A new Physical constant from the ratio of the reciprocal of the “Rydberg constant” to the Planck length

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Abstract
This study presents a unique set of solutions, using empirically determined physical quantities, in achieving a novel dimensionless constant $\alpha \left(1/R_\infty \right)/PL$ from the ratio of the inverse of the Rydberg constant to the Planck length. It is henceforth shown that the Lorentz Scalar coming into play, which we dub the Parana constant, necessitates us to interpret the Gravitational constant $G$ as being neither universal nor Lorentz Invariant. Just the same, the elementary charge in the MKS system should not by itself be considered as Lorentz Invariant, but the term $e^2/\varepsilon_0$, including its powers, ought to be. That being the case, the “Rydberg constant” must not, according to the present undertaking, be deemed a ubiquitous magnitude either, but the ratio of its reciprocal to Planck length would, in effect, be. The Parana constant is furthermore shown to exhibit meaningfulness as the proportion of the Planck mass to the electron rest mass. Throughout our derivations, we take the opportunity to reveal interesting features and deliberate over them.

Keywords
Rydberg constant, Planck length, Lorentz Invariance, Lorentz Scalar, Dimensionless Physical constant, Parana Constant.

1. Introduction

The Universal Matter Architecture (UMA) scaffolding [1, 2, 3, 4, 5, 6, 7] developed a few decades ago by the third co-author, which led him to derive Yarman’s Approach [8, 9] for all force interactions that resulted in YARK (Yarman-Arik-Kholmetskii) gravitation theory [10, 11, 12, 13, 14, 15, 16] (where a symbiosis between Quantum Mechanics and gravitation was harmoniously achieved — with the associated gravitational field energy becoming a non-vanishing quantity in all possibly definable reference frames), makes certain that the “theoretical speed of light in vacuum” $c_0$ (exactly 299,792,458 m/s) is “truly” a universal constant just like the Planck constant $h$ (having the dimensions Js or $m^2$ kg s\(^{-1}\)) and the elementary charge in the CGS unit system (bearing the dimensions statCoulomb = cm\(^{3/2}\) m\(^{1/2}\) s\(^{-1}\)). In other words, these quantities remain rigorously invariant under Lorentz Transformations when embedded within even a strong gravitational field of an immensely massive stationary body, or any other type of field an object under consideration would interact with.

However, as it shall soon be disclosed, “the elementary charge in the MKS system” — with this simply meaning a
specific Ampere times 1 second, where the MKS unit of electrical current “Ampere” is determined solely via the arbitrary assignment of the value of the magnetic permeability of classical vacuum in history as $\mu_0 = 4\pi \times 10^{-7}$ N/A² owing to the choice of placing two parallel electrical wires of ideal length and of negligible cross-section a meter apart to achieve $2 \times 10^{-7}$ Newtons attraction force between them (so long as a homogenous flow of charged particles in each is maintained) — should not by itself be thought of as Lorentz Invariant or constant, but the term $\varepsilon_0$ as well as its exponents, ought to be (where $\varepsilon_0$ denotes the permittivity of free space) [17].

In a similar vein, the present undertaking necessitates an interpretation of the Gravitational constant $G$ (possessing the dimensions $m^3 \, kg^{-1} \, s^{-2}$; and otherwise understandable as “acceleration of surface area per unit mass” from an extrapolation of its dimensions $[L^2/M * [L/T^2]]$) as being neither universal nor Lorentz Invariant [cf. 18, 19, 20, 21]. That being the case, we may refer to it as $G_{\oplus}$ in order to delineate a flexible Earth-bound magnitude.

Finally, with the elementary charge value in the MKS system and the “Gravitational constant” failing to represent universal quantities, followed by the implication that the electron rest mass ($m_{e0}$) should too get altered commensurately with force intensity as per Yarmans’s Approach [22, 23], we find the “Rydberg constant” to be not at all a ubiquitous invariant either.

The principal way to demonstrate these facts will be through the set of derivations below of a novel physical constant $\alpha_{(1/R,c)PL}$ as the ratio of the reciprocal of the “Rydberg constant” to the Planck length [24]. It will be seen that this new Lorentz Scalar has profound meaning for natural philosophy.

### 2. Inverse of the Rydberg constant to the Planck length: A new dimensionless invariant quantity

To begin with, let us equate, by a factor of $1/n$, the reciprocal of the known Rydberg constant (making thus $1/10973.731.5705365$ meters) to the known Planck length in the MKS unit system (considered to be 1.61639446559731E-35 meters) — all while remembering that $\varepsilon_0$ is classically (that is to say, following the era of J. C. Maxwell [25]) derived from the magnetic permeability of vacuum $\mu_0$ owing to the relationship $c^2 = \frac{1}{\varepsilon_0 \mu_0}$ as $\frac{10^7}{4\pi |c|^2}$ Farads per meter, where $|c|^2$ is the modulus of the square of the speed of light in empty space (just the number $299,792,458^2$).

