# Gravitoelectromagnetism. II.
# Speed of Light in Gravitational Field

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## Abstract

From four Maxwell-Hertz equations for the vacuum and two modified material equations, two equations describing the propagation of the electromagnetic wave with the slower speed, the stronger the gravitational field, were obtained.

**Keywords:** Maxwell-Hertz equations, material equations, Minkowski space-time, Schwarzschild space-time, conformally flat space-time, wave equation, Black Hole Universe.

## 1. Introduction

In the theory of relativity, the speed of light is determined by the disappearance of the square of the differential of space-time distance. The general theory of relativity examines the deformations of space-time through the masses. From the general form of the metrices of such space-times it follows that the speed of light is the smaller the stronger the gravitational field and the speed of light is only constant in the conformally flat space-times.

Due to the simplicity in metric equations considered in this work we will limit ourselves to only two variables: spatial and temporal. The c symbol will mean the standard value of the speed of light.

From the four Maxwell-Hertz equations for the vacuum and two modified material equations, we get two equations describing the propagation of the electromagnetic wave with the slower speed, the stronger the gravitational field.

## 2. Speed of light in Minkowski space-time

$$\begin{tikzcd}
(ds)^2=(dx)^2-c^2(dt)^2 \arrow[ddd] & \\
 & (ds)^2=0 \\
 & v_{light}^2 \equiv \left(\frac{dx}{dt}\right)^2 \\
v_{light}^2=c^2 &
\end{tikzcd}$$

## 3. Speed of light in conformally flat space-time

$$(\mathrm{ds})^2 = [\Phi(\mathrm{x},\mathrm{ct})]^2\left[(\mathrm{dx})^2 - \mathrm{c}^2(\mathrm{dt})^2\right]$$

$$\Bigg\downarrow \quad \begin{aligned} &(\mathrm{ds})^2 = 0 \\ &\mathrm{v}^2_{\mathrm{light}} \equiv \left(\frac{\mathrm{dx}}{\mathrm{dt}}\right)^2 \end{aligned}$$

$$\mathrm{v}^2_{\mathrm{light}} = \mathrm{c}^2$$

## 4. Speed of light in the gravitational field according to Einstein

Albert Einstein in the papers [1, 2, 3, 4, 5] discussed the problem of the definition of the speed of light in the gravitational field, proposing the following expression:

$$\mathrm{v}_{\mathrm{light}} = \mathrm{c}\left(1 - \frac{|\Phi|}{\mathrm{c}^2}\right)$$

$\Phi$ – gravitational potential

## 5. Speed of light in Schwarzschild space-time

Schwarzschild metric is sometimes written [6] in the following form:

$$\mathrm{ds}^2 = \left\{\frac{\mathrm{x}^2}{\mathrm{r}^2}\left[\left(1 - \frac{\mathrm{r_S}}{\mathrm{r}}\right)^{-1} - 1\right] + 1\right\}\mathrm{dx}^2 + \left\{\frac{\mathrm{y}^2}{\mathrm{r}^2}\left[\left(1 - \frac{\mathrm{r_S}}{\mathrm{r}}\right)^{-1} - 1\right] + 1\right\}\mathrm{dy}^2 +$$

$$+\left\{\frac{\mathrm{z}^2}{\mathrm{r}^2}\left[\left(1 - \frac{\mathrm{r_S}}{\mathrm{r}}\right)^{-1} - 1\right] + 1\right\}\mathrm{dz}^2 - \left(1 - \frac{\mathrm{r_S}}{\mathrm{r}}\right)\mathrm{c}^2\mathrm{dt}^2 +$$

$$+\frac{2}{\mathrm{r}^2}\left[\left(1 - \frac{\mathrm{r_S}}{\mathrm{r}}\right)^{-1} - 1\right](\mathrm{xy\,dxdy} + \mathrm{xz\,dxdz} + \mathrm{yz\,dydz}), \quad \mathrm{r_S} = \frac{2\mathrm{GM}}{\mathrm{c}^2},$$

$$\Bigg\downarrow \quad \mathrm{x} = \mathrm{r}, \quad \mathrm{y} = \mathrm{z} = 0, \quad \mathrm{dx} = \mathrm{dr}, \quad \mathrm{dy} = \mathrm{dz} = 0$$

$$(\mathrm{ds})^2 = \left(1 - \frac{\mathrm{r_S}}{\mathrm{r}}\right)^{-1}(\mathrm{dr})^2 - \left(1 - \frac{\mathrm{r_S}}{\mathrm{r}}\right)\mathrm{c}^2(\mathrm{dt})^2$$

$$\Bigg\downarrow \quad \begin{aligned} &(\mathrm{ds})^2 = 0 \\ &\mathrm{r_S} \equiv \frac{2\mathrm{GM}}{\mathrm{c}^2} \\ &\mathrm{v}^2_{\mathrm{light}} \equiv \left(\frac{\mathrm{dr}}{\mathrm{dt}}\right)^2 \end{aligned}$$

$$\mathrm{v}^2_{\mathrm{light}} = \mathrm{c}^2\left(1 - \frac{\mathrm{r_S}}{\mathrm{r}}\right)^2 = \mathrm{c}^2\left(1 - \frac{2\mathrm{GM}}{\mathrm{rc}^2}\right)^2$$

