Why it is hard to understand – and, therefore, *explain* – quantum math

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Abstract : If mathematics is the queen of science, then physics might well be the king. It successes are obvious. However, as a science, physics may have failed in one regard, and that is to *explain* what its basic concepts – such as state vectors, wavefunctions, and transformation matrices – actually *represent*. When studying quantum mechanics, it is, effectively, hard to keep up the initial enthusiasm, and those who branch out to other fields – which is most of us – quickly end up going through the motions only: we regurgitate models and equations and know how to solve the standard problems, so as to pass the exam, but then forget about them as soon as possible.

This paper explores a very intuitive sentiment about the issue: the wavefunction is a rather 'flat' mathematical object – it is two-dimensional, basically – so it can't do the trick, perhaps. In contrast, Maxwell's equations have real vectors in them, which is why a deeper or more intuitive understanding of electromagnetism comes relatively easily. Indeed, when everything is said and done, we are just human beings living in three-dimensional space, and that is why vector equations (or systems of vector equations), as a mathematical tool, make sense to us. This paper further explores this sentiment.

It also offers a way out by, predictably, presenting yet another possible physical interpretation of the wavefunction. More importantly (for the reviewer of this paper, at least), this paper offers a sensible response to the mainstream view that three-dimensional physical interpretations of the wavefunction cannot make any sense because of the weird 720° symmetry of the wavefunction when describing spin-1/2 particles (fermions or – for all practical purposes – electrons). The author does so by analyzing (1) Dirac's belt trick more in detail – and what it implies in terms of the *interaction* between the observer and the object – as well as (2) Feynman's derivation of the transformation matrices for spin-1/2 two-state systems.

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[1] Websites: https://readingfeynman.org/ and https://readingeinstein.blog/.

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Introduction

Gauss called mathematics "the queen of sciences." Gauss was also a physicist, and inspired many other physicists – such as Boltzmann, Maxwell and Planck. Most notably, perhaps, Gauss' divergence theorem underpins our understanding of the propagation mechanism of an electromagnetic wave – a wonderful interplay between circulation and flux, linear and circular motion: two equations ($\partial\mathbf{B}/\partial t = -\nabla\times E$ and $\partial\mathbf{E}/\partial t = c^2\nabla\times\mathbf{B}$) that capture (almost) anything we know about electromagnetism.[2]

If math is the queen, physicists might want to claim physics is the king: "The Schrödinger equation has been one of the great triumphs of physics. By providing the key to the underlying machinery of atomic structure it has given an explanation for atomic spectra, for chemistry, and for the nature of matter."[3] And there are, of course, many more recent claims to fame, such as the experimental verifications of the theoretical Higgs field in CERN's LHC accelerator in July 2012, or the detection of gravitational waves by Caltech's LIGO Lab. We may also add recent advances in nuclear fusion or nanotechnology. And, yes, the mass-energy equivalence formula ($E = mc^2$) may well be the best-known formula in the world.

The $E = mc^2$ formula is probably also one of the least understood formulas in the world. Indeed, physics, as a science, may have failed in one regard, and that is to *explain* what the basic concepts in physics – such as state vectors, wavefunctions, and transformation matrices – actually *are*. Indeed, physicists themselves have had uninterrupted discussions on the reality of the wavefunction for almost a hundred years now (Schrödinger published his equation in 1926). It is, therefore, not surprising that some outsiders should think that most physicists do not understand – in a deep or intuitive sense, that is – what they are talking about. The eminent physicist Richard Feynman, whom Bill Gates referred to as "the best teacher he never had"[4], admitted so much when he wrote the following in his introduction to quantum mechanics:

> "Atomic behavior appears peculiar and mysterious to everyone—both to the novice and to the experienced physicist. *Even the experts do not understand it the way they would like to.*"[5]

Let us be honest: even the mathematically inclined outsiders – such as engineers, or quantitative social scientists, such as myself – will admit quantum math is wholly unattractive. In contrast, we all agree Maxwell's equations are wonderfully beautiful. We understand vectors, and we understand why we need *two* of them to explain what is, of course, *one force* ($\mathbf{F} = q\mathbf{E} + q(\boldsymbol{v}\times\mathbf{B})$). And when we combine them with an analysis of the dynamics of the Poynting vector, we start to see how a two-dimensional oscillation might propagate through space by constantly borrowing and returning energy from one place to another.

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[2] The $\partial\mathbf{B}/\partial t = -\nabla\times E$ and $\partial\mathbf{E}/\partial t = c^2\nabla\times\mathbf{B}$ are Maxwell's equations in empty space, when there are no charges or currents: $\boldsymbol{j}$ and ρ are, therefore, zero. We will refer to **E** and **B** as *real* vectors – a term which may be loosely defined as vectors in three-dimensional space which transform according to Lorentz' transformation rules and, therefore, respect relativity.

[3] Richard Feynman, *Lectures on Physics*, Volume III (1966), p. 19-18

[4] For a more balanced discussion on Feynman's qualities as a teacher, see: https://mathblog.com/was-richard-feynman-a-great-teacher/, accessed on 12 July 2018.

