Introduction

The 2018 Physics Today essay competition invites participants to identify a ‘significant advance’ in his or her field since the millennium that deserves wider recognition among non-experts, and to write an essay that describes the advance, how it was made, and why it’s important[1]. This essay takes quantum mechanics to be the field of interest, introducing ‘non-experts’ to a new synthesis of math and physics, of geometry and fields, a computationally precise yet intuitive representation of wavefunctions and their interactions at all scales, allowing for a common sense interpretation of quantum phenomena and resolution of most if not all quantum paradoxes. It’s all about the wavefunction, the foundation, fundamental, quantum philosophy, quantum logic. As yet we are all non-experts.

The Advance

The advance is a synthesis of two threads - geometric extension of Clifford algebra[2–6], and generalization of impedance quantization[7–9]. The first extends the wavefunction beyond point particle quarks and leptons to the full Pauli wavefunction of 3D space, and models wavefunction interactions by the geometric product of Geometric Algebra (GA). The second introduces impedance quantization to quantify amplitude and phase of energy transmission during Geometric Wavefunction Interactions (GWI). The model is naturally gauge invariant, finite, and confined.

Two of the four historical threads of figure 1, the two outer threads introduced here, are mostly unknown to the two inner threads they encompass. Their mainstream absence follows from a series of historical accidents[10].

Not shown is the web of connections between ‘gauge’ and ‘string’ theories, both being quantum field theories. This is unlike the two threads comprising the synthesis of math and physics in geometric wavefunction interactions. Neither of itself is a quantum field theory, but rather only their synthesis. Connections with early gauge/string theory milestones are reasonably well understood in the GWI model, later milestones not yet so much.

Figure 2 presents an outline of this section. The first subsection introduces the fundamental geometric objects of Euclid. The second takes them to comprise components of the vacuum wavefunction. The third assigns topologically appropriate quantized electric and magnetic fields to wavefunction components, and the fourth introduces impedance quantization of electromagnetic wavefunction interactions. The last illustrates how the scattering matrix of single measurement observables, the particle physicist’s holy grail, emerges from the geometric algebra of wavefunction interactions[11, 12].

FIG. 1: Four threads of the geometric wavefunction.
Geometry - Fundamental Geometric Objects

To gain appreciation of GA as applied to wavefunctions and their interactions, one begins with the geometry of Euclid - point, line, plane, and volume elements. Perhaps not coincidentally, these are the fundamental geometric objects of the particle physicist’s Clifford algebra in three dimensions, the Pauli algebra, less abstract and more intuitive than mainstream matrix representations. Pauli matrices are the three basis vectors of space in GA, Dirac matrices the four of spacetime. Geometric representation is physical space, physical spacetime.

Space - the Vacuum Wavefunction

What remains to have a vacuum wavefunction model in the geometric representation is to specify orientation of wavefunction components shown in figure 2. The scalar is customarily taken to be the point charge quantum in quantum field theory. It has the time orientation degree of freedom, the sign of phase advance of its oscillations - minus or plus, clockwise or counter clockwise, left hand or right, particle or antiparticle. Vectors and pseudovector bivectors have the three orientational degrees of freedom of space. The pseudoscalar trivector is volume charge, the topological dual of the scalar[13]. These comprise the eight component Pauli vacuum wavefunction.

Fields - Five Fundamental Constants

To assign quantized electric and magnetic fields to the fundamental geometric objects of the 3D Pauli wavefunction is straightforward, requires only five fundamental constants - electric charge quantum \( e \), permittivity of space \( \varepsilon_0 \), speed of light \( c \), Planck’s constant \( h \), and electron mass \( m_e \). Of the five, only combinations of the first four are needed to quantize the eight topologically appropriate fields. Compton wavelength of the electron, the lightest rest mass particle (excepting perhaps the neutrino), sets the scale of space[7].

