
\gamma^0 \delta^0 & \gamma^1 \delta^0 & \gamma^2 \delta^0 & \gamma^3 \delta^0 \\
\gamma^1 \delta^1 & \gamma^2 \delta^1 & \gamma^3 \delta^1 & \gamma^4 \delta^1 \\
\gamma^2 \delta^2 & \gamma^3 \delta^2 & \gamma^4 \delta^2 & \gamma^5 \delta^2 \\
\gamma^3 \delta^3 & \gamma^4 \delta^3 & \gamma^5 \delta^3 & \gamma^6 \delta^3
\end{array} \right]
\]

10 different products

\(\begin{array}{c}
\text{sym.} \\
\text{sym.} \\
\text{sym.}
\end{array} \)

5 different products

\(\begin{array}{c}
\text{sym.} \\
\text{sym.} \\
\text{sym.}
\end{array} \)

\(\text{sym. and the third products are redundant}\)

generates Poincare group \(\mathbb{R}^{1,3} \times O(1,3)\)

(2 \times S^1 \times S^1 \times SU(2) = 2 \times T^2 \times SU(2)\) with \(T^2 = \text{Torus}\)

\(\mathbb{R}^3 \text{ Clifford-Torus}\)

This flat torus is a subset of the unit 3 - sphere \(S^3\).
The Clifford torus divides the 3 - sphere into two congruent solid tori.
The Clifford - Torus embedded in \(S^3\) becomes a minimal surface.

The second curvature tensor \(C_{\text{spin}}\) is determined by the first curvature tensor \(C_{em}\) because its components are a mix of the components of \(C_{em}\).

The vacuum part of the extended Einstein equation then is:

\(\text{Vacuum} = A g + i \Lambda^\nu g^\nu\)

\(\Lambda\ldots\text{cosmological constant}\)

\(\Lambda', \ldots\text{second cosmological constant}\)

\(\Lambda\) and \(\Lambda'\) comes from the Okt Quinteinpotential (see picture).

\(g = \text{comes from the first curvature tensor}\)

\(g^\nu - \psi \text{ comes from the second curvature tensor}\)

then

The vacuum energy density is

\[
\text{Vacuum Energeticity} = \frac{c^4}{8 \pi G} \left[ \Lambda + i \Lambda' \right] + \left[ \begin{array}{c}
\gamma^0 \\
\gamma^1 \\
\gamma^2 \\
\gamma^3
\end{array} \right] \left[ \begin{array}{c}
-1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & -1
\end{array} \right] \left[ \begin{array}{c}
\gamma^0 \\
\gamma^1 \\
\gamma^2 \\
\gamma^3
\end{array} \right] = \frac{c^4}{8 \pi G} \left[ \begin{array}{c}
\gamma^0 \\
\gamma^1 \\
\gamma^2 \\
\gamma^3
\end{array} \right] \left[ \begin{array}{c}
\gamma^0 \\
\gamma^1 \\
\gamma^2 \\
\gamma^3
\end{array} \right]
\]

\(\begin{array}{c}
\gamma^0 \\
\gamma^1 \\
\gamma^2 \\
\gamma^3
\end{array} \)

\(\gamma^0 \delta^0 = \text{generates flat expanding spacetime}\)

\(\gamma^1 \delta^1 = \text{generates } Z \times \text{ spinning Torus } \mathbb{T}_{2} = S^1 \times S^1\)

\(\mathbb{T}^2 = \text{ Flat Clifford-Torus}\)

\(\gamma^2 \delta^2 = \text{generates flat twisting and expanding vacuum}\)

\(\gamma^3 \delta^3 = \text{generates flat twisting and expanding vacuum}\)

Hint: \(\Lambda'\) comes from the third part of the Okt Quinteinpotential.

