Again about the "Twin paradox" 1)

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Since the existence of the twin paradox is often denied when discussing the theory of relativity, it is necessary to dwell on this once again. It is shown that formal SRT and GRT means do not provide a solution to the twin paradox.

The essence of the twin paradox

We consider two twin brothers. One of them went to space travel, another remained on the Earth. Upon the return of the astronaut, brothers compare travel times, each with its own clock. The question arises in connection with the use of Lorentz transformations to estimate the travel time for both. According to these transformations, if we consider the brother-Earthling motionless, the time of travel on his watch will be longer than the time on the testimony of the brother-astronaut. This fact is interpreted as the fact that the brother-cosmonaut will arrive younger than his twin-Earthling. However, if we consider the brother-astronaut in the reference system, associated with the spacecraft, the situation will be the opposite: the brother-Earthling will be younger. By means of a special theory of relativity to answer the question - who in reality will be younger, it is impossible due to the symmetry of the reference systems of the twin brothers. Thus, the source of the twin paradox is the symmetry of the twin brother's reference systems, and its subject classical formulation in STR fixes the inability to answer the question - who will remain younger as a result?

To better understand what kind of symmetry we are talking about, let's look at another illustrative example. To do this, we introduce a thermodynamic time measurement scale.

Let the cosmonaut decide to boil the kettle. The time spent on boiling was equal to the interval of the eigentime 2) \( \tilde{\tau} \). According to the direct Lorentz transformations, this interval corresponds to the coordinate time interval by hours of the Earthling

\[
\Delta t = \frac{\tilde{\tau}}{\sqrt{1 - V^2}}. \tag{i}
\]

Let the Earthling boil the kettle now. The ratio between the interval of eigentime spent on boiling \( \tilde{\tau} \) and the corresponding interval of coordinate time by the astronaut's clock is established by the inverse Lorentz transformations:

\[
\Delta t' = \frac{\tilde{\tau}}{\sqrt{1 - V^2}}. \tag{ii}
\]

From general considerations, namely those from which the Lorentz transformations themselves are derived (homogeneity and isotropy of space-time relations, their continuity), we must say that the boiling processes themselves, as thermodynamic processes, the cosmonaut's and the Earthling's, according to the principle of relativity and equality of inertial reference systems, should proceed completely identical under identical conditions. It follows that the ratio \( \tilde{\tau} = \tilde{\tau}' \) must be satisfied. This makes it possible to measure the intervals of their eigentime with the help of "thermodynamic clock". This is also supported by the consequences of STR and GTR: the eigentime is an invariant of 4-coordinate transformations. It follows from (i) and (ii) that \( \Delta t = \Delta t' \).

Thus, the eigentimes \( \tilde{\tau} \) and \( \tilde{\tau}' \) coincide, the coordinate times \( \Delta t \) and \( \Delta t' \) obtained by direct and inverse Lorentz transformations also coincide, and the relations between the intervals of eigenvalues and coordinate times are symmetric when replacing one reference frame with another.

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1) I beg your pardon for my not very good English! The original text on Russian: http://vixra.org/pdf/1804.0345v1.pdf

2) Eigentime of a moving material body is the time measured in the frame of reference in which the body rests. Coordinate time is the time obtained by transforming the space-time coordinates in the transition from one reference system to another.
A common bond of intervals of coordinate time $t$ unshaded reference frames with its eigentime $\tau'$ shaded reference system is expressed by the formula

$$\tau' = t \sqrt{1 - V^2},$$

where $V$ is the speed of one reference frame relative to another. This ratio reflects the STR's general conclusion that when comparing the readings of two hours, those whose readings are compared with two different ones will always be lagging behind (see Appendix). This effect can be considered as purely kinematic and as not directly related to the nature of the time itself.

Let us reinforce this conclusion with the following remark. To do this, quantize thermodynamic scale measuring the time of the brothers by cups of tea that they have drank.