[5] Richard Feynman, *Lectures on Physics*, Volume III (1966), p. 1-1

1. Why state vectors don't feel real

In his famous *Lecturs on Quantum Mechanics*, Feynman tells his readers they can skip the chapter on transformations from one *representation* to another. It's worth quoting him here: "This chapter is a rather long and abstract side tour, and it does not introduce any idea which we will not come to by a different route in later chapters. You can, therefore, skip over it, and come back later to it if you are interested."[6]

I have returned to it because I realize these transformations bother us – outsiders: mathematically inclined non-professionals who try to understand and keep up – profoundly. Maxwell's equations give us a relativistically correct description of electromagnetic waves because they are written as *vector equations*. I am not talking abstract *state* vectors here but *real* vectors: three-dimensional objects (the electric field vector **E** and the magnetic field vector **B**) with a magnitude and a direction in the *real* (three-dimensional) space that we imagine to be living in. Of course, we know that the three numbers that *describe* those vectors depend on the reference frame, but we don't worry about that because we know we can do these Lorentz transformations whenever we would want to swap reference frame, and we *understand* those transformations – because we *understand* relativity theory – *somehow*, at least. 😊 In contrast, while the quantum-mechanical state vectors are defined in terms of a base – or a *representation* as it is referred to in quantum mechanics – the transformation rules are complicated and non-intuitive. Physicists have accepted that – because they need to get through their exercises, lectures, exams or some doctoral thesis, perhaps – but outsiders haven't: most of us would probably like to think of state vectors as some sort of *projection* of a deeper reality.

As a young man, looking at study options, I once had a discussion with an engineer – and, yes, it was a discussion on quantum physics, the stuff that has fascinated me all of my life, and will continue to do so. He was a smart, cultured and sophisticated man. He told me he and most, if not all, of his fellow students had been attracted by the same, but they all ended going through the motions only when studying quantum physics: they would regurgitate the models and equations, and know how to solve the standard problems, but then forget about it as soon as possible. I will never forget his intuitive conclusion on the subject-matter: he told me he instinctively felt the wavefunction was like a 'flat' object, so it couldn't do the trick. What he meant to say was that, as a mathematical object, it has two degrees of freedom only: a 'real' and an 'imaginary' dimension – both of which are, obviously, equally real – or equally *necessary* in the description, I would say.

Some 25 years later, I have come to the conclusion he might be right. The wave equation – Schrödinger's equation, that is – is two-dimensional as well. It has to be, of course, because its solutions – the wavefunctions – are two-dimensional only. As such, it strongly suggests that the whole mathematical framework already incorporates some line of sight between the observer and the object, and the discussion on the reality of the wavefunction may then be usefully focused on the relation between the real and imaginary part of the wavefunction and that line of sight.

The line of sight, or the line of motion? In the frame of reference of the object itself – i.e. an electron orbital in the context of Schrödinger's equation – it must be the line of sight, so let us stick to that for the time being.

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[6] Richard Feynman, *Lectures on Physics*, Volume III (1966), p. 6-1

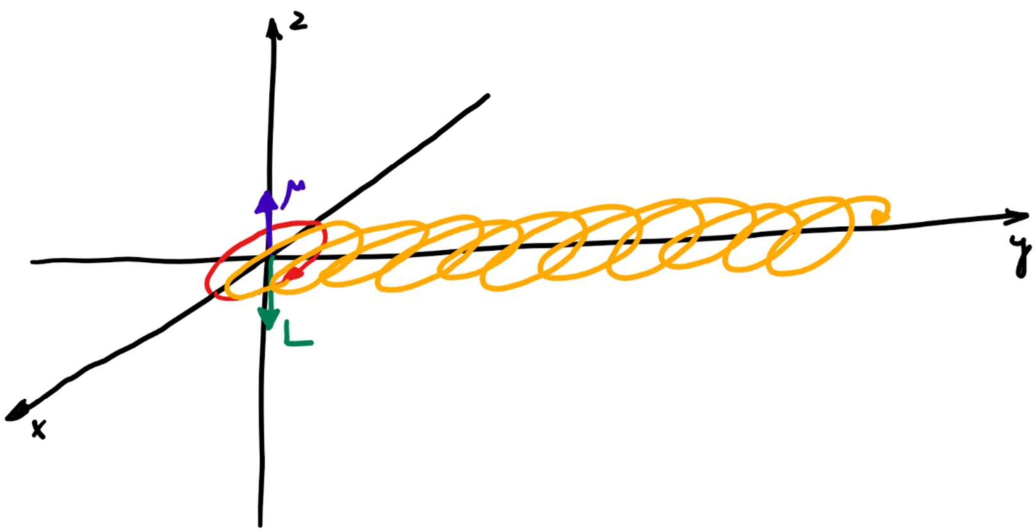
2. Why a physical interpretation of the wavefunction makes sense

In any physical interpretation of the wavefunction, the real and imaginary component of the wavefunction will, obviously, be perpendicular to each other. In fact, we may remind the reader that this is probably *not* a matter of interpretation: the real and imaginary part of a complex number are orthogonal *by definition*: $a \cdot e^{i \cdot \theta} = a \cdot (\cos\theta + i \cdot a \cdot \sin\theta)$. That is how Euler *defined* them.[7] Hence, we have two obvious possibilities here:

1. The real and imaginary component of the wavefunction are *both* perpendicular to the line of sight.
2. Only one of them is perpendicular. The other is in the same direction.