\(\Lambda' = \frac{\Lambda}{\varphi} \text{ with } \varphi \ldots\text{golden ratio}\)

\[
\text{Dirac-Spinor} = \left( \begin{array}{c}
\gamma^0 \\
\gamma^1 \\
\gamma^2 \\
\gamma^3
\end{array} \right) \text{ Weyl-Spinor} = \left( \begin{array}{c}
\gamma^0 + \gamma^1 \\
\gamma^0 - \gamma^1 \\
\gamma^2 + \gamma^3 \\
\gamma^2 - \gamma^3
\end{array} \right) \text{ or } \left( \begin{array}{c}
\gamma^0 \\
\gamma^1 \\
\gamma^2 \\
\gamma^3
\end{array} \right)
The question now is how does the spin of particles act on the second curvature tensor? The second curvature tensor has 5 different excitability values. 

The complete curvature Tensor for a free Spin 0 particle:

\[
\hat{f}(t) = R_{\alpha \beta} e^{\gamma \tau \omega} = R_{\alpha \beta} e^{\gamma \tau \omega}
\]

Example free Diracparticle

First we want to rearrange the Diracspinor by a matrix. This makes the relation between Diracparticles like electrons and the curvature tensors better visible.

\[
\psi_D = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \psi_D \quad \text{Dirac Spinor} \quad \begin{pmatrix} \psi_{\downarrow} \\ \psi_{\uparrow} \end{pmatrix}
\]

\[
\begin{pmatrix} \psi_{\uparrow} \\ \psi_{\downarrow} \end{pmatrix} \quad \text{Spin UP} \quad \begin{pmatrix} \psi_{\downarrow} \\ \psi_{\uparrow} \end{pmatrix} \quad \text{Spin DOWN}
\]

<6.1> The complete curvature Tensor for a free Spin 0 particle:

\[
\frac{8\pi G}{c^3} \left( T_{\alpha \beta} + \frac{i}{\sqrt{2}} S_{\alpha \beta} \right) = \frac{c}{G \hbar} \left( \begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right) + \frac{i}{\sqrt{2} \hbar} \left( \begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right)
\]

<6.2> The complete curvature Tensor for a free standstill Electron is:
6.3 The complete curvature Tensor for a Photon (massless vectorboson) is:

\[
\frac{8\pi G}{c^4} (T_{\mu\nu} + \frac{i}{c} S_{\mu\nu}) = \frac{e}{G h} \left[ \begin{array}{cccc}
\omega_1^2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & \omega_2^2 & 0 \\
0 & 0 & 0 & \omega_3^2
\end{array} \right] + \frac{i}{c^2 \phi} \left[ \begin{array}{cccc}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{array} \right]
\]

6.4 The complete curvature Tensor for a Graviton (massless tensorboson) is:

\[
\frac{8\pi G}{c^4} (T_{\mu\nu} + \frac{i}{c} S_{\mu\nu}) = \frac{e}{G h} \left[ \begin{array}{cccc}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\omega_1^2 & 0 & 0 & 0 \\
\omega_2^2 & 0 & 0 & 0
\end{array} \right] + \frac{i}{c^2 \phi} \left[ \begin{array}{cccc}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{array} \right]
\]
Getting a closed form for the extended General Relativity EGR.

we know that our energy – momentum curvature tensor

$$\text{Real}(C_{\text{em}}) = R_{\mu \nu} - \frac{R}{2} g_{\mu \nu} + \Lambda g_{\mu \nu}$$

and that

$C_{\text{spin}}$ is defined by multiplication of tensor elements of $C_{\text{em}}$.

The question now is how can we express $C_{\text{spin}}$ analogous to $C_{\text{em}}$ above as terms of Riemann – Geometric?

For that we define the operator for 4 x 4 matrices or tensors:

$$\mathcal{T} = T + T^\dagger$$

as the transpose in the big diagonal and then in the small diagonals from top right to bottom left:

$$\begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{pmatrix} \rightarrow \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{pmatrix} \rightarrow \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{pmatrix}$$

and a

special simple matrices multiplication

$$C = A \cdot B$$

with

$$c_{ij} = \begin{cases} 
+ a_{i,j} b_{j,i} & \text{if } i = j \\
-a_{j,i} b_{i,j} & \text{if } i \neq j
\end{cases}$$

then it is easy to see that

$$(A + B) = A + B$$

with

$$C_{\text{em}} = \frac{c}{G \, h \, c} \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{pmatrix} \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{pmatrix} \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{pmatrix} \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{pmatrix}$$