If, say, an astronaut during the interval of his eigentime $\hat{\tau}'$, measured by "thermodynamic hours", boils the kettle $n$ times, then under the same conditions and the Earthman for his eigentime also boils the kettle $n$ times. Under the Lorentz transformations as well as under the general continuous transformations, the discrete number of events that occurred in the time intervals $\tau$ and $\tau'$ will remain the same, since the number already occurred events are absolute invariants: if they occurred in one frame of reference, they must be fixed by observers and other systems associated with the original system of not too extravagant transformations of 4-coordinates. This allows us to measure the intervals of his time not only by means of "thermodynamic clock" showing a continuous time but in discrete "cups of tea drunk", which means the actual change of the topology of the scale of time measurement. The first scale represents the traditional continuous time scale, the second – discrete.

There is a situation when, for example, the Earthling has "aged" by eigentime $\tilde{\tau}'$ (several cups of tea), will be sure, according to the Lorentz transformations, that his vis-a-vis has grown old by the coordinate time interval $\Delta t'$, and $\Delta t = \Delta t' \neq \tau = \tau'$. The situation will be completely symmetrical about the observation and measurement of each of the twins.

This is directly contrary to the "certainty principle", because it makes it difficult to answer the question: how Long does it actually take to boil, or is it $\Delta t = \Delta t'$ or is it $\hat{\tau} = \hat{\tau}'$? Thus, the symmetry of the connection between the boiling eigentime of one of the brothers and of the time of its vis-a-vis is obvious (relations (i) and (ii) and is clearly demonstrated. It is this fact that we call symmetry, which leads to the paradox in the STR: we cannot say for sure - by what hours the kettle will boil or which of the twins will be younger? The existence of a” contradiction " or non-agreement is not in doubt. It's the same now applies his permission. And here the question naturally arises: what time do the twin brothers live, what time in this case to take for the "present" - coordinate or own?

The "common " solution of the paradox is the following: the fact of asymmetry of the reference systems of the Earthman and the astronaut is fixed. Since the cosmonaut launches rocket engines, then without any kinematic and dynamic grounds for the "present" time, the coordinate time is clearly taken, that is, in fact, the preference for "rejuvenation" is given without any reason to the brother-cosmonaut.

Meanwhile, to measure the time spent by a person, it is more natural to choose a discrete scale of measurement, since the measurement of the time lived by a person, for example, the real number of heartbeats is more adequate than the same measurement by an abstract continuous time. In addition, the natural age of man is measured by discrete years, that is, a discrete number of revolutions of the Earth around the Sun. As for the already occurred events, their number is invariant for any reference systems. In this regard, we can see that eigentime (because it is an invariant of arbitrary continuous transformations) is more adequate for measuring the age of a person.

Let's leave for a while the final decision on which of the times – eigentime or coordinate's considered real and will answer the question:
Why the inclusion of acceleration by the brother-cosmonaut cannot be considered as the cause of the symmetry breaking of the reference systems of the cosmonaut and the Earthman?

As before, we will distinguish the coordinate time \( t \) and eigentime \( \tau \) and first of all we will describe in general the relations between the scales of the axes of the considered times.

Let the clock of coordinate times associated with the systems of the Earthling (\( S \)) and the astronaut (\( S' \)) be synchronized by \( t = t' = 0 \), and the origin of the spatial coordinates at this single time coincide: \( x = x' = y = y' = z = z' = 0 \). The axis of the coordinate time has the same status as that of the spatial coordinate axes. The relationship between the coordinate times of the twin brothers is described by the 4-coordinate transformations at \( S \leftrightarrow S' \) transitions.

Eigentime \( \tau \) is measured by stationary local hours (relative to the frame of reference of the observer). In general, the scales of the segments of the eigentime axis \( \tau \) do not coincide with the scales of the segments of the coordinate time axis \( t \), and the relationship between them is described by the formula \( d \tau = \sqrt{g_{00}} dt \), so that we have time differentials: \( dt \neq d\tau \).

Since for the pseudo-Euclidean metric \( STR \ g_{00} = 1 \), in this particular case, the scales of the segments of the Earthling eigentime axis \( \tau \) coincide with the scales of the segments of the axis of its coordinate times \( t \), so \( d\tau = dt \).