The Stern-Gerlach experiment told us the spin of an electron will always be up or down (as shown below) and, therefore, I like to think the second possibility is the more likely one (think of the *x*-axis as the line of sight here), but that is an early assumption only.

Figure 1: Could this be an electron?



Mainstream physicists will immediately reject easy physical interpretations of the wavefunction (such as the one above) because of their training and – more to the point – because of the weird 720° symmetry of the wavefunction. Let me tackle the first objection first. I know most physicists hate philosophers, but I think it is about time physicists show how one can get, through the application of an operator, an *observable* (such as the (average or expected) linear or angular momentum) out of the wavefunction if… Well… If it doesn't have any physical meaning. If it is not *real*, somehow, then how do we get *real* data out of it?

[7] In fact, one may argue whether or not Euler (or Cardano, Descartes, Newton, or Leibnitz before him) did actually recognize the physical or geometrical meaning in them, as they coined the term 'imaginary' for their second component (as opposed to 'real', for the first component). In any case, this is not a discussion on the history of the math and the question is, therefore, rather moot.

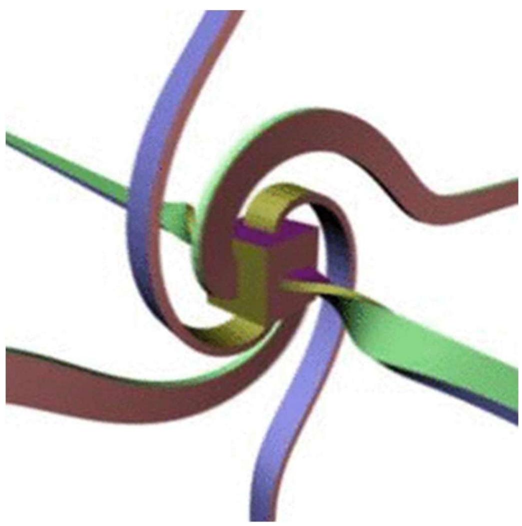
Now, physicists and philosophers will both ask me: what is real? My answer is: "I don't know what's real, but anything that has a magnitude, a direction (linear or rotational), and something to grab on, is *real enough* for me." 😉 That brings me to the second question.

3. What is the reality of an object with a 720° symmetry?

I must assume you have looked at a number of illustrations of objects that are supposed to have a symmetry of 720°. Animations are better than simple pictures in this regard. Indeed, there are many nice videos on Dirac's belt trick (with an actual leather belt or any other object that has belt-like characteristics) or, more generally, on 720° symmetries. I can reference one I like in particular: https://www.youtube.com/watch?v=JDJKfs3HqRg. It is a physics student who is turning a glass of water in his hand – not once, but twice, as one does in tricks like this. The video clearly demonstrates a 360° turn is *not* like we are walking around the object (the glass of water, in this case) to get back to where we were. No. *We are turning the object around by 360°!* That's a very different thing than (1) looking at it, (2) walking around it, and (3) looking at it again. In case you doubt this statement, you should actually do the trick yourself (I found doing the trick myself is remarkably illuminating).

If you don't like the reference above, I warmly recommend the 3D animated versions of the belt trick made by Jason Hise (2016)[8]. The still below is a frozen image of one of these.

Figure 2: Dirac's belt trick with six belts (five are redundant)



The point is, in all of these illustrations, there is no independent object. The object is always *connected* to the observer, or the outside world – be it through Dirac's belt or, in the referenced video (https://www.youtube.com/watch?v=JDJKfs3HqRg), through an arm with a wrist and an elbow. 😉 Both have a definite orientation in space: the observer has a notion of what is up and down and – more importantly – *does* something to the object. Hence, it is *definitely* not like we are making a full 360° turn around the object. No. *We are turning the object* , instead. That's an entirely different operation. In fact,

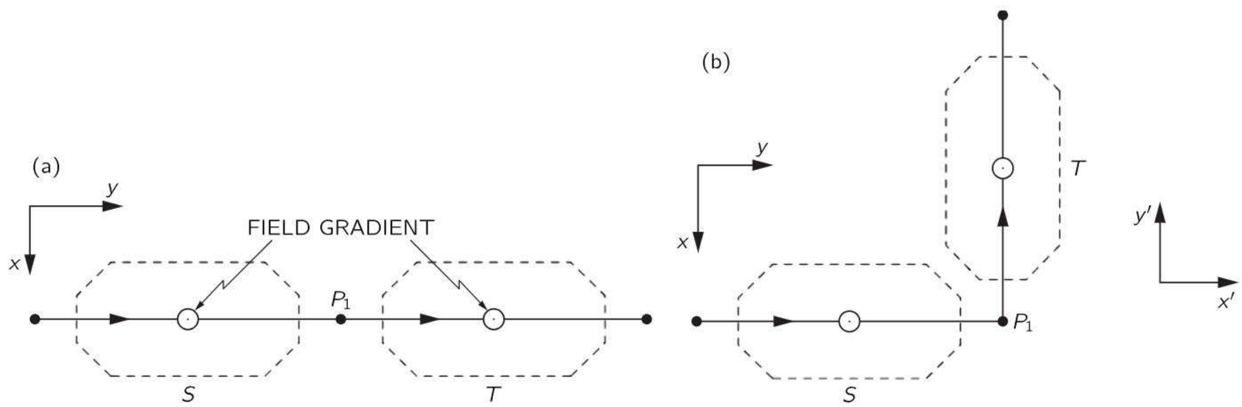
[8] See: https://commons.wikimedia.org/wiki/User:JasonHise, accessed on 13 June 2018.