Therefore:

$$\frac{8 \times h^5 \times c^5 \times 10^{28} \times 2 \pi}{m_e \times e^4 \times 2 \pi c^5} = n^2, \quad (1c)$$

$$\frac{(4 \times 4 \times 4) \times h^5 \times c^5 \times 10^{28} \times \pi}{m_e \times e^8 \times 2 \pi c^5} = n^2, \quad (1d)$$

$$\frac{h^5 \times c^5 \times 10^{28} \times \pi}{m_e \times e^8 \times 2 \pi c^5} = n^2, \quad (1e)$$

$$\frac{h^5 \times 10^{28} \times \pi}{m_e \times e^8 \times 2 \pi c^5} = n^2, \quad (1f)$$

$$\frac{n^2}{\pi^2} = \frac{n^2}{2 \times 10^{28} \times \pi^5 \times c^5} = n^2, \quad (1g)$$

$$\frac{n^2}{m_e \times e^4 \times 1 \text{ meter}^2} = \frac{n^2}{10^{28} \times 2 \pi c^5}, \quad (1h)$$

where

$$\frac{8 \times h^5 \times c_0 \times 10^{28} \times \pi}{m_e \times e^4} = \sqrt{\frac{G \varepsilon_0}{2 \pi c^5}}, \quad (1a)$$

$$\frac{8^2 \times h^6 \times c^2 \times 10^{5} \times \pi^4}{n^2 \times m_e \times e^8 \times (4\pi |c|^2)^3 \text{ meter}^4} = \frac{G \varepsilon_0}{2 \pi c^5}, \quad (1b)$$

$$\frac{8^2 \times h^6 \times c^2 \times 10^{5} \times \pi^4}{n^2 \times m_e \times e^8 \times (4\pi |c|^2)^3 \text{ meter}^4} = \frac{G \varepsilon_0}{2 \pi c^5}, \quad (1b)$$

$$v = \frac{\sqrt{\frac{10^{28} \times \pi^4}{2 \pi c^5 \times 299,792,458^3}}}{2 \pi c^5} = 1.4302940891066 \text{ m/s}$$

(indicating here an “enigmatic velocity”), with $n$ corresponding to our novel dimensionless quantity

$$\alpha_{(1/R,c)PL} = 5.63765262613852E+27$$

(i.e., Prana constant) and $1/n$ thus making

$$\alpha_{PL(1/R,c)} = 1.7737879775656E-28$$

if we rely on the latest values (without highlighting the related measurement uncertainties)

$$m_e = 9.1093835611E-31 \text{ kg}, \quad (2a)$$
\( h = 6.62607004081 \times 10^{-34} \text{ Js}, \)

\( c_0 = 299,792,458 \ (\text{m/s}), \)

\( e = 1.602176620898 \times 10^{-19} \ \text{C}, \)

\( \varepsilon_0 = 8.854187817620 \times 10^{-12} \ (\text{F/m}), \)

\( G_\Theta = 6.6754518 - 11 \ (m^3 \text{kg}^{-1} \text{s}^{-2})[19]. \)

Upon the revelation we have landed on at this stage (with \( G_\Theta \) representing an Earth-bound quantity), it is right away possible to crosscheck the one-to-one dimensional correspondence of the LHS to the RHS of the relationship emerging from Eq. (1h)

\[
\frac{\mathcal{C}_8^8}{v^8} = \frac{m^8 \cdot k^4 \cdot \varepsilon^4}{s^8};
\]

(3a)

ergo,

\[
\mathcal{C}_8 = \frac{m^8 \cdot k^4 \cdot \varepsilon^4}{s^8},
\]

(3b)

\[
\mathcal{C}_0 = \frac{8 \cdot m^8 \cdot k^4 \cdot \varepsilon^4}{s^8},
\]

(3c)

\[
\mathcal{C} = \sqrt{\frac{8 \cdot m^{12} \cdot k^4 \cdot \varepsilon^4}{s^8 \cdot \text{meter}^4}},
\]

(3d)

\[
\mathcal{C} = \frac{m^{(3/2)} \sqrt{k^4 \varepsilon^4}}{s}.
\]

(3e)

The last proportionality is none other than the MKS equivalent of the Lorentz Invariant CGS unit \( \text{StatCoulomb} \) (otherwise christened the “electrostatic unit” or ESU in the literature). The exact transformation to ESU (bearing the dimensions of \( \sqrt{hc} \)) is achieved via:

\[
\frac{\sqrt{10^7} \mathcal{C}_0}{\sqrt{100^2 \times 1000^2 \times 4 \pi \varepsilon_0}} = 10 \cdot |c_0| \cdot \frac{m^{(3/2)} \sqrt{k^4 \varepsilon^4}}{s}, \]