However, this is not so for the time \( t' \) and eigentime \( \tau' \) of cosmonaut, since the cosmonaut metric is different from the pseudo-Euclidean one, and the scales of the segments of the times \( \tau' \) and \( t' \), as already mentioned, \( dt' \neq \tau' \) and are described by the ratio according to \(^3\) using the expression for the time component of the metric tensor:

\[
\sqrt{g_{00}} \approx 1 + 2\varphi(x')
\]

Since the twin brothers, each of them, "live" in their eigentime, it is natural to measure the age of each of them in their eigentime. And the symmetry breaking is to be found in difference of the values of the intervals of these times. The intervals of coordinate times at arbitrary transformations of 4-coordinates, of course, will vary.

Let the astronaut turn on the rocket engines at some point of time and start its accelerated movement.

In general case, the "lengths" of eigentimes during the passage of the path between the moments of switching on and off the rocket engines of the astronaut, calculated on the coordinates of the Earthman and the astronaut and lived by both of them during the acceleration of the ship, coincide with the same segment of the world line interval (s), which is invariant under 4-coordinate transformations. And if measure the time twin brothers living during the rocket's acceleration, by their eigentimes, then the discussion of this paradox would be stopped, including the pseudo-Riemann metric, since symmetry is transformed into the identity and invariance of the twin brothers eigentimes. The decision is purely "kinematic", using only of the point Lorentz transformations, and with nothing other than the relative speed. It does not even involve any physical bodies or processes by which time should be measured.

Should be noted that the classical notions of speed, length, and time intervals in quantum mechanics are rather ambiguous from the topology point of view, so the microlevel's description of physical processes raises its own questions. And the results of experiments with real \( \pi \) mesons and their interpretation again return us to the problem of asymmetry. However, here we can refer to the formula (iii): in fact, in such experiments we measure the coordinate time of \( \pi \)-meson, which according to (iii) is always "longer" than the eigentime.

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\(^3\) L.D. Landau, E.M. Lifshitz. *Theoretical physics. v. II*; §84, (84.1)

\(^4\) This determines the possibility of synchronization of spatially spaced clocks in inertial reference systems.

Now (GTR, GTR), not yet resolved the question about total physical commensurability of the coordinate time and eigentime we should continue the discussion of the paradox.

Let us consider a concrete example illustrating the role of acceleration and the obtained equivalent gravitational field on the "rate" of the eigentimes of both brothers, the result of which will give an answer to the question.

We will consider the length of the world line between the event of the beginning of the acceleration of the ship and the event of the termination of the acceleration, and when it will say about some eigentimes, it will be implied the intervals namely between these two events.

Let's introduce terms:

- $\tau$ – cosmonaut's eigentime calculated from the data of the Earthling (from the coordinates of the laboratory reference system $S$);
- $\tau'$ – the eigentime of the Earthling calculated from the data of the astronaut (from the coordinates of the reference system $S'$ associated with the astronaut).

We have in the frame of reference of the earth with the pseudo-Euclidean metric of 4-space STR:

$$ds^2 = dt'^2 - (dx^1)^2 - (dx^2)^2 - (dx^3)^2 = (dt')^2 = (d\tau)^2, \tag{1}$$

where $(t, x^1, x^2, x^3)$ are the coordinates of the astronaut in the system of reference of the Earthling, which are functions of time $t$. In the formula (1) it is taken into account that the cosmonaut's eigentime $\tau$ coincides with the coordinate time $t'$ of the system $S'$.

The ratio of (1) allow to find the eigentime $\tau$ the astronaut's by the coordinate time $t$ of Earthman:

$$\tau = \int_{\text{world line}} \sqrt{1 - V(t)^2} \, dt \tag{2}$$

This result allows us to answer the question: there is no acceleration in the formula (2) to calculate the cosmonaut's eigentime. The formula is valid to calculate the eigentime $\tau$, of the astronaut moving with arbitrary velocity $V(t)$ in the reference frame of the earth. This is first.