and

$$C_{\text{spin}} = \frac{1}{G \, h \, c^2} \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{pmatrix} \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{pmatrix} \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{pmatrix} \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{pmatrix}$$

it follows that

$$C_{\text{spin}} = \frac{G \, h \, c}{c^2} \text{Real}(C_{\text{em}}) \cdot \text{Real}(C_{\text{em}})^{\dagger} = \frac{i}{\hbar} \text{Real}(C_{\text{em}}) \cdot \text{Real}(C_{\text{em}})^{\dagger} \hbar \text{ Planck length}$$
Take care that the multiplication of the tensor is not the normal tensormultiplication or matricesmultiplication. It is the above defined simple multiplication $C_{ij} = +/\kappa A_{ij} B_{ij}$.

$\frac{\partial}{\partial \varphi} C_{\mu\nu} - \frac{i}{\varphi} \frac{\partial}{\partial \varphi} C_{\mu\nu} = \frac{8\pi G}{c^4} (T_{\mu\nu} + \frac{i}{\varphi} S_{\mu\nu})$

we get the final compact result for the extension of General Relativity by

\begin{align*}
K_{\mu\nu} + \frac{i}{\varphi} \frac{\partial}{\partial \varphi} K_{\mu\nu} = \frac{8\pi G}{c^4} (T_{\mu\nu} + \frac{i}{\varphi} S_{\mu\nu})
\end{align*}

with

$S_{\mu\nu} = \frac{1}{2} T_{\mu\nu} T_{\mu\nu} \ldots$ Spin tensor

$K_{\mu\nu} = R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} + \Lambda g_{\mu\nu}$

$\varphi$-golden ratio

The real part is the known General Relativity.
The imaginary part is the Spinextension of GR.

Hint: The Energy-Stress tensor is still symmetric with or without Spin!

The question now is what is it good for?
In the same way as energydensity, momentumdensity also warps spacetime spin or spindensity warps a $\mathbb{R}^4$ Torus.

Take care that the multiplication of the tensor is not the normal tensormultiplication or matricesmultiplication. It is the above defined simple multiplication $C_{ij} = +/\kappa A_{ij} B_{ij}$.

**<7> Candidates for the dark matter in the universe.**

The Oktoquintenfield can be divided into 2 areas (left and right).
The left one has 4 SU(5) Bosons which are over the timespace (curvature) fields.

I suppose that the Planckparticles with planckmass come into being by the Symmetriebreak $SU(5) \times U(1) \times U(1) \rightarrow U(1)$ and the decomposition (Protons, electrons, ...) of it comes from the other Symmetriebreak $SU(2) \times U(1) \rightarrow U(1)$ which only acts on the left half of the Oktoquintenfield.

I suppose that the 4 W-Bosons in the left half (4 of the 20 charged SU(5) Bosons) split into protons, electrons and so on and the other SU(5) Bosons in the right half keep planckparticles.

So the left W-Bosons are the reason for the visible matter and the right W- and Z-Bosons are the reason for the dark matter.

As seen in <4> we get a particle Lambda which is also an additional candidate for dark matter!