Interaction Impedances

Given that, in quantum field theory, wavefunction fields are quantized, it is unavoidable that impedances of Geometric Wavefunction Interactions (GWI) are likewise quantized at the Compton wavelength.

There are two essential points. First, what matters are not absolute impedances, but rather their relative values, whether impedances are matched. In this they are like the energy whose transmission they govern. Absolute values of energy are said to be (singularities complicate the question) of no consequence, only differences.

The second essential point recognizes fundamental differences between scale-dependent and invariant impedances. Invariant impedances include vector Lorentz of quantum Hall and Aharonov-Bohm effects, centrifugal, chiral, Coriolis, and three body. Associated potentials are inverse square, the forces often termed ‘fictitious’[14]. Resulting motion is perpendicular to applied force. They cannot do work, cannot communicate information, communicate only quantum phase, not a single measurement observable. Like gravity, they cannot be shielded. They are the acausal channels of non-local entanglement, correspond to the rotation gauge fields of gauge theory gravity[9, 15–18].

Scale dependent impedances include Coulomb, scalar Lorentz, and dipole-dipole, with \( 1/r \) and \( 1/r^3 \) potentials. They are causal and local, communicate both amplitude and phase, correspond to translation gauge fields.

A partial impedance network of the interaction of two Pauli wavefunctions is shown in figure 3. Correlations of unstable particle coherence lengths with nodes of the impedance network follow from the fact that impedances must be matched in both amplitude and phase for the energy transmission essential to particle decay. Phase information is contained in particle lifetimes/coherence lengths. Amplitudes and branching ratios are calculated from mismatches[19].
FIG. 3: Correlation of lifetimes/coherence lengths of unstable particle spectrum\cite{20,22} with impedance nodes of the network generated by excitation of the lightest rest mass manifestation of the vacuum wavefunction\cite{23}.

**Spacetime and the S-matrix**

Wavefunction interactions are modeled by the geometric product, the sum of dot and wedge products shown in figure 4. The product multiplies not numbers or symbols, but fundamental geometric objects, changing their dimensionality, their ‘grade’. For instance, the product of two vectors is scalar and bivector, of two lines point and plane.

While not essential to this essay, it should be noted that grade increasing operations break topological symmetry\cite{24}. This is a possible origin of the topological inversion of pseudovector and pseudoscalar of the Pauli algebra, and thereby of dark matter\cite{25}.

Figure 5 show the S-matrix generated by taking the geometric product of two Pauli wavefunctions\cite{20}. As in the Dirac equation, wavefunctions at top and left can be associated with electron and positron. Their interaction generates a 4D Dirac algebra of flat Minkowski spacetime, arranged in odd transition (yellow) and even eigenmodes (blue) by grade/dimension. Mode impedances indicated by symbols (triangle, square,...) are plotted in figure 3.
How it was Made

The value of practical experience, the wisdom of the body, is not to be underestimated. Initial realization of a then-nebulous essential concept came early\cite{29}. A basic vibratory piledriver/extractor comprises two synchronized counter-rotating eccentric weights, transforming 2D rotor motion into 1D motion of their containment, in some sense a mechanical analog of Dirac electron/positron bivectors’ counter-rotating phases. Design, construction, and operation of such a device was fertile ground for emergence of a background-independent analysis\cite{30} of the two body problem and Mach’s principle. Absence of what it revealed from mainstream physics motivated what followed.

Thirty-five years of weekends and evenings in libraries and on the web passed before meaning of that analysis became clear, before understanding came from musical acoustics that what is calculated in the analysis is mechanical impedance\cite{31}. Conversion to electromagnetic impedances followed from analysis of electromechanical oscillators\cite{7}.