The answer may seem too abstract (although logically sufficient for the case of pseudo-Euclidean metrics), since it is the astronout who will turn on, and the Earthman, being locally in the pseudo-Euclidean metric, will only detect the result of the engines at the beginning of the accelerated motion of the rocket.

However, it should be noted that the astronaut will notice the movement of the Earthling with acceleration. According to the Einstein's equivalence principle, it will detect the occurrence and presence of an equivalent gravitational field, and as a result — the acceleration of the Earthling in this field.

Let we show that at any moment of time and when the Earthling moves in an equivalent gravitational field, the "deceleration" of time will be determined by the velocity, not by the acceleration and exactly by the formula similar to (2). And this second. It should be noted that the accelerating system of cosmonaut reference in a small time interval can be considered as instantly inertial. This feature allows us to note the equivalence of accelerated motion in the inertial frame of reference (from the point of view of the Earthling) and the motion of the Earthling in the gravitational field (from the point of view of the astronaut), that is, the Einstein principle of equivalence; the very same movement of the Earthling occurs by inertia.

To describe the motion in the gravitational field, we must use the means of GTR. Let us consider this in more detail.
In comparison with the General solutions of the problems by means of GTR, our problem is simplified, since we are interested only in the time components of the metric tensor: Earthling \( g_{00} \) and cosmonaut \( \dot{g}_{00} \), related to the systems of the twin brothers reference, in one of which the space-time metric is pseudo-Euclidean. The relationship between these values is most easily obtained by considering the expressions for the time interval fragments, which (interval) is an invariant of the 4-coordinate transformations.

So, let the cosmonaut describes in the system of reference of the Earthling \( S \) an arbitrary trajectory: \( x(t), y(t), z(t) \).

In general case, the relationship between the differentials of intervals, eigentimes and coordinates of reference systems \( S \) and \( S' \), linking pairs of close events, is written in the form:

\[
d s^2 = (d\tau')^2 = \dot{g}_{00}(dt')^2 = g_{00}'(dt')^2 = g_{00}'(dt')^2 + 2g_{a0}dx'^a dt' + g_{a\beta}dx'^a dx'^\beta; \quad (\alpha, \beta = 1,2,3),
\]

\( \varepsilon \partial e \left(t', x^1, x^2, x^3\right) \) — the event coordinates in the reference frame of the astronaut \( S' \).

The right part of the formula (3) allows us to establish a connection between the differentials of the coordinates of the earthling moving in an accelerated manner with respect to the system \( S' \) and, in particular, the differential of its coordinate time \( dt' \), with the differential \( d\tau \) of the eigentime.

However, we are interested in the left part of equality (3) — the connection between the coordinate time of the cosmonaut \( t' \) and the eigentime of the Earthling \( \tau' \), according to Landau — "the true time" (see 2):

\[
d\tau' = \sqrt{\dot{g}_{00} dt'}
\]

For a pseudo-Euclidean metric, for example, in the system of reference of the Earthling, according to (1) and (4), \( d\tau = dt \), which means the independence of the scales of the coordinate axes of the eigentime and the coordinate time from the spatial coordinates, that is, the eigentime "flows" equally at all points of the pseudo-Euclidean space. However, according to (4), in general case this is not so: eigentime \( \tau' \) due to the dependence of the right part (4) on the spatial coordinates can "flow" in different ways at different points in space.

The cosmonaut, when calculating his own time, will have to take into account that he, like the Earthling is in the gravitational field, since the movement of the Earthling is not constant, and the components of the metric of the cosmonaut \( \dot{g}_{00} \) is determined by the potential \( \varphi \) (5):

\[
g_{00}' \approx 1 + 2\varphi(x')
\]

From (5), for example, for a homogeneous field \( \varphi (x^a) \) by means of identical transformations, it follows:

\[
g_{00}' \approx 1 + 2\varphi(x') = 1 - 2a' x' = 1 - (a' t')^2 = 1 - (V'(t'))^2
\]

Here \( a' \) is the acceleration experienced by the Earthling in an equivalent gravitational field. The product of this acceleration for the distance \( x' \), taken with the opposite sign coincides with the potential difference between the points of motion in the equivalent gravitational field.