it actually *is* an operation – in contrast to making a full 360° walk around the object to then look at it from the same spot as before, which is *not* an operation[9]).

4. Feynman's version of Dirac's belt trick

Feynman himself acknowledges the above criticism (turning an object around is not the same as going around it) in the mentioned (in)famous *Lecture* on transformations – which is a "long and abstract side tour", as he puts it, but very interesting. In fact, the thought experiments in this chapter come tantalizing close to a physical interpretation of the wavefunction of spin-1/2 particles. I've elaborated on that in a previous paper[10], so let me just summarize the basic line of thought here. In the illustration below[11] (Feynman, III, 6-3), Feynman compares the physics of two beam splitters with a different relative orientation: in (a), the angle is 0°, while in (b) we have a (right-handed) rotation of 90° about the z-axis.

Figure 3: A rotation of 90° about the *z*-axis



He then proves – using geometry and logic only - that the probabilities and, therefore, the magnitudes of the amplitudes for the particle to have its spin 'up' or 'down' (denoted as C^+ and C^- and C'^+ and C'^- in the *S* and *T* representation respectively) must be the same, but the amplitudes must have different phases, noting – in his typical style, mixing academic and colloquial language – that "there must be some way for a particle to tell that it has turned a corner in (b)."

The various interpretations of what *actually* happens here may, in my humble view, shed some light on the heated discussions on the reality of the wavefunction – and of quantum states (or *state* vectors).[12] We know, from theory and experiment, that the amplitudes are different. To be precise, for the given difference in the relative orientation of the two apparatuses (90°), we know that the amplitudes are given by $C'^+ = e^{i\cdot\phi/2}\cdot C^+ = e^{i\cdot\pi/4}\cdot C^+$ and $C'^- = e^{i\cdot\phi/2}\cdot C^- = e^{i\cdot\pi/4}\cdot C^-$ respectively. The more subtle question here is the following: is the *reality* of the particle in the two setups the same?

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[9] We obviously make abstraction of time passing by here. The argument of the wavefunction will also change as a function of time, of course. We assume *our* 360° turn *around* the object is done instantaneously.

[10] *Reflections on the Reality of the Wavefunction*, http://vixra.org/abs/1712.0201, accessed on 12 June 2018.

[11] Richard Feynman, *Lectures on Physics*, Volume III (1966), p. 6-6

[12] The alert reader will note that we are discussing a two-state system here, as opposed to, say, an electron orbital. Hence, the *meaning* of the wavefunction is definitely not the same. However, this objection is not fundamental.

Feynman notes that, while “the two apparatuses in (a) and (b) are different”, “the probabilities are the same.” He refrains from making any statement on the particle itself: is or is it not the same? The common-sense answer is obvious: of course, it is the same! The particle is the same, right? In (b), it just took a turn – so it is just going in some other direction. That’s all. However, common sense is seldom a good guide when thinking about quantum-mechanical realities. Also, from a more philosophical point of view, one may argue that the reality of the particle is not the same: something might – I’d say: *must* – have happened to the electron because, when everything is said and done, the particle did take a turn in (b). It did not in (a).

What did cause it to make a turn in (b). Feynman is equally vague on that. I suspect it’s our experimental set-up: we just *make* it make a turn. And then we are surprised we have a different wavefunction? *Surely, you’re joking, Mr. Feynman!* 😊

Is he or isn’t he? I strongly feel he was on the fence about it all, but his position – and the orthodox interpretation of quantum math – may have prevented him from fully exploring what he was exploring here. Feynman notes that, if we rotate the T apparatus by 360°, the system seems to be indistinguishable from the zero-degree situation. However, the amplitudes will be different: $C'^{+} = e^{i\cdot\phi/2}\cdot C^{+} = e^{i\cdot\pi}\cdot C^{+} = -\,C^{+}$ and $C'^{-} = e^{i\cdot\phi/2}\cdot C^{-} = e^{i\cdot\pi}\cdot C^{-} = -\,C^{-}$. Both amplitudes are multiplied by −1. Feynman says the following in this regard (Feynman, III, 6-3): “It is very curious to say that, if you turn the apparatus 360°, you get new amplitudes. They aren’t really new, though, because the common change of sign doesn’t give any different physics.” However, in a footnote – a *crucial* footnote, in my view – he acknowledges the *reality* of the situation might not be the same. Referring to a continuity assumption he had used earlier, he notes the following:

> “If something has been rotated by a sequence of small rotations whose *net* result is to return it to the original orientation, it is possible to define the idea that it has been rotated by 360° – *as distinct from zero net rotation* – if you have kept track of the whole history.”