(4a)

\[
\frac{\sqrt{10^9} \varepsilon_{\text{ESU}}}{\sqrt{4 \pi \varepsilon_0}} = 4.80320467329146 \times 10^6 \sqrt{\varepsilon} \frac{cm^{(3/2)} \sqrt{F}}{s} \text{ (StatC)}. \]

(4b)

Hence, we have straightforwardly ascertained that it is not \( 1.602176620898 \times 10^{-19} \ \text{C} \) that is a universal constant, but instead

\[
\frac{\sqrt{10^9} \times 1.602176620898 \times 10^{-19} \ \text{C}}{\sqrt{4 \pi \times 8.854187817620 \times 10^{-12} \ \text{F/m}}} \]

that equates to Newtons force times surface area in squaremeters (i.e., \( Nm^2 \)), which otherwise bears the dimensions of \( hc \) the way these appear in the classical expression of the Fine Structure constant

\[
\alpha = \frac{e^2}{2 \varepsilon_0 hc};
\]

with the factor \( 10 \) in \( \sqrt{10^7+2} \) to counterbalance the \( 10 \) on the RHS of Eq. (4a) obviously not representing anything physical in the conversion.

Proceeding from Eq. (1g), we are able to find two alternative equalities that yield the same dimensionless value for \( \alpha_{(\text{UR } \varepsilon_0)\text{PL}} \):

\[
n^2 = \left(\frac{h^5 \cdot \varepsilon^4 \cdot \text{meter}^{-4}}{G \cdot m_0^2 \cdot e^8 \cdot \pi^3 \cdot |c|^3 \cdot \text{second}^3}\right) \cdot \frac{10^{28} \cdot \text{meter}^5}{2 \cdot \pi^3 \cdot |c|^3 \cdot \text{second}^3}, \]

(5a)

\[
n^2 = \left(\frac{h^5 \cdot 10^{7+4}}{G \cdot m_0^2 \cdot e^8 \cdot \pi^3 \cdot |c|^3 \cdot \text{second}^3 \cdot \text{meter}^4}\right), \]

(5b)

\[
n^2 = \left(\frac{10^7 \cdot 10^{7+3} \cdot h^5 \cdot |c|^3}{G \cdot m_0^2 \cdot e^8 \cdot 2 \cdot \pi^3 \cdot |c|^6 \cdot \text{second}^5 \cdot \text{meter}^4}\right) \cdot \left(\frac{10^{7+3} \cdot \pi^3}{4 \pi |c|^3 \cdot \text{meter}^3}\right), \]

(5c)

\[
n^2 = \left(\frac{4^3 \cdot 10^7 \cdot |c|^3 \cdot h^5 \cdot \varepsilon^4 \cdot \text{meter}^4}{G \cdot m_0^2 \cdot e^8 \cdot 2 \cdot \pi^3 \cdot |c|^6 \cdot \text{second}^5 \cdot \text{meter}^4}\right) \cdot \left(\frac{10^{7+3} \cdot \pi^3}{4 \pi |c|^3 \cdot \text{meter}^3}\right), \]

(5d)

\[
n^2 = \left(\frac{32 \cdot 10^7 \cdot c_0^3 \cdot h^5 \cdot \varepsilon_0^3 \cdot \pi \cdot \varepsilon^4 \cdot \text{meter}^4}{G \cdot m_0^2 \cdot e^8 \cdot \pi^3 \cdot \text{second}^5 \cdot \text{meter}^4}\right). \]

(5e)

\[
\alpha_{(\text{UR } \varepsilon_0)\text{PL}} = \left(\frac{32 \cdot 10^7 \cdot \text{meter} \cdot \varepsilon_0 \cdot c_0^3 \cdot h^5}{G \cdot m_0^2 \cdot e^8 \cdot \pi \cdot \varepsilon^4 \cdot \text{second} \cdot \Omega}\right). \]

(5f)
\[ n = \sqrt{\frac{32 \times 10^7 \text{meter} \times \epsilon_0^3 \times c_0^3 \times h^5}{G \times m_e^2 \times e^8 \times \mu_0}}, \]

\[ n = \sqrt{\frac{32 \times 4\pi \times \epsilon_0^3 \times c_0^3 \times h^5}{G \times m_e^2 \times e^8 \times (4\pi \times 10^{-7} \text{ meter}^3)}}, \]

\[ \alpha_{\alpha^{1/R \times }/\text{PL}} = \sqrt{\frac{128\pi \epsilon_0^3 \times h^5 \times \epsilon_0^3}{G \times m_e^2 \times e^8 \times \mu_0}}; \]

and accordingly,

\[ n = \frac{8}{Z_0} \sqrt{\frac{2\pi \epsilon_0^3 \times h^5 \times \epsilon_0^2}{G \times m_e^2 \times e^8 \times \frac{\mu_0}{\mu_0}}}, \]

\[ \alpha_{\alpha^{1/R \times }/\text{PL}} = \frac{8h^3 \epsilon_0}{Z_0 m_e e^4} \times \sqrt{\frac{\epsilon_0^3}{Gh}}; \]

where we once more return to the beginning of this section, since the impedance of free space is also \( Z_0 = 1/(\epsilon_0 c_0) \) — thus providing us with the reciprocal of the Rydberg constant on the LHS and the inverse of the Planck length on the RHS of Eq. (5k)’s multiplier.