During those years grad school, the Michigan spin physics group\cite{32}, and MIT Bose condensation group\cite{33} deepened the physics appreciation. Twenty-five years at Brookhaven brought a gut-level sense of phase and coupled modes from implementing phase-locked feedback loops for betatron tune and coupling in RHIC, Tevatron, and LHC\cite{34}. Design of electron synchrotron light source instrumentation\cite{35} brought realization that what governs the flow of energy in the photon-electron interactions of QED, the photon near-field impedance\cite{36}, is also absent from the mainstream. This motivated the generalization of impedance quantization\cite{8}.

The last essential step closed the circle, starting around 2010 with gradual transition from a focus on calculation to quantum foundations, a focus on basics of the unobservable wavefunction and wavefunction interactions that spark ongoing proliferation of the quantum philosopher’s interpretations\cite{11,38,39,40}.
Why it’s Important

The preface to the 50th anniversary second edition of Professor Hestenes’ original text makes four claims for innovation in SpaceTime Algebra, summarized here:

- STA enables a unified, co-ordinate free formulation for all of relativistic physics.
- Pauli and Dirac matrices are basis vectors in space and spacetime, no connection to spin.
- STA reveals that the unit imaginary in quantum mechanics has its origin in spacetime geometry.
- STA reduces the mathematical divide between classical, quantum, and relativistic physics, particularly rotors.

To these we add the following claims of the Geometric Wavefunction Interaction model:

- gauge invariance - Impedances shift phases. GWI is naturally gauge invariant.
- finiteness - Impedance mismatches provide natural QED cutoffs. GWI is naturally finite.
- confinement - Confinement is the flip side of finiteness. GWI is naturally confined.
- background independence - In STA, motion is described with respect to the object in question. Similarly, in the two body problem motion is with respect to one of the two. There is no background. GWI is naturally background independent.
- photon-electron interaction - Dirac spoke to the core of the model in asserting that “Until we have a really satisfactory explanation of how electrons and photons interact, it will hardly be possible to go on and explain the other particles.” With GWI we have a satisfactory explanation.
- gravitation - Matching quantized impedances at the Planck scale reveals an exact identity between electromagnetism and gravity.
- quantum interpretation - GWI wavefunctions exist as electromagnetic fields configured as geometric objects in 3D space, interacting via Maxwell’s equations via a network of quantized impedances. Wavefunctions and their interactions can be visualized. This permits resolution of many if not all paradoxes found in proliferating worlds of quantum interpretations.

Discussion

Feynman’s gifts included great talent for expressing the passion of the physicist: “...people who have studied (physics) far enough to begin to understand a little of how things work are fascinated by it and this fascination drives them on in this investigation that the race is making into its own environment.”

Just as an intuitive understanding of electron and photon interactions is the foundation of a proper understanding of quantum mechanics, and a proper understanding of quantum mechanics is the foundation of this investigation we are making into our own environment, our philosophical insight is the foundation of it all.

As Carlo Rovelli recently pointed out, no one says this better than Einstein himself:

“A knowledge of the historic and philosophical background gives that kind of independence from prejudices of his generation from which most scientists are suffering. This independence created by philosophical insight is - in my opinion - the mark of distinction between a mere artisan or specialist and a real seeker after truth.”

Put more strongly, it is sometimes said that: “Scientists do not do anything unless they first get permission from philosophy.”

Quantum Mechanics seems to ask “Why the quantum?” However this might better be phrased as “Why the wavefunction?” For this we have no answer, except perhaps this - it seems to work.
Conclusion

With an understanding of geometric wavefunction interactions it seems possible to have a model that is naturally gauge invariant, finite, and confined. It seems possible to understand gravitation as impedance mismatched electromagnetism, and similarly weak and strong nuclear forces as confinement by reflections from impedance mismatches. It seems possible to have an intuitive, real world understanding of the measurement problem, to remove the ambiguities and resolve the outstanding quantum paradoxes. Finally, deBroglie wavelength is Doppler sideband of Compton[44]. One cannot help but feel excitement over what applications might arise in condensed matter.

Acknowledgements

We thank David Hestenes for the gift of Geometric Algebra.

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