**<8> Some important points of the Oktoquintenpotential**
To get the maxima, minima and the zeropoints of the potential we have to substitute
\[ z = \phi^2 \] it is enough (because of symmetry) to take a look on the positive \( \phi \)'s,
and solve the cubic equations in the bracket
\[
V(\sqrt{z}) = z \left( \frac{\tau_1}{2} + \frac{\tau_2}{4} z + \frac{\lambda}{8} z^2 \right) \text{ and }
V'(\sqrt{z}) = \sqrt{z} \left( \tau_1 + \tau_2 z + \lambda z^2 \right)
\]
We will make it short and write the results.
First the Zeropoints:
\[
z_1 = u + v = -\sqrt{\frac{2C}{3}} \sqrt{\frac{\sqrt{27} - i \sqrt{5}}{32} + \frac{\sqrt{27} + i \sqrt{5}}{32}} \]
\[
z_2 = \epsilon_1 u + \epsilon_2 v
\]
\[
z_3 = \epsilon_2 u + \epsilon_1 v
\]
Where \( \epsilon_1 = -\frac{1}{2} + I \cdot \frac{\sqrt{3}}{2} \) and \( \epsilon_2 = \frac{1}{2} - I \cdot \frac{\sqrt{3}}{2} \)
then
\[
z_1 = -0, 990839414 \times 2 \sqrt{\frac{2C}{3}}
\]
\[
z_2 = 0, 378460979 \times 2 \sqrt{\frac{2C}{3}}
\]
\[
z_3 = 0, 612372435 \times 2 \sqrt{\frac{2C}{3}}
\]
then the zeropoints are
\[
\phi_1 = -0, 995409169 \times \frac{8C}{3}
\]
\[
\phi_2 = 0, 615196699 \times \frac{8C}{3}
\]
\[
\phi_3 = 0, 782542290 \times \frac{8C}{3}
\]
\[
\phi_2 = c = 0, 615196699 \times \sqrt{\frac{8C}{3}} = \sin(37, 966214178) \times \frac{8C}{3}
\]
\[
\phi_2 = 37, 966214178^\circ
\]
\[
\phi_3 = c \cdot \sqrt{\phi} = 0, 782542290 \times \sqrt{\frac{8C}{3}} = \sin(128, 5066061932) \times \frac{8C}{3}
\]
\[
\phi_3 = 128, 5066061932^\circ
\]
Then the Maxima and the Minima:
\[
z_1 = u + v = \sqrt{\frac{C}{3}} \left( \sqrt{\frac{\sqrt{37} - i \sqrt{27}}{64} + \frac{\sqrt{37} + i \sqrt{27}}{64}} \right)
\]
\[
z_2 = \epsilon_1 u + \epsilon_2 v
\]
\[
z_3 = \epsilon_2 u + \epsilon_1 v
\]
Finally we have two positive results:
\[
z_{\text{max}} = 0, 233475630 \times \frac{2C}{3}
\]
and
\[
z_{\text{max}} = 0, 725352944 \times \frac{2C}{3}
\]
and one negative
\[
z_1 = -\left( z_{\text{max}} + z_{\text{min}} \right)
\]
Then because of \( z = \phi^2 \)
\[
\phi_{\text{min}} = 0, 483193160 \times \frac{4C}{3} \text{ and }
\]
\[
\phi_{\text{max}} = 0, 8516765489 \times \frac{4C}{3}
\]
In cubic equations the real zeropoints comes from the cos(\( \alpha \)) or from sin(90 - \( \alpha \)) of angles (see https://en.wikipedia.org/wiki/Cubic_function).
Then for \( \phi_{\text{min}} \) we get an angle \( \alpha_{\text{min}} \):
\[
\phi_{\text{min}} = 0, 483193160 \times \frac{4C}{3} = \sin(28, 89160846) \times \frac{4C}{3}
\]
\[
\alpha_{\text{min}} = 28, 89160846 \text{ degrees is very near to the Weinbergangle}
\]
\[
sin^2(\alpha_{\text{min}}) = sin^2(28, 89160846) = 0, 233475630
\]
and for \( \phi_{\text{max}} \) we get an angle
\[
\phi_{\text{max}} = 0, 8516765489 \times \frac{4C}{3} = \sin(121, 60508985) \times \frac{4C}{3}
\]
\[
\alpha_{\text{max}} = 121, 60508985 \text{ degrees}
\]
Geometric interpretation of the roots (zeropoints) in cubic equations with 3 real zeropoints

\[ \phi_{\text{min}} = 0, 483193160 \times \sqrt[3]{\frac{4}{3}} \] and

\[ \phi_{\text{max}} = 0, 8516765489 \times \sqrt[3]{\frac{4}{3}} \]

In cubic equations the real zeropoints come from the \( \cos(n) \) or from \( \sin(90-n) \) of angles (see https://en.wikipedia.org/wiki/Cubic_function).

Our second extreme value \( L_1 \) is at \( \theta_1 \). With the relation above we can calculate the third extreme value \( L_2 \).

\[ \phi_{\text{max}} = 1, 762669... \times \theta_1 \]

Conclusions

Dark Energy comes from the Oktoquintenpotential (the second term in the potential).

Dark Matter could be the W, Z Bosons of the SU(5) Symmetry and the Lambda Boson from the mixing.