As a consequence of our derivations up to this point — with the speed of light in empty space fixed at the onset as well as the Planck constant pinned down to a singular value for all reference frames — the variance of the electron rest mass must be on par with the variance of \( G \) (e.g., “conformal”) to ensure the Lorentz Scalarity of \( \alpha_{\alpha^{1/R \times }/\text{PL}} \); seeing as \( \mu_0 \) too has been determined by hand before all else. This will be easy to demonstrate via the exact dimensional proportionality out of Eq. (5j)

\[ \alpha_{\alpha^{1/R \times }/\text{PL}} = \sqrt{\frac{128\pi c_0^5 h^5 \epsilon_0^4}{G_0 m_e^2 e^8}} \]

yielding

\[ \frac{c^2 s^2}{m^4 k^2 g^2} \equiv \frac{c^2 s^2}{m^4 k^2 g^2} \]

since it is established that 1 Henry = \( \frac{s^2}{F} = \frac{m^2 \cdot kg}{C^2} \), insofar as leading us directly from Eq. (6b) to Eq. (6c). Given that the \( \mu_0 = \frac{m \cdot kg}{C^2} \) portion of the RHS is absolute by metrological norm, the only remaining option to preserve the Lorentz Scalar property of Eq. (5i) — whence Eq. (6a) is obtained — is to allow for \( G \) and \( m_e^2 \) to vary oppositely and conformally; seeing as the magnetic permeability value (i.e., inductance per length) — while nailed down to a singular number — turns out to be totally arbitrary on account of the fact that the constancy of a self induced electromotive force is contingent upon the geometry of the individual elements of a circuit configuration (e.g., a solenoid).

While the reader can easily notice that \( G m_e^2 \) is dimensionally identical to \( hc \) (i.e., \( Nm^2 \) or Newtons force times surface area in squaremeters), neither \( G \) by itself is dimensionally commensurate with either \( h \) or \( c \), nor is \( m_e^2 \) the dimensional analogue of either \( h \) or \( c \). Therefore, this synopsis neatly serves to illustrate how the “Gravitational constant” and electron rest mass squared must vary in opposite directions by the same amount to preserve the Lorentz Scalar structure of the Parana constant; with the end result that neither the Rydberg constant nor the Planck length actually signifies a universally unchanging guideline, because the former depends on \( m_e^2 \) and the latter depends on \( G_0 \) by definition at the most fundamental level.

One other important thing to notice about the proportionality in Eq. (6c) is how each side happens to be the dimensional analogue of \( 1/hc^2 \), with the exception of \( C^2 \) in the numerators. Due to the presence of elementary charge squared in Coulombs thereat, one cannot say that the RHS and LHS of Eq. (6c), the way they make up Eq. (6a), are individually Lorentz Invariant; just as it cannot be said that the RHS and LHS of Eq. (1i)’s multiplier — the former of which is dimensionally \( \left( \frac{m^2 \cdot kg}{C^2} \right) \) while the latter of which is its exact reciprocal — are Lorentz Invariant each (because the first part before the multiplier in Eq. (1i) is dimensionally commensurate with \( h^6c^2 \) times \( L^2 \), while the latter is its exact inverse; where the presence of an additional squaremeter destroys the Lorentz Invariance of the individual terms in question).

Yet, recall that the Parana constant \( \alpha_{\alpha^{1/R \times }/\text{PL}} \) is both a dimensionless universal quantity and a Lorentz Scalar by construct, since any leftover units of the abovementioned kind are anyway cancelled out at the end as required.

Continuing forward from Eq. (5i), and keeping in mind that the Fine Structure constant \( \alpha \) is \( \frac{e^2}{2 \mu_0 c^2} \), one can derive even better alternative equalities for the Parana constant whose co-
cancelling terms are, in fact, Lorentz Invariant:

\[ n = \sqrt{\frac{128 \cdot (2 \cdot 4 \cdot 4 \cdot 4) \pi \cdot c_0^3 \cdot h^5 \cdot \varepsilon_0^3}{G \cdot m_e^2 \cdot e^3 \cdot \mu_0}}, \quad (7a) \]

