The Dirac Operator for Lie Groups

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1 Recalls of Lie group theory

A Lie group $G$ is a differentiable manifold with differentiable group structure [W]. The tangent fiber bundle at any point is the Lie algebra, due to the product of the Lie group. So a vector field is a map $m : G \rightarrow g$ of $G$ in the tangent space at unity. The Killing form is an invariant Riemann metric over the group.

2 The Dirac operator over a Lie group

Let be an orthonormal basis $E_i$ of the Lie algebra $g$.

Definition 1. The Dirac operator $\mathcal{D}$ for the Lie group $G$ is acting over the vector fields:

$$\mathcal{D}(m) = \sum_i [E_i, \nabla E_i(m)]$$

with $\nabla$ the Levi-Civita connection over the Riemann manifold $G$ and $[,]$ the Lie bracket of vector fields.

Theorem 1. The definition is independant of the choice of the basis.

Demonstration 1.
The choice of another basis $E'_i$ define an orthogonal matrix, so:

$$E'_i = \sum_j a_{ij} E_i$$

and as $\sum_j a_{ij} a_{kj} = \delta^k_i$, the Kronecker symbol, the Dirac operators are identical.

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References