These are weird philosophical questions. Is an apparatus *that has been turned* 360° a different apparatus? Is an electron that takes a complete turn – over 360° – a different electron?

My answer is positive: it *is* different, and so that is why we should not be surprised that the wavefunction is different. We definitely meddled: Dirac’s belt – or whatever other *physical* connection between the observer and the object that is being observed – is definitely there! Feynman acknowledges the same: rotating something by 360° is distinct from zero net rotation. In fact, we should not be surprised that the wavefunction does *not* return to its original value after one 360° turn, but wonder why it returns to its original value after *another* turn of 360°! Why after two turns? Why not after three, or four? Or some rotation in-between, like one-and-a-half or three and three-quarters? Now *that* is a deep symmetry.

Am I a loner here? I don’t think so. 😊 So… Well… What could we be thinking of?

5. A physical interpretation of the wavefunction

Let us look at Schrödinger's equation's once more. Feynman's summary interpretation of Schrödinger's equation is the following:

> "We can think of Schrödinger's equation as describing the diffusion of the probability amplitude from one point to the next. […] But the imaginary coefficient in front of the derivative makes the behavior completely different from the ordinary diffusion such as you would have for a gas spreading out along a thin tube. Ordinary diffusion gives rise to real exponential solutions, whereas the solutions of Schrödinger's equation are complex waves."[13]

Feynman further formalizes this in his *Lecture on Superconductivity*, in which he refers to Schrödinger's equation as the "equation for continuity of probabilities".[14] His analysis there is centered on the *local* conservation of energy, which makes *me* think Schrödinger's equation might be an energy diffusion equation. I've written about this *ad nauseam* in the past, and so I'll just refer you to one of my other papers here for the details[15], and limit this post to the basics, which are as follows.

The wave equation (so that's Schrödinger's equation in its non-relativistic form, which is an approximation that is good enough) is written as:

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m_{eff}}\nabla^2\psi + V\psi$$

The resemblance with the standard diffusion equation (shown below) is obvious:

$$\frac{\partial\phi}{\partial t} = D\nabla^2\phi + S$$

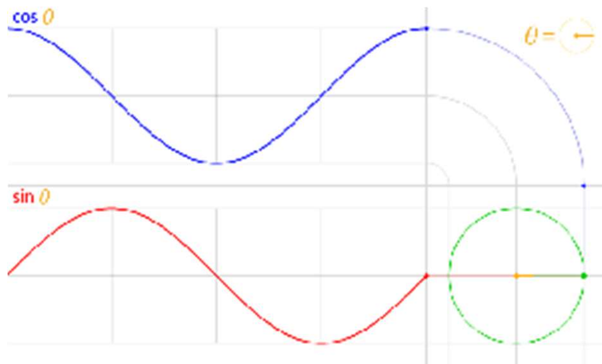
As Feynman notes, it is just that imaginary coefficient that makes the behavior quite different. *How* exactly? Well… You know we get all of those complicated electron orbitals (i.e. the various wave *functions* that satisfy the equation) out of Schrödinger's differential equation. We can think of these solutions as (complex) *standing waves*. They basically represent some *equilibrium* situation, and the main characteristic of each is their *energy level*. I won't dwell on this because – as mentioned above – I assume you master the math. Now, you know that – if we would want to interpret these wavefunctions as something real (which is surely what *I* want to do!) – the real and imaginary component of a wavefunction will be perpendicular to each other. A general geometric representation of the *elementary* wavefunction $\psi(\theta) = a\cdot e^{-i\cdot\theta} = a\cdot e^{-i\cdot(E/\hbar)\cdot t} = a\cdot\cos[(E/\hbar)\cdot t] - i\cdot a\cdot\sin[(E/\hbar)\cdot t]$ is shown below:

[13] Richard Feynman, *Lectures on Physics*, Volume III (1966), p. 16-4

[14] Richard Feynman, *Lectures on Physics*, Volume III (1966), p. 21-3 and 21-4

[15] See: http://vixra.org/author/jean_louis_van_belle_maec_baec_bphil, accessed on 12 June 2018.

Figure 4: Euler's formula



The 90° angle makes me think of the similarity with the mathematical description of an electromagnetic wave. Let me quickly show you why. For a particle moving in free space – with no external force fields acting on it – there is no potential (U = 0) and, therefore, the Vψ term – which is just the equivalent of the the *sink* or *source* term S in the diffusion equation – disappears. Therefore, Schrödinger's equation reduces to:

$$\partial\psi(\mathbf{x}, t)/\partial t = i\cdot(1/2)\cdot(\hbar/m_{eff})\cdot\nabla^2\psi(\mathbf{x}, t)$$

Now, the key difference with the diffusion equation – let me write it for you once again: $\partial\phi(\mathbf{x}, t)/\partial t = D\cdot\nabla^2\phi(\mathbf{x}, t)$ – is that Schrödinger's equation gives us *two* equations for the price of one. Indeed, because ψ is a complex-valued function, with a *real* and an *imaginary* part, we get the following equations:

