Refutation of the modern, general, and strong Löwenheim–Skolem theorem

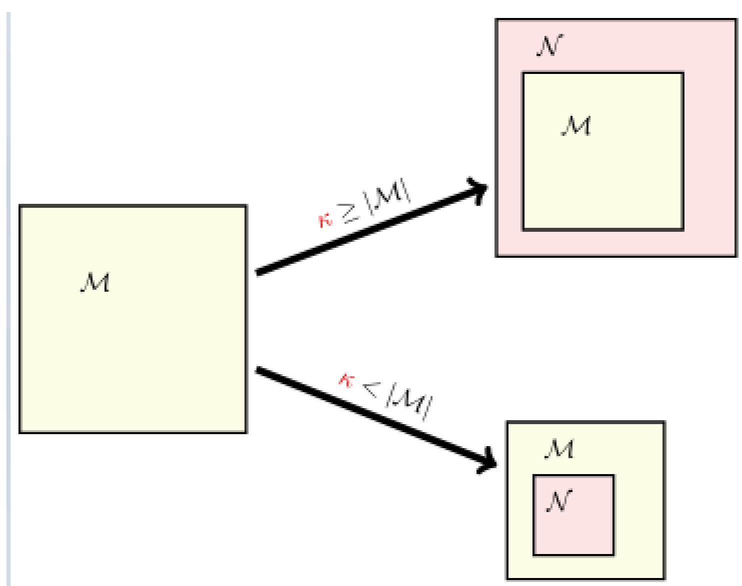
Abstract: We evaluate the equation for the modern, general, and strong Löwenheim–Skolem theorem. It is not tautologous, hence refuting the upward and downward parts. These form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B); ~(y < (z@z)) |y|.
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: en.wikipedia.org/wiki/Löwenheim–Skolem_theorem

Precise statement
Illustration of the Löwenheim–Skolem theorem



The modern statement of the theorem is both more general and stronger than the version for countable signatures. In its general form, the Löwenheim–Skolem theorem states that

for every signature σ, every infinite σ-structure M and every infinite cardinal number κ ≥ |σ|, there is a σ-structure N such that |N| = κ and
if κ < |M| then N is an elementary substructure of M;
if κ > |M| then N is an elementary extension of M. (1.1.1)

LET p, q, r, s: κ , M, N, σ.

((#s&q)&~(#s>~((s@s)>#p)))>((%s&r)>(~((s@s)>(#r=p)) &
(((~((s@s)>#p)<q)>(r>q))&~((q>r)>(~((s@s)>#p)>q))))) ;
TTTT TTTT TTTT TTCC (1.1.2)

Remark 1.1.2: Eq. 1.1.2 is *not* tautologous, hence refuting the modern, general, and strong theorem.

The theorem is often divided into two parts corresponding to the two [k, |M| conditions] above. The part of the theorem asserting that a structure has elementary substructures of all smaller infinite cardinalities is known as the downward Löwenheim–Skolem theorem. .. The part of the theorem asserting that a structure has elementary extensions of all larger cardinalities is known as the upward Löwenheim–Skolem theorem. ..