\[ n = \sqrt{\frac{(2 \cdot 4 \cdot 4) \cdot 4 \pi \cdot c_0^3 \cdot h^4 \cdot \varepsilon_0^2}{G \cdot m_e^2 \cdot \alpha^4 \cdot e^2 \cdot \mu_0 \cdot (2 \cdot 4 \cdot \varepsilon_0 \cdot c_0)}}, \quad (7b) \]

\[ n = \sqrt{\frac{(2 \cdot 2^2 \cdot 2^2) \cdot 4 \pi \cdot h \cdot c_0}{2^2 \cdot 2^2 \cdot G \cdot m_e^2 \cdot \alpha^4 \cdot \mu_0 \cdot \varepsilon_0^2 \cdot \varepsilon_0^2}}, \quad (7c) \]

\[ \alpha_{(UR \_c)} = \frac{\sqrt{8 \pi \hbar \alpha}}{\sqrt{G m_e \alpha^2}} \quad (7d) \]

wherefrom we obtain

\[ n = \left( \frac{\sqrt{8 \pi \cdot 2 \pi}}{\alpha^2 \cdot m_e} \right)^2 \cdot \left( \frac{\hbar \alpha}{G} \right), \quad (7e) \]

\[ \alpha_{(UR \_c)} = \frac{4 \pi}{\alpha^2} \cdot \frac{m_{\text{Planck}}}{m_e}, \quad (7f) \]

or else

\[ \alpha_{(UR \_c)} = \frac{m_{\text{Planck}}}{m_e} \quad (7g) \]

with \( m_{\text{Planck}} \) being the equivalent of Planck mass \( \sqrt{\frac{\hbar \alpha}{G}} \times 4\pi/\alpha^2 \), making \( 5.13555401557385 \times 10^{-3} \text{ kgs} \). As we had indicated previously, the numerator to the denominator of the RHS of Eq. (7d) squared cancels out exactly as \((Nm^2)\alpha/(Nm)^3\) when the \( \alpha^2 \) term is fully expanded (or just \( Nm^2/Nm^2 \) when it is not), making the co-cancelling terms Lorentz Invariant because they both possess solely the dimensions of a power of the similitude of \( \hbar c \). This would also be the case if we picked another route from Eq. (7c) to obtain an \( e^2/\varepsilon_0 \) term as follows:

\[ n = \sqrt{\frac{4 \pi \cdot 2 \cdot 4 \cdot \hbar \cdot c_0}{G \cdot m_e^2 \cdot \alpha^2}}, \quad (8a) \]

\[ n = \sqrt{\frac{4 \pi \cdot 4 \pi \cdot e^2}{G \cdot m_e^2 \cdot \alpha^2}}, \quad (8b) \]

\[ n = \sqrt{\frac{4 \pi \cdot 4 \pi \cdot e^2}{G \cdot m_e^2 \cdot \alpha^2}}, \quad (8c) \]

\[ \alpha_{(UR \_c)} = \frac{4 \pi}{\alpha^2} \cdot \frac{e^2}{(G m_e^2 \cdot 4 \pi \varepsilon_0)}, \quad (8d) \]

Again, by the time we land at Eq. (8d), the square of the ratio after the multiplicator on the RHS has the proportion \( Nm^2/Nm^2 \) (owing to \( e^2/\varepsilon_0 \) being the counterpart of \( Gm_e^2 \) dimension-wise). Consequently, here too has it been shown that the co-cancelling terms are Lorentz Invariant just like with the squared numerator to the squared denominator of the RHS of Eq. (7d) being \((Nm^2)^2/(Nm)^3\) (or just \( Nm^2/Nm^2 \) if we ignore the expanded contribution of the square of the Fine Structure constant in the denominator).

The equality above is otherwise descriptive of a mass that one can associate with YARK theory of gravity using the relationship

\[ \alpha_{(UR \_c)} = \frac{4 \pi}{\alpha^2} \cdot \frac{\sqrt{\alpha m_{\text{Planck}}}}{m_e} = \frac{4 \pi}{\alpha^2} \cdot \frac{m_{\text{MAXX}}}{m_e}, \quad (9a) \]

because

\[ \sqrt{\frac{e^2}{G m_e^2 \cdot 4 \pi \varepsilon_0}} = \sqrt{\frac{\hbar c}{2 \pi G m_e^2 \cdot 2 \pi \varepsilon_0} \frac{e^2}{2 \pi G m_e^2 \cdot 2 \pi \varepsilon_0} = \sqrt{\alpha m_{\text{Planck}}} \quad \frac{m_{\text{MAXX}}}{m_e}, \quad (9b) \]

whereby \( G m_{\text{MAXX}}^2 \) translates to \( \frac{e^2}{4 \pi \varepsilon_0} \) when we equate the square-rooted part of Eq. (8d) with \( \sqrt{\alpha m_{\text{Planck}}} \) out of Eq. (9a):