From (4) and (6) follows

\[
\tau' = \int_{x}^{x'} \sqrt{1 - V'(t')^2} dt'.
\]

Footnote 1: For the pseudo-Euclidean metric of Earthling \( g_{00} = 1 \).
The transformations of coordinate velocities to GTR are determined by the formulas of the 4-coordinate transformation. At the same time, neither the classical nor the relativistic law of velocity addition retains its form—it can take a rather arbitrary form. At the same time, the solutions of the Hilbert-Einstein equations allow for arbitrary transformations of the space-time coordinates with a change in the metric. We use this fact to determine the type of 4-coordinate transformation so that in (3) there would be a relation:

$$2g'_{a0}dx'^a dt' + g'_{a\beta}dx'^\alpha dx'^\beta = 0; \quad (\alpha, \beta = 1, 2, 3)$$

Then, in one-dimensional motion along the axis $x'^1$ we get

$$\frac{dx'^1}{dt'} = -2 \frac{g'_{10}}{g'_{11}},$$

that will determine the law of motion $V'(t')$ in (7). We see the complete identity of formulas (2) and (7). Thus $\tau = \tau'$, since they represent the length of the same world line ($c = 1$).

Thus, using approximate formulas (5), (6) and (9), we determined the component of the metric tensor of the cosmonaut metric $g'_{00}$ through the potential $\varphi$. As a result of the application of these formulas, we obtained the ratio (7), which gives an approximate solution for the eigentime of the Earthling, at least in a homogeneous field.

Uniform acceleration can be achieved only in the local time case and cannot be considered as possible for long time intervals. Therefore, the solution with constant acceleration in a homogeneous gravitational field must be considered as a special case and the approximation in which the formula (6) is given.

With sufficiently smooth metric and 4-coordinate transformation functions, the pseudo-Riemann space-time can be converted locally to the equivalent pseudo-Euclidean. Therefore, the conclusions for a homogeneous field will have sufficient generality.

To establish the exact fact of the symmetry of the twin brothers in relation to the acceleration and equivalent to the gravitational field and its transformation into the identity of their eigentimes, the approximate formulas are not enough. But here the essential circumstance is that the eigentime at $c = 1$ is the length of the world line, that is, the interval of the world line in the pseudo-Riemann 4-space-time between the moments of switching on and off the rocket engines. The interval is an invariant of 4-coordinate transformations, i.e. $\tau = \tau'$.

In addition, Lorentz transformations, with time-dependent speed of the astronaut reference frame can accurately solve the problem about the time intervals of the fragments of the journey of the astronaut and Earthling. However, to obtain such a solution, it is necessary first of all to present the thrust force of rocket engines and the equation of motion of an astronaut in a covariant form.

Thus, the segments of the twin brothers' eigentimes corresponding to the live-time of travel during the astronaut's acceleration are equal to $\tau = \tau'$, and the inclusion of the acceleration does not violate the symmetry due to the appearance of an equivalent gravitational field in the other.

However, the second question arises:

1. What is the symmetry of GTR, if this formula is derived from the invariance of the interval and pseudo-Euclidean space-time, that is, in fact, for the case of STR?

Indeed, the travel time transformation formulas (2) and (7) are derived from the pseudo-Euclidean space-time relations. But the paradox of gemini arose in STR. The consideration of symmetry in general relativity, requires further explanation. But there is another logic.
Generalization of inertial reference systems in GTR is the so-called geodetic systems, that is, systems, which are not affected by the forces of non-gravitational origin-unaccounted in the space-time metric.

It is known that the general transformations of the pseudo-Riemann space do not change the properties of geodesic reference systems (as well as the inertial property of a given reference system does not change when the observer moves from his inertial system to another inertial). In addition, in GTR, in the "pure case", there is no need even to ensure the return of the astronaut, as when considering the problem in STR: it is possible to set closed trajectories, such as the movement of the Moon around the Earth. Because of this reference system, associated with the astronaut and the Earthman are completely symmetrical and equal with respect to the properties of geodetic. And among them it will be impossible to find a dedicated candidate to make him "younger".

