A Very Simple Single Electron Lamb Shift Approximation

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Abstract

The Lamb shift was discovered by Willis Lamb and measured for the first time in 1947 by Lamb and Rutherford [1, 2, 3] on the hydrogen microwave spectrum. We suggest that the Lamb shift can be approximated by a very simple function that seems accurate enough for most experimenters working with elements where relativistic effects of the electron are minimal, that is up to element 80 or so. Even if our new approximation does not show anything new in physics, we think it can be useful for experimenters and students of quantum physics and chemistry; now everyone can calculate the Lamb shift on the back of an envelope.

Key words: Lamb shift, hydrogen like atom, single electron, simple approximation formula.

A Very Simple Lamb Shift Approximation Formula

In this short note we present a simple approximation for the Lamb shift for one electron in any atom. The only input needed in this formula is the atomic number. We compare it with far more complex methods presented in the literature and we provide a table of Lamb shift observations to show that our approximation is quite accurate.

We suggest that the Lamb shift can be approximated very well with the following function (the output will be in electron volts)

$$E_l = \frac{X \times Z^Y}{100000} \tag{1}$$

where Z is the atomic number and X and Y are constants. Optimization indicates that setting $X = 4.2$ and $Y = 3.53$ gives accurate results. Table 1 shows predictions from our very simple formula compared to predictions generated by much more complex calculations from quantum electrodynamics, see [4] and also [5]. Our model gives minimal physical insight. It can best be described as a curve fitting function. However, we will claim that even simple curve fitting functions can be useful. For example, if someone asks you about the Lamb shift in a job interview, you can calculate it in your head, or at least on the back of an envelope. In addition, students with just an introductory course in physics and little or no knowledge of quantum electrodynamics can get some basic intuition about the approximate size of the Lamb shift for a single electron.

Our approximation formula also provides good predictions for higher elements. Recently, [6] measured the Lamb shift for hydrogen-like lead and gold using microcalorimeters. They measured (211 ± 42) eV for hydrogen-like gold and (260 ± 53) eV for hydrogen-like lead. Quantum electrodynamics predicts a Lamb shift of 205.2 for gold and 244.6 for lead. The Lamb shift prediction we get from our approximation formula for hydrogen-like gold, element 79, is

$$E_l = \frac{4.2 \times 79^{3.53}}{100000} \approx 209.83 \tag{2}$$

and for lead, element 81, it is

$$E_l = \frac{4.2 \times 81^{3.53}}{100000} \approx 229.19 \tag{3}$$

We conclude that our approximation seems quite accurate and likely accurate enough for most experimenters. Our formula is very simple, and we think could be useful for both experimenters as well as students wanting to do quick back on the envelope Lamb shift calculations. However, for higher order elements above 80 or so relativistic effects starts to kick in and the approximation is no longer accurate.

Assuming a rod of length

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Element	Element Number	Johnson & Soff[a]	Simple Approximation
Hydrogen H	1	0.00003	0.00004
Helium He	2	0.00045	0.00049
Lithium Li	3	0.00198	0.00203
Beryllium Be	4	0.00564	0.00560
Boron B	5	0.01264	0.01232
Carbon C	6	0.02433	0.02345
Nitrogen N	7	0.04218	0.04041
Oxygen O	8	0.06778	0.06474
Fluorine F	9	0.10280	0.09811
Neon Ne	10	0.14894	0.14231
Sodium Na	11	0.20792	0.19923
Magnesium Mg	12	0.28170	0.27087
Aluminum Al	13	0.37196	0.35931
Silicon Si	14	0.48082	0.46675
Phosphorus P	15	0.61014	0.59546
Sulfur S	16	0.76202	0.74781
Chlorine Cl	17	0.93833	0.92626
Argon Ar	18	1.14142	1.13334
Potassium K	19	1.37240	1.37167
Calcium Ca	20	1.63463	1.64394
Scandium Sc	21	1.92922	1.95292
Titanium Ti	22	2.26026	2.30145
Vanadium V	23	2.62602	2.69246
Chromium Cr	24	2.89507	3.12892
Manganese Mn	25	3.48029	3.67559
Zinc Zn	30	6.42991	6.87840
Zirconium Zr	40	16.87449	19.85448
Tin Sn	50	36.01793	44.83695
Neodymium Nd	60	68.36588	88.37679
Ytterbium Yb	70	122.83293	136.91322
Mercury Hg	80	217.84340	219.35975
Thorium Th	90	401.83863	332.44924
Fermium Fm	100	809.00865	482.22452
Darmstadtium Ds	110	1853.59070	675.09590

Table 1: This Table shows our Lamb shift approximation as well as more accurate calculations based on quantum electrodynamics by Johnson and Soff. The outputs are in electron volts (eV)

[a]The original Johnson and Soff values must be multiplied by 0.000123986 to get them into to the unit system used here.

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