1. $Re(\partial\psi/\partial t) = -(1/2)\cdot(\hbar/m_{eff})\cdot Im(\nabla^2\psi)$
2. $Im(\partial\psi/\partial t) = (1/2)\cdot(\hbar/m_{eff})\cdot Re(\nabla^2\psi)$

In case you would wonder where these equations come from, they can be easily derived from noting that two complex numbers $a + i\cdot b$ and $c + i\cdot d$ are equal if, and *only* if, their real and imaginary parts are the same. Now, the $\partial\psi/\partial t = i\cdot(\hbar/m_{eff})\cdot\nabla^2\psi$ equation amounts to writing something like this: $a + i\cdot b = i\cdot(c + i\cdot d)$. Now, remembering that $i^2 = -1$, you can easily figure out that $i\cdot(c + i\cdot d) = i\cdot c + i^2\cdot d = -d + i\cdot c$. [Now that we're getting a bit technical, let me note that the m_{eff} is the *effective* mass of the particle, which depends on the medium. For example, an electron traveling in a solid (a transistor, for example) will have a different effective mass than in an atom. atom. In free space, we can drop the subscript and write $m_{eff} = m$.]

The equations above make me think of the equations for an electromagnetic wave in free space (no stationary charges or currents):

1. $\partial\mathbf{B}/\partial t = -\nabla\times\mathbf{E}$
2. $\partial\mathbf{E}/\partial t = c^2\nabla\times\mathbf{B}$

Now, these equations – and, I must therefore assume, the other equations above as well – effectively describe a *propagation* mechanism in spacetime, as illustrated below:

Figure 5: Propagation mechanisms

$$\mathrm{Re}(\partial\psi/\partial t) = -(1/2)\cdot(\hbar/m_{\mathrm{eff}})\cdot\mathrm{Im}(\nabla^2\psi)$$

$$\mathrm{Im}(\partial\psi/\partial t) = (1/2)\cdot(\hbar/m_{\mathrm{eff}})\cdot\mathrm{Re}(\nabla^2\psi)$$

$$\partial\mathbf{B}/\partial t = -\nabla\times\mathbf{E}$$

$$\partial\mathbf{E}/\partial t = c^2\nabla\times\mathbf{B}$$

You know how it works for the electromagnetic field: it's the interplay between circulation and flux. Indeed, circulation around some axis of rotation creates a flux in a direction perpendicular to it, and that flux causes this, and then that, and it all goes round and round and round. Something like that. I will let you look up how it goes, *exactly*. The principle is clear enough. Somehow, in this beautiful interplay between linear and circular motion, energy is borrowed from one place and then returns to the other, cycle after cycle.

Now, we know the wavefunction consist of a sine and a cosine: the cosine is the real component, and the sine is the imaginary component. Could they be equally real? Could each represent *half* of the total energy of our particle? I firmly believe they do. The obvious question then is the following: why wouldn't we represent them as *vectors*, just like **E** and **B**? I mean… Representing them as vectors (I mean *real* vectors – something with a magnitude and a direction, in a *real* vector space – as opposed to these *state* vectors from a Hilbert space) would *show* they are real, and the transformation matrices we need to go from one (representational) *base* to another might become more intuitive. In fact, that's why vector notation was invented (sort of): we don't need to worry about the coordinate frame. It's much easier to write physical laws in vector notation because… Well… They're the *real* thing, aren't they?

What about dimensions? Well… I am not sure. However, because we are – arguably – talking about some pointlike charge moving around in those oscillating fields, I would suspect the dimension of the real and imaginary component of the wavefunction will be the same as that of the electric and magnetic field vectors **E** and **B**. We may want to recall these:

1. **E** is measured in *newton per coulomb* (N/C).
2. **B** is measured in newton per coulomb divided by m/s, so that's (N/C)/(m/s).

The weird dimension of **B** is because of the weird force law for the magnetic force. It involves a vector cross product, as shown by Lorentz' formula:

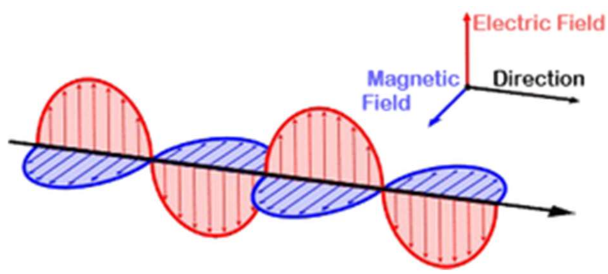
$$\mathbf{F} = q\mathbf{E} + q(\boldsymbol{v}\times\mathbf{B})$$

Of course, it is only *one* force (one and the same physical reality), as evidenced by the fact that we can write **B** as the following vector cross-product: $\mathbf{B} = (1/c)\cdot\boldsymbol{e}_x\times\mathbf{E}$, with $\boldsymbol{e}_x$ the unit vector pointing in the *x*-direction (i.e. the direction of propagation of the wave). [Check it, because you may not have seen this expression before. Just take a piece of paper and think about the geometry of the situation.] Hence, we may associate the $(1/c)\cdot\boldsymbol{e}_x\times$ *operator*, which amounts to a rotation by 90 degrees, with the s/m dimension. Now, multiplication by *i* also amounts to a rotation by 90° degrees. Hence, if we can agree on a suitable convention for the *direction* of rotation here[16], we may boldly write:

$$\mathbf{B} = (1/c)\cdot\boldsymbol{e_x}\times\mathbf{E} = (1/c)\cdot i\cdot\mathbf{E}$$