\[ \frac{\sqrt{\alpha m_{\text{Planck}}}}{m_e} = \sqrt{\frac{e^2}{G m_e^2 \cdot 4 \pi \varepsilon_0}}, \quad (10a) \]
As a corollary, we have demonstrated how the “Universal Gravitational constant” and electron rest mass squared must vary in opposite directions by the same conformal factor to preserve the Lorentz Scalar property of our novel dimensionless Parana constant, given that neither the “Rydberg constant” nor the Planck length actually signifies a universally unchanging principle — because the former depends on \( m_e^2 \) and the latter depends on \( G \) by definition at a most fundamental level (with these solely representing Earth-bound flexible quantities as elaborated in [10, p. 565] for both YARK and either Special or General Relativity). This is all the more so since lightspeed in empty space, as well as vacuum permeability from which the permittivity of free space is inferred by virtue of \( c^2 = \frac{1}{\mu_0 \varepsilon_0} \), are all fixed by hand to strictly absolute quantities — thus making it impossible, come what may, for \( G \) and \( m_e^2 \) to be universal. In other words, while \( G m_e^2 \) is dimensionally identical to \( h c \) (i.e., \( Nm^2 \) or Newtons force times surface area in square meters), neither \( G \) by itself is dimensionally commensurate with either \( h \) or \( c \), nor is \( m_e^2 \) the dimensional analogue of either \( h \) or \( c \).

The major contribution of this study is the culmination reached in both Eq. (7g) and Eq. (9a), where we have emphasized the Parana constant in direct relation to firstly the “Parana mass” divided by the electron rest mass \( m_{0e} \), and secondly the “YARK mass” over the same electron rest mass. The ratio of \( m_{\text{YARK}}/m_{0e} \) yields \( 2.04080623273918E+21 \), which would remain Lorentz Invariant as long as the numerator and the denominator vary conformally in the same direction.

Significantly though, Eq. (9a) relates the YARK mass (given as \( \sqrt{\alpha m_{\text{Planck}}} \)) to the electron rest mass (where \( \alpha \) is the Fine Structure constant) in such a way that the attraction between two YARK bodies of \( 1.85904867479047E-09 \) kgs each precisely parallels the electric force between a proton and an electron.

As illustrated on the RHS of the multiplier of Eq. (8d) on the way to Eq. (10c), \( G m_{\text{YARK}}^2 \) translates to \( \frac{e^2}{4\pi \varepsilon_0} \); and these terms, when altogether under a square root as shown in Eq. (8d), bear the dimensions of \( \sqrt{Nm^2/Nm^2} \). Therefore the co-cancelling terms are already Lorentz Invariant. This by itself seems to betoken the importance of \( 2.04080623273918E+21 \) as a dimensionless subsidiary to \( \alpha_{(UR, e)}^{(pl)} \) if one is at a liberty to ignore the factor \( 4\pi/\alpha^2 \).

While one may argue that the “Rydberg constant” expression includes an electron rest mass which cannot be expressed separately in terms of conventional constants such as \( e, h, c \), and \( \alpha \), still (as is well known) the mass to charge ratio of the electron is a measurable quantity [27], [28] — thence, the electron’s rest energy (thus, its rest mass) ought verily to be considered a universal parameter under the ideal conditions of empty space. Even more fundamentally, the “Parana ratio of lengths” (e.g., Eq. (1a)) leads to the “Parana ratio of masses” (e.g., Eqs. (7f-g), (9a)); in other words, just as the meter unit necessarily drops off of the former, so does the kilogram unit drops off of the latter — thus rendering the Parana constant independent of the electron mass by virtue of

\[
m_{\text{YARK}}^2 = \frac{e^2}{G m_e^2 4\pi \varepsilon_0} \text{.} \tag{10b}
\]

\[
m_{\text{YARK}} = \sqrt{\frac{e^2}{G 4\pi \varepsilon_0}} \text{.} \tag{10c}
\]

This mass \( 1.85904867479047E-09 \) kgs is geared in such a way that the gravitational force reigning in between a pair of them is equal to the electric force \( F_C \) reigning in between a proton and an electron, with both pairs situated at the same arbitrary distance. Thus, \( m_{\text{YARK}} \) gains usefulness as the generator of an attraction with respect to another \( m_{\text{YARK}} \) as though by a solitary proton over a single electron.

One should add that Newton’s second law (mass x acceleration) would, at any rate, apply to either the proton mass or the electron mass undergoing motion — though, it may be more sensible to assume the presence of just an electric force in between these charges while no gravitational force likely emerges at the atomic level. The Parana constant expressed via Eq. (9a) acquires a deeper meaning in that sense. Such a proportionality involving two fundamental masses (e.g., the YARK mass divided by the electron rest mass) would attribute to \( \alpha_{(UR, e)}^{(pl)} \) exceptional importance at the micro-scale. The equivalence of the Parana constant to the ratio of the YARK mass over the electron rest mass strengthens too the notion that the Planck length — although assembled through a mere dimensional analysis — cannot all the way be arbitrary. Therefore, our novel Parana constant not only happens to be a Lorentz Scalar, but also must come about as a fundamental constant of nature.