The inclusion of the rocket engine once again brings the system of reference of the astronaut from the class of freely moving systems (geodetic). Again there is asymmetry and the ability to mark one of the twins.

Yes, and the asymmetry appears, and the ability to distinguish the reference system. But it is no longer possible to calculate your eigentime available means: we go beyond the means of accounting FOR the characteristics of free movement. To account for this factor and subsequent correct conclusions, it is necessary to introduce a covariant expression of the emerging new force into the astronaut's equations of motion and continue the analysis further to obtain the result "until the number".

Thus, we have returned in essence to the original formulation of the paradox, only instead of inertial systems we have geodesics. The symmetry of the reference systems has been preserved, although, perhaps, at different coordinate time intervals, but the solution of the paradox is achieved, as in the previous case, by considering one's eigentime as the true time of life of gemini during the trip of the cosmonaut brother and waiting for his brother-Earthling. In addition, it should be expected that with the correct definition and introduction of an external force in the equations of motion, the symmetry will remain with respect to the transformations of 4-coordinates in the form of covariance of the equations of motion.

Thus, neither STR nor GTR deprives the condition of the problem of the symmetry property of the situation with the twin brothers. In STR, this symmetry is relative to the Lorentz transformations, in GTR — symmetry is relative to the general transformations. GTR itself was "invented" for the generalization of Lorentzian relativity to general relativity principle, which determined the impossibility of the twin paradox in the framework of general relativity. The solution of the problem of twin paradox by means GTR for pseudo-Euclidean metrics allows to get rid of tricks associated with the consideration of additional inertial reference systems to ensure the return of the astronaut to Earth.

Formulas (2) and (7) show that the "flow of time" is not determined by acceleration, it is determined by the velocity of motion. However, the velocity parameter is a topological link between continuous space and continuous time. In this case, the twin paradox should be considered as a topological paradox associated with the representation of space-time relations as continuous.

A simple resolution of the paradox is achieved by quantizing the time axis or by transitioning to an affine evolutionary time for calculating the age of twin brothers. This approach allows us to explain the phenomenon associated with the comparison of the lifetimes of the \( \pi \)-mesons resting in the laboratory frame of reference and arriving from space and answer the following question:

\[ \text{Are the results of experiments with } \pi \text{ mesons a confirmation of the time dilation effect?} \]

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The following fact shows the effect of the apparent "time dilation". It is known that $\pi$-mesons arriving from space with a relativistic speed, have the "time of life" significantly more than its laboratory counterpart-twin. We have already considered the resolution of this phenomenon with (iii. See, for example, the Application: "time delay" always occurs in those hours whose readings are compared with the readings of the other two). The explanation of this fact is also possible on the basis of the concept of the point evolutionary time that is eigentime.

The eigentime of the $\pi$-meson, like any elementary particle, is discrete and consists of two events: birth and decay. Between these events, the elementary particle is in the state of identity with itself. According to the time measurement procedure described in [1], all these intermediate states between birth and decay must be "glued" to the state of birth of the particle. The discrete pair of events - birth and decay, as absolute ones, will be unambiguously fixed in any frame of reference both in number (pair) and in results (birth and decay).

The macroscopic coordinate time measured in the experiment is represented by a continuous set of continuum's power. Lorentz transformations, like any more General point ones, transform the coordinate time interval, which includes only two events of the particle's own life, into one of the new elements of the class of power-equivalent continuous sets, again including a pair of States of birth and decay. This allows us to dissect the causes for the lack of general physical measurability - coordinate's and eigen's times in the point's evolutionary.

The difference between the topologies of the set of particle states (the discreteness of eigentimes) and the mapping of states to the continuous coordinate time axis (the continuity of the coordinate time representation) leads to an inadequate interpretation of the situation associated with the interpretation of the own life time of the elementary particle.
However, the phenomenon with meson twins, as it is easy to see, is simply solved by quantization of the eigentime and transition to the affine evolutionary time on the coordinate axis.