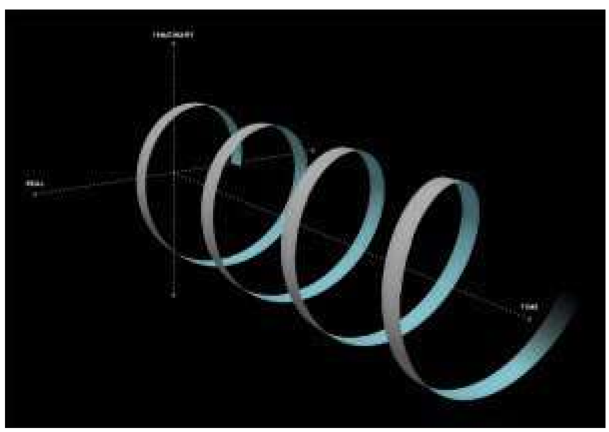
This is, in fact, what triggered my geometric interpretation of Schrödinger's equation about a year ago now. I have had little time to work on it, but think I am on the right track. Of course, you should note that, for an electromagnetic wave, the magnitudes of **E** and **B** reach their maximum, minimum and zero point *simultaneously* (as shown below). So their *phase* is the same.

Figure 6: EM wave (circularly polarized)



In contrast, the phase of the real and imaginary component of the wavefunction is not the same, as shown below.

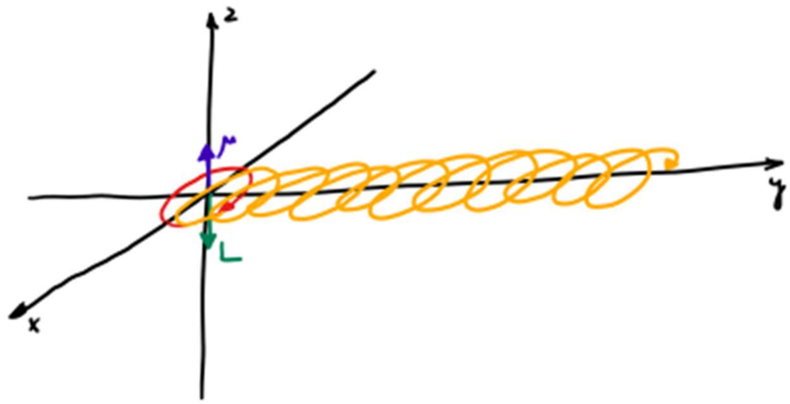
Figure 7: The elementary wavefunction



In fact, because of the Stern-Gerlach experiment, we should probably be thinking of a motion like this:

[16] The convention would also need to include our understanding of the directions of the real and imaginary component of the wavefunction vis-á-vis the line of sight between the observer and the object or, possibly, its direction of motion.

Figure 8: The flywheel (or flying saucer?) model of an electron



But that shouldn't distract you. The question here is the following: could we possibly think of a new formulation of Schrödinger's equation – using *vectors* (not *state* vectors – objects from an abstract Hilbert space – but *real* vectors) rather than complex algebra? I think we can, but then I wonder why the *inventors* of the wavefunction – Heisenberg, Born, Dirac, and Schrödinger himself, of course – never thought of that. Or… Well… Perhaps they did.

One of the more thoughtful commenters on my physics blog[17] - where I first ventilated the thoughts above – noted the following: "Schrodinger's equation is just a re-use of the wave equation that comes from harmonic motion of something tangible, which is why it is rather bizarre to see quantum physicists interpreting it as a merely a probability wave. But when we consider the fact that real strings are "tied" to something – an endpoint of sorts – and must interact with it on a particular plane of existence, the analogy should hold true for the Schrodinger equation. The borders (what the wave is "tied" to) is potential energy, and therefore, the wave itself – by virtue of the fact that it must interact on the same plane of existence – should also be an energy wave, which seems to be what you're saying (albeit you've come to these conclusions mathematically rather than analogously)."

I am sure considerations like this must have crossed the minds of the mentioned geniuses and, therefore, some more historical research is probably in order.

[17] https://readingfeynman.org/2018/06/11/should-we-reinvent-wavefunction-math/#comments, accessed on 13 June 2018.

Conclusions

Above, we explained why quantum math comes across as obscure and difficult. The basic difficulty is that the standard representation of the wavefunction (and wavefunction) depends on the pre-establishment of a 'line of sight', i.e. the line between the object and the observer – or the line of motion of the object – and, hence, that already establishes (part of) the reference frame. In contrast, electromagnetic waves – or any other physical wave – are usually represented by vectors (real vectors, not *state* vectors) which have an independent reality, i.e. they do *not* depend on the reference frame.