Meaningful relationships between the Parana mass and “Planck mass” as well as the YARK mass and “Planck mass” can be tackled in later studies.

3. Discussions

To summarize, Eqs. (1i), (5f), (5i) along with its sibling (6a), followed by (7d), (7f), (8d) and (9a) all yield the exact same dimensionless quantity

\[
\alpha_{(UR, e)}^{(pl)} = 5.63765262613852E+27 \text{;}
\]

with its reciprocal thereby making

\[
\alpha_{(pl)}^{(UR, e)} = 1.77378789775656E-28 \text{.}
\]

Notice that the ability to represent the Parana constant via co-cancelling Lorentz Invariant components in Eqs. (7d) and (8d) reinforce our conjecture that it is, in fact, not the “Rydberg constant” nor the Planck length that are ubiquitously invariable quantities, but instead \( \alpha_{(UR, e)}^{(pl)} \) (in the same spirit as the Fine Structure constant — insofar as one may trust that the latter is indeed a universal invariant).
of the interconnectivity of our derivations.

Besides the posited universality of the electron rest mass in vacuum \((m_{\text{e}0})\) along with the constancy of the “utmost theoretical speed of light” \((c_0)\), the importance of maintaining Lorentz Invariance in the presence of gravity is analogous to its proofs in the case of electromagnetism (e.g., the scalar product of constituent Electric and Magnetic Fields turning out to be Lorentz Invariant) \([29, \text{p. 63-65}]\). Any unity between the atomistic world and the macroscopic world would definitely require it; as is the situation with the Universal Matter Architecture (UMA) scaffolding upon which YARK theory of gravity is built. In this respect, the Lorentz Scalar that we dubbed the Parana constant — with this implying a special dimensionless number (just like the Fine Structure constant) which does not get affected by the uniform translational motion of an object or any isotropic/anisotropic field with which it engages — becomes significative.

Further elaborations can be made with regards to establishing a “Parana length” and a “Parana period of time” through exactly the same philosophy presented here; where \(m_{\text{max}}/m_e\) might be used to evoke the dimensionless quantity \(\alpha_{(1/R_e)}/\text{Parana Length}\) that we already implied as a possible subsidiary to the Parana constant, with its value being \(2.0408062327918E+21\). To rephrase, “Parana length” would equate exactly to Planck length times \(4\pi/\alpha^{(5/2)}\) from the definition

\[
L_{\text{Parana}} = m_e \left( \frac{8 \cdot \hbar^3 \cdot c_0 \cdot \varepsilon_0^2}{m_e \cdot e^4} \right)/m_{\text{max}} = 4.46523063172899E-29 \text{ meters}; \quad (11)
\]

which is thus larger than the Planck length by a factor of about \(10^6\). Whereas the latter quantity might not have much substance after all — for it comes out of just a dimensional fitting — Parana length, in contrast, seems to bear significance since it relates the “Rydberg constant” to the ratio of \(m_{\text{max}}/m_e\); which, in turn, appears to have a crucial meaning as we have discussed at length.

Lastly, it is possible to concoct a “Parana time” as the ratio of the reciprocal of the “Rydberg constant” to the speed of light in empty space, yielding

\[
\frac{1}{c_0} \cdot \frac{1}{R_e} = 3.0396596914557E-16 \text{ seconds}
\]

that is otherwise expressible as \(2\hbar/(\alpha^2 c_0^2 m_{\text{e}0})\) from a direct derivation out of Eq. (6a) via extracting the Planck time \(\sqrt{\hbar/c_0^2}\) — which is longer than Planck time by precisely as much as the Parana constant.

4. Conclusion

The Rydberg constant, as quite well known, was measured during the early days of atomic spectroscopy and attracted much attention while scientists initially puzzled over it. Niels Bohr’s great success was to express it in terms of the charge of the electron, its mass, the Planck constant, and the velocity of light in empty space in just the way exemplified (albeit in reciprocal form) on the LHS of Eq. (1a).

As explained throughout this contribution, all said quantities appearing in the Rydberg expression — except the electron’s mass — are definitely universal constants (with \(e\) in the CGS system commensurate to \(e/\sqrt{\varepsilon_0}\) in the MKS system; with the implication that the Coulomb unit by itself is not Lorentz Invariant, but that the StatCoulomb unit is).