Eigentime as an invariant of continuous 4-coordinate transformations and its relation to coordinate time play a key role in the resolution of the twin paradox. Therefore, it is very useful to consider the properties of this connection in the three hypostases of the organization of matter. The known image of Boron [2] is supplemented with the left column (see Fig. 1).

The complexity of the description of space-time relations at the micro level (right column) in *PCM-topology* is explained by the fact that for the primary elements of geometry or topology in quantum mechanics there are no point prototypes. However, in the example with π-mesons, it is possible to consider a pair of events of birth and decay of a particle as discrete. Without going into the structure of the processes of birth and decay as some internal processes, we can consider them as elementary and complete events on the axis of their own time, that is, as a point for geometry. This allows the particle's eigentime to be displayed on the continuum axis, but in a discrete scale. It is obvious that a pair of events, which is included in a limited continuous set (axis segment) as a number equal to two, will remain an invariant with respect to any continuous axis transformations in the classical topology, despite the fact that the same transformations will change the scales and scales of the axis itself. A limited set, which is already considered as the interval of the continuous time axis of the macro level, containing a pair of events of birth and decay, and after the transformation will also contain this pair of events, but at a changed scale. Thus, the discrete eigentime will remain an invariant under continuous transformations, in particular, under transformations of macroscopic coordinate time. This allows the particle's eigentime to be displayed on the continuum axis, but in a discrete scale. It is obvious that a pair of events, which is included in a limited continuous set (axis segment) as a number equal to two, will remain an invariant with respect to any continuous axis transformations in the classical topology, despite the fact that the same transformations will change the scales of the axis itself. A limited set, which is already considered as the interval of the continuous time axis of the macro level, containing a pair of events of birth and decay, and after the transformation will also contain this pair of events, but at a changed scale. Thus, the discrete eigentime will remain an invariant under continuous transformations, in particular, under transformations of macroscopic coordinate time.

Returning to Fig. 1, note the intermediate position of classical theory between mega- and microlevel descriptions. In contrast to the micro-level case, the possibility of conceptual construction of physics on the basis of the concept of point physical bodies and dimensionless time intervals appears at the macrolevel. These point features and intervals (events) can be compared to the basic geometry elements in the *PMC-topology* under certain conditions. The same conditions are realized in the approximation of the sizes and ages of "bricks" that make up the actual physical body and events that are obviously larger than the characteristic parameters in the microcosm, that is, to represent these "building blocks" as having no dimensions in comparison to the macrobodies. In this case, it becomes possible to describe the space-time relations between material points as continuous relations in *PMC-topology*.

Regarding the transformations of the coordinates describing the relative positions of 4 points of the Minkowski space it is necessary to say the following. An important point here is the question of the choice of invariants of transformations that should become standards in the arithmetization of space-time relations. To date, this is either two independent standards of length and time (classic, middle column), or a single standard signal propagation speed standard, which is taken as the speed of light in vacuum (STR, left column). Accordingly, there are two types of 4-coordinate transformations: Galileo's and Lorenz's. For both types of transformations, eigentime is invariant. However, the coordinate's times are transformed in different ways. If the identity of eigentime and coordinate's times is preserved for Galileo's transformations, the Lorentz transformations violate this identity; the same can be said about the General transformations of 4-coordinates of pseudo-Riman space-time in GTR: they do not preserve the equality of their eigentime and

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7) *PMC-topology* – point-metric classical topology.

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coordinate's times. Thus, in the general case between classical and relativistic coordinate continuous times there are "scissors". It is these "scissors" that gave rise to the twin paradox in STR when comparing the coordinate times of different reference systems.

Classical standards of length and time have a common area of applicability with a single relativistic standard of speed at low speeds and weak gravitational fields. The identity of eigentime and coordinate times according to the results of the classical method of arithmetization of space-time relations determines the consistent use of the universal concept of time in physics as continuous in classical physics (at the macrolevel).