## 6. Idem per idem

Below we give an example of incorrect determination of the speed of light based on the Schwarzschild metric.

$$(ds)^2 = \left(1 - \frac{r_S}{r}\right)^{-1}(dr)^2 - \left(1 - \frac{r_S}{r}\right)c^2(dt)^2$$

$$\Big\downarrow \quad \begin{array}{l} (ds)^2 = 0 \\ r_S \equiv \dfrac{2GM}{c^2} \end{array}$$

$$c^2 = \left(\frac{dr}{dt}\right)^2\left(1 - \frac{r_S}{r}\right)^{-2} = \left(\frac{dr}{dt}\right)^2\left(1 - \frac{2GM}{rc^2}\right)^{-2}$$

The above equation is an algebraic equation of the fourth degree with respect to c.

$$c^4 - \left[\frac{4GM}{r} + \left(\frac{dr}{dt}\right)^2\right]c^2 + \frac{4G^2M^2}{r^2} = 0$$

## 7. Maxwell-Hertz equations and modified material equations in the presence of a gravitational field in a vacuum

$$\boxed{\begin{array}{l} \mathrm{rot}\mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t} \\ \mathrm{div}\mathbf{B} = 0 \end{array}} \qquad \boxed{\begin{array}{l} \mathrm{rot}\mathbf{H} = \dfrac{\partial \mathbf{D}}{\partial t} \\ \mathrm{div}\mathbf{D} = 0 \end{array}} \qquad \boxed{\begin{array}{l} \mathbf{D} = \varepsilon_0\varepsilon_G\mathbf{E} \\ \mathbf{B} = \mu_0\mu_G\mathbf{H} \end{array}}$$

**E** – vector of electric field intensity
**D** – vector of electric induction
**B** – vector of magnetic induction
**H** – vector of magnetic field intensity
$\varepsilon_0$ – vacuum permittivity
$\mu_0$ – vacuum permeability

$$\varepsilon_0\mu_0 = \frac{1}{c^2} \quad \text{or} \quad c = \frac{1}{\sqrt{\varepsilon_0\mu_0}}$$

$\varepsilon_G$ – relative electric permittivity of the gravitational field
$\mu_G$ – relative magnetic permeability of the gravitational field
$\varepsilon_0$ – describes the electric properties of the vacuum
$\mu_0$ – describes the magnetic properties of the vacuum
$\varepsilon_G$ – describes the electric properties of the gravitational field
$\mu_G$ – describes the magnetic properties of the gravitational field

From four Maxwell-Hertz equations and two modified material equations, assuming that

$$\frac{\partial \varepsilon_G}{\partial t} = 0, \quad \frac{\partial \mu_G}{\partial t} = 0,$$

after troublesome transformations, two equations can be obtained whose left sides will be in the form of the wave equation.

Wave equation for vector **E**

$$\boxed{\nabla^2\mathbf{E}-\mu\varepsilon\frac{\partial^2\mathbf{E}}{\partial t^2}=(\mathrm{grad}\,\mu)\times\frac{\partial\mathbf{H}}{\partial t}-\mathrm{grad}\left[\frac{1}{\varepsilon}(\mathbf{E}\cdot\mathrm{grad}\,\varepsilon)\right]}$$

Wave equation for vector **H**

$$\boxed{\nabla^2\mathbf{H}-\mu\varepsilon\frac{\partial^2\mathbf{H}}{\partial t^2}=-(\mathrm{grad}\,\varepsilon)\times\frac{\partial\mathbf{E}}{\partial t}-\mathrm{grad}\left[\frac{1}{\mu}(\mathbf{H}\cdot\mathrm{grad}\,\mu)\right]}$$

From the above two equations it follows that:

$$\boxed{\frac{1}{v_{\mathrm{light}}^2}=\varepsilon\mu=\varepsilon_0\mu_0\varepsilon_G\mu_G \quad \text{or} \quad v_{\mathrm{light}}^2=\frac{c^2}{\varepsilon_G\mu_G}}$$

In the case of the Schwarzschild space-time in a vacuum:

$$\varepsilon_G\mu_G=\left(1-\frac{r_S}{r}\right)^{-2}\approx\left(1-2\frac{r_S}{r}\right)$$

$r_S$ – Schwarzschild radius
$r$ – distance from the center of the source mass

On the surface of the Earth: $\frac{r_S}{r}\approx 1{,}5\times10^{-9}$.

In the Black Hole Universe [7, 8]:

$$\varepsilon_G\mu_G=\left(1-\frac{r^2}{R^2}\right)^{-2}$$

$r$ – distance from the Black Hole Universe
$R$ – radius of the Black Hole Universe

## 8. Final remarks

I proposed that gravity "enter" the vacuum material equations through the relative electric permittivity of the gravitational field and relative magnetic permeability of the gravitational field.
In [9] I have given which physical contents contain generally covariant Maxwell-Hertz equations, written in a modified three-dimensional form.

## References

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