We followed up on these thoughts by offering a physical interpretation of the wavefunction. To be precise, we offered a physical interpretation of the matter-wave for an electron.

This interpretation is remarkably similar to the interpretation of the electromagnetic wave. In fact, I suggest their *physical* dimension is the same. What are the differences? One crucial difference is, of course, the phase difference between the two components. There is also the different orientation of the angular momentum, which is explained by our interpretation of the direction of the real and imaginary component of the wavefunction *vis-á-vis* the line of sight, or the line of motion of the particle.

Last but not least, the model of an electron that is offered here combines the idea of an orbital and, importantly, the pointlike *charge* that is whizzing around in it. Hence, the oscillation effectively pushes and pulls that charge around. Even if we think of that charge having zero 'rest mass' (whatever that may be), it should create an 'electromagnetic mass' (mass is just a measure of inertia), and that may explain why an electron cannot travel through space at the speed of light.

In such interpretation, the key difference between the EM wave and the matter-wave is just: (1) geometry (the physical dimensions of the 'components' of both waves would be the same: we're talking an oscillating electromagnetic field in both cases), and (2) the presence – or, in the case of a photon, the absence – of a pointlike charge in that whirling oscillation.

What is the whirling motion of the pointlike charge? The *Zitterbewegung*, so to speak? I elaborated on that in a previous paper, and so I should refer there.[18] However, for the convenience of the reader, I will provide a short summary here. The model is, basically, a flywheel model. It is inspired by the mathematical similarity between the E = $m \cdot a^2 \cdot \omega^2$ (the energy of *two* oscillators in a particular 90° degree combination) and the E = $m \cdot c^2$ formula. If this were to represent something real, then we need to give some meaning to the $c = a \cdot \omega$ identity that comes out of it. Now, if we assume, just for fun, that E and m are the energy and mass of an electron, then the *de Broglie* relations suggest we should equate ω to E/ħ. As for *a,* the *Compton scattering radius* of the electron ($\hbar/(m \cdot c)$ would be a more likely candidate than, say, the Bohr radius, or the Lorentz radius. Why? Because we're not looking at an electron in orbit around a nucleus (Bohr radius), and we're also not looking at the size of the *charge* itself (classical electron radius), because we assume the charge is *pointlike*. We get:

$$a \cdot \omega = [\hbar/(m \cdot c)] \cdot [E/\hbar] = E/(m \cdot c) = m \cdot c^2/(m \cdot c) = c$$

Wow! Did we just *prove* something? No. We don't prove anything here. We only showed that our $E = m \cdot a^2 \cdot \omega^2 = m \cdot c^2$ equation *might* (note the emphasis: *might*) make sense. Let me show you something else. If this *flywheel model* of an electron makes sense, then we can, obviously, also calculate

[18] See: *The Wavefunction as an Energy Propagation Mechanism*, http://vixra.org/abs/1806.0106, accessed on 13 June 2018.

a *tangential velocity* for our charge. The tangential velocity is the product of the *radius* and the *angular* velocity: $v = r \cdot \omega = a \cdot \omega = c$.

Wow! Did we just *prove* something? Is an electron nothing but a point charge that is spinning around some center at… Well… The speed of light?

Maybe. But probably not. We need to explain the *mass* of our electron here, and that's not so easy because we say it is basically a possibly massless point charge going up and down, and back and forth. So we need to calculate the *equivalent* mass of the energy of that oscillation. We are, of course, talking about the *electromagnetic* mass of a charge, but I am not aware of any model that does that what we want to there, and that is to calculate the electromagnetic mass of a charge that is simply moving up and down, and back and forth, or left and right, in some harmonic two-dimensional oscillation. There is also the added complication that an oscillating charge should radiate its energy away, so we would need an explanation of why that is *not* happening. Hence, the situation is very complicated. We need a formula for the electromagnetic mass of a zero mass charge oscillating along one axis – let's denote that by m_{elec} – and that mass would be the *effective* mass for an oscillation that is perpendicular to the first one. Fortunately, the two motions are, effectively, independent (because of the 90° angle between them). Having said that, it is easy to see that the final calculation might be quite complicated.

Having said that, we believe the basic ideas might be valid:

1. A charge with zero rest mass will acquire some *electromagnetic* mass when linearly oscillating.
2. This electromagnetic mass provides an *anchor* for a linear oscillation in a direction that is perpendicular to the original one.
3. Maxwell's propagation mechanism for an electromagnetic wave may ensure such two-dimensional oscillation is sustainable and *propagates* itself, so to speak.

Does this make sense? Probably. It is difficult to summarize a ten-page paper in just a few paragraphs. Hence, I will refer you, once again, to my other paper(s).[19] 😉

References

This paper discusses general principles in physics only. Hence, references were limited to references to basic physics textbooks and online material only. All of the illustrations in this paper are open source or have been created by the author. The author hopes the reviewer(s) will give him, once again, the benefit of the doubt. As the author wrote this paper for his son, who is currently struggling with physics as part of the first year of engineering studies, he also hopes the reviewer(s) will forgive his casual tone.

[19] See: http://vixra.org/author/jean_louis_van_belle_maec_baec_bphil, accessed on 13 June 2018.