The invariance of eigentime at arbitrary the point transformations of 4-coordinates resolves the paradox and allows to preserve the metric measurability of eigentime, leaving only the property of the affine ordering (left column) of the coordinate's time at its continuity.

Thus, the general analysis of the data Fig.1 lets say: the transition from macro- to micro- lost the continuity of their eigentime while maintaining continuity of the coordinate (macro-) time; during the transition from macro- to mega- lost physical measurability of the coordinate time according to classical standards, while preserving the measurability of its eigentime on a single standard speed. In addition, when discrete presentation of their eigentime, "scissors" between the coordinate and eigentime also disappear when you measure time in discrete dimensionless units in the number of distinguishable events.

When analyzing the properties of flying and resting p-mesons, their eigentimes acquire the property of discreteness, which leads to a change in the topological properties of space-time relations at the micrlevel (the right column). In particular, there are difficulties in determining the velocities as derivatives of such time. This is exacerbated by the interpretation of the results of experiments A. Aspect [3].

What can be said about the paradox, the appearance of which was demonstrated by two "Cheshire cats" using the results of the Aspect's experiments. Recall that the need to comply with the principle of genetic identity involves maintaining the integrity of the system when it changes.

There are two possible solutions to the photon version of the problem:

1. We must consider a two-photon system as consisting of two integral subsystems (photons).
2. We must consider it as a single unity, non-localized point system, described by a single state function. The linearity of the wave equation allows us to work with the aggregate wave function obtained as superpositions of solutions for the first and second photons.

If the first leads to a contradiction with STR, the second – to a contradiction with the classical topology.

Thus, summarizing all of the above about the data Fig.1, it can be concluded that the space-time paradoxes at the micro level have a topological nature, and at the mega-level arise due to the lack of a new standard of measurements of space-time relations.

Links
[3]. Касимов В.А. Некоторые топологические парадоксы СТО. Новосибирск. 2014 г. [https://www.academia.edu/32427340/]
[4]. Касимов В.А. О постулате постоянства скорости света в СТО. Новосибирск. 2015 г. [https://www.academia.edu/32427342/]
[5]. В. Касимов. О втором постулате СТО. Новосибирск. 2015 г. [https://www.academia.edu/32452588/]

Again about the "Twin paradox"
Appendix

Inertial reference system $S'$ with a clock $c$ placed at the origin, moving with constant speed $\vec{V}$ relative to the stationary system $S$. The system $S$ contains a pair of its clocks, (a) − at the origin and (b) − at a distance $d = x$ from the origin along the axis $X$. Clock $c$ of the system $S$ first fly past the clock $a$, then−past the clock $b$. The $a$ and $b$ clocks of the system $S$ are pre-synchronized. Synchronize clock $c$ with clock $a$ to the time of their meeting each other. The spatial point of this meeting is taken as the origin of the two systems.

We are interested in two events:

1. event 1 - meeting hours $c$ with hours $a$;
2. event 2-meeting hours $c$ with hours $b$.

The coordinates of the first and second events in the system $S$:

$x_1 = 0$, $t_1 = 0$; $x_2 = d$, $t_2 = d/V$.

The coordinates of the same events in the system $S'$ are calculated according to the synchronization and Lorentz transformation condition:

$x' = \frac{x - Vt}{\sqrt{1 - V^2}}$, $t' = \frac{t - Vx}{\sqrt{1 - V^2}}$,

what gives:

$x'_1 = 0$, $t'_1 = 0$; $x'_2 = 0$, $t'_2 = (d/V) * \sqrt{1 - V^2}$.

The difference between the hours will be

$t_2 - t'_2 = (d/V) * (1 - \sqrt{1 - V^2})$,

which is always positive, i.e. $t_2 > t'_2$.

*****

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Again about the “Twin paradox”
V.A. Kasimov. Again about the "Twin paradox" (English version)

Abstract

As often in discussions of the theory of relativity denied the existence of the twin paradox, there is a need to dwell on this again. It is shown that the formal means of SRT and GRT a paradox twins are not resolved.