Refutation of quantum gates: Hadamard; Pauli-X, -Y, -Z; Toffoli; and Fredkin

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We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated *proof* value, `F` as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal.

LET: p, q, r, s: probability, |0⟩, |1⟩, √2; ~ Not; & And; + Or; - Not Or;
> Imply, greater than; < Not Imply, less than; = Equivalent; @ Not Equivalent;
% possibility, for one or some; # necessity, for all or every;
(p@p) ordinal 0; (%p>#p) ordinal 1; (%p<#p) ordinal 2; (p=p) ordinal 3.

From: en.wikipedia.org/wiki/Quantum_logic_gate for the Hadamard gate (H):

Basis states (basis vectors) as qubits are defined as:

|0⟩ to (|0⟩+|1⟩)/√2 and (0.1.1)

q>((q+r)\s) ; `TTTT TFTF TTTT TFTF` (0.1.2)

|1⟩ to (|0⟩−|1⟩)/√2 (0.2.1)

r>((q-r)\s) ; `TTTT TTTT TTTT TTTT` (0.2.2)

"... which means that a measurement will have equal probabilities to become 1 or 0" (0.3.0)

We write Eq. 0.5.0 to mean: the measurement of the basis states imply a combined probability of]0,1[. (0.3.1)

(p>(p@p))&(p<(%p>#p)) ; `FFFF FFFF FFFF FFFF` (0.3.2)

We evaluate the following gates: Hadamard; Pauli-X, -Y, -Z; Toffoli; and Fredkin.

Hadamard (H) gate: |0⟩ to (|0⟩+|1⟩)/√2; and |1⟩ to (|0⟩−|1⟩)/√2. (1.1)
(p=((q>((q+r)\s))&(r>((q-r)\s))))>((p>(p@p))&(p<(%p>#p))) ;
`TFTF TFTF TFFT TFFT` (1.2)

Pauli-X gate: |0⟩ to |1⟩; and |1⟩ to |0⟩. (2.1)
(p=((q>r)&(r>q)))>((p>(p@p))&(p<(%p>#p))) ; `TFFT FTTF TFFT FTTF` (2.2)

Pauli-Y gate: LET s=i; |0⟩ to i|1⟩; and |1⟩ to −i|0⟩. (3.1)
(p=((q>(s&r))&(r>(~s&q))))>((p>(p@p))&(p<(%p>#p))) ;
`TFFT FTFT TFFT FTFT` (3.2)

Pauli-Z gate: |0⟩ to |0⟩; and |1⟩ to −|1⟩. (4.1)
(p=((q>q)&(r>~r)))>((p>(p@p))&(p<(%p>#p))) ; `TFFT FTFT TFFT FTFT` (4.2)

Toffoli (CCNOT): $|a, b, c\rangle$ to $|a, b, c\oplus ab\rangle$. (5.1)

((p=(((q=r)=(%p>#p))>(s=(s@(q&r)))))+(p=(q@r)))>((p>(p@p))&(p<(%p>#p))) ;

FFTF TFFN FFTF TFFN (5.2)

Fredkin (CSWAP):

$C_{out}=C_{in}$; O_1=(Not C And I_1) Or (C And I_2); O_2=(C And I_1) Or (Not C And I_2) (6.1)

(p=((r=((~q&r)+(q&s)))&(s=((q&r)+(~q&s)))))>((p>(p@p))&(p<(%p>#p))) ;

TFTF TFFT TFTF TFFT (6.2)

As rendered, Eqs. 1.2, 2.2, 3.2, 4.2, 5.2, and 6.2 are *not* tautologous. This means the following quantum gates are refuted: Hadamard; Pauli-X, -Y, -Z; Toffoli; and Fredkin.