In effect, the Rydberg constant to be associated with an excited Hydrogen atom’s emission on the surface of the Sun ought to get decreased as much as the “gravitational redshift” coming into play. Recall that this gravitational redshift is the same in General Theory of Relativity (GTR) and in YARK theory of gravity up to a third order Taylor expansion (although they are based on totally different philosophies from the ground up). In other words, while the shift is due solely to curvature in GTR, it is due to a rest mass decrease owing to static binding in YARK (as implied by the law of energy conservation embodying the mass and energy equivalence of the special theory of relativity); where the Rydberg constant in gravitation gets diminished on account of the decrease in the rest mass of the electron — which, in turn, leads to a redshift in frequency.

The “Rydberg constant”, at the same time, must undergo variance through a uniform translational motion; for mass (per se) — or just the same, rest energy if one lets \(c\) be unity — gets influenced in the first place as such. What all this means is that, the “Rydberg constant” is not really a universal constant after all, especially when a reference frame at rest scrutinizes (from its locality) the empirical outcomes gathered in another non-inertial reference frame.

To summarize, below are the results we landed at which might be deemed the most important ones:

- The Parana constant could be expressed as the ratio of what we dubbed the “Parana mass” to the electron rest mass as shown in Eqs. (7f-g).
- The Parana mass too gains meaningfulness by virtue of being proportional to the “YARK mass” (cf. Eq. (9a)), where the proportionality constant is nailed to the inverse of the \((5/2)\)nd power of the Fine Structure constant; with the “YARK mass” signifying the mass that generates, when acted upon by its twin at a given distance, a gravitational force equal to what would be exerted by a proton over an electron (or vice versa) separated by the same distance from each other.
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Table 1. List of significant Para-an constant equalities in the text all yielding the same value $5.63765262613852E+27$.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Equality</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eq.1i</td>
<td>$\alpha (1/R_\infty)/PL = \left(\frac{\hbar^3 v}{G\phi}\right) \ast \left(\frac{1 \Omega^2}{m_e e^4 # 1 \text{ meter}^2}\right)$</td>
</tr>
<tr>
<td>Eq.5f</td>
<td>$\alpha (1/R_\infty)/PL = \frac{32 \pi \times 10^7 \times \text{meter} \times \epsilon_0 \times c \times \hbar}{G \times m_e \times e^8 \times \text{second} \times \Omega}$</td>
</tr>
<tr>
<td>Eq.5i &amp; 6a</td>
<td>$\alpha (1/R_\infty)/PL = \frac{128 \pi \times c \times \hbar^3 \times \epsilon_0}{G \times m_e \times e^8 \times \mu_0}$</td>
</tr>
<tr>
<td>Eq.7d</td>
<td>$\alpha (1/R_\infty)/PL = \frac{8 \pi \times \hbar \times c \epsilon_0}{\sqrt{G \times m_e \times \alpha^2}}$</td>
</tr>
<tr>
<td>Eq.7f</td>
<td>$\alpha (1/R_\infty)/PL = \left(\frac{4 \pi}{\alpha^5} \times \frac{m_{\text{Planck}}}{m_e}\right)$</td>
</tr>
<tr>
<td>Eq.8d</td>
<td>$\alpha (1/R_\infty)/PL = \left(\frac{4 \pi}{\alpha^5 \times \sqrt{G \times m_e^2 \times 4 \pi \epsilon_0}}\right)$</td>
</tr>
</tbody>
</table>

- The abovementioned case evokes that, though obtained through a mere dimensional analysis, “Planck length” could well bear a physical meaning. However, determining the shortest distance in the universe may require some more elaboration and fine-tuning. The shortest distance in empty space in the universe then appears to be surely proportional to the Planck length.

- Next to the “Para-an mass”, we were moreover encouraged to introduce a “Para-an length” and a “Para-an time”; whose proportionality with the “Planck length” and “Planck time” respectively seems worthy of further deliberation in subsequent studies, since the proportionality constant of concern in each case is related to the Para-an constant possessing the value $5.63765262613852E+27$.

- Last, but not the least, we took the opportuniy to reveal interesting features with regards to “universal constants” and ponder over them during our derivations. One important item is that the “Gravitational constant” (unlike the elementary charge expressed in the CGS system and the Planck constant) is not at all Lorentz Invariant, and therefore cannot be a ubiquitous magnitude just as recent doubts evince.

The various expressions for the same dimensionless magnitude we came up with, christened herein the Para-an constant, upon an exercise where we embarked from the ratio of the inverse of the “Rydberg constant” to the Planck length are all Lorentz Scalars (cf. Table 1) — and, in this sense, either thoroughly universal, or (most likely) proportional to ubiquitous constants. The same is indeed true for also the ratio of the “YARK mass” to the electron mass (which, by construct, comes to be proportional to the division of the inverse of the Rydberg constant by the Planck length).

Notice once again that, in Table 1,

$$v = \sqrt{\frac{10^{28} \times \text{meter}^5}{2 \times \pi^3 \times 299792458^3 \times \text{second}^5}} = 1.43024900891066 \text{ m/s}$$

with $\alpha$ being the Fine Structure constant.

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