Proofing the Collatz Conjecture by Mathematical Induction

Zhang Tianshu
Zhanjiang city, Guangdong province, China
Email: chinazhangtianshu@126.com
china.zhangtianshu@tom.com

Abstract

In this article, the author applies the mathematical induction, classifies positive integers, and passes operations according to the operational rule, to achieve the goal that proves the Collatz conjecture finally.

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1. Introduction

The Collatz conjecture is also variously well-known 3n+1 conjecture, the Ulam conjecture, Kakutani’s problem, the Thwaites conjecture, Hasse’s algorithm, and the Syracuse problem etc. Yet it is still both unproved and un-negated a conjecture ever since named after Lothar Collatz in 1937.

2. Basic Concepts and Criteria

The Collatz conjecture states that take any positive integer n, if n is an even number, then divide n by 2; if n is an odd number, then multiply n by 3 and add 1. Repeat the above process indefinitely, then no matter which positive integer you start with, it will eventually reach a result of 1. Let us regard above-mentioned operational stipulations as the operational rule of the conjecture or the operational rule for short.
Start with any positive integer to operate successive emerging positive integers by the operational rule, afterwards, regard consecutive integers plus synclastic arrowheads among them as an operational route.

If an integer (or an integer’s expression) \( P_{ie} \) exists at an operational route, then may term the operational route “an operational route of \( P_{ie} \)” or “the operational route \( P_{ie} \)”. Two operational routes of \( P_{ie} \) branch from an integer (or an integer’s expression) after pass operations of \( P_{ie} \).

Before make the proof, be necessary to prepare two theorems concerned, so as to reach certain of conclusions by them.

**Theorem 1** If a positive integer or a positive integer’s expression at operational route \( P_{ie} \) suits the conjecture, then \( P_{ie} \) suits the conjecture.

For examples: (1) If \( 27+2^3\eta \) suits the conjecture, and \( P_{ie}=31+3^2\eta \), where \( \eta \geq 0 \), then from \( 27+2^3\eta \rightarrow 82+3 \times 2^3\eta \rightarrow 41+3 \times 2^2\eta \rightarrow 124+3 \times 2^2\eta \rightarrow 62+3 \times 2\eta \rightarrow 31+3^2\eta \rightarrow 27+2^3\eta \), conclude that \( 31+3^2\eta \) suits the conjecture. (2) If \( 4+3\mu \) suits the conjecture, and \( P_{ie}=5+2^2\mu \), where \( \mu \geq 0 \), then from \( 5+2^2\mu \rightarrow 16+3 \times 2^2\mu \rightarrow 8+3 \times 2\mu \rightarrow 4+3\mu \rightarrow 5+2^2\mu \), conclude that \( 5+2^2\mu \) suits the conjecture.

**Proof** Suppose \( C_{ie} \) suit the conjecture at an operational route of \( P_{ie} \). If \( C_{ie} \) appears before \( P_{ie} \), then the operations of \( C_{ie} \) via \( P_{ie} \) reached 1 already. If \( C_{ie} \) appears behind \( P_{ie} \), then the operations of \( P_{ie} \) pass \( C_{ie} \), afterwards continue along operational route of \( C_{ie} \) to reach 1.

**Theorem 2** If a positive integer or a positive integer’s expression at operational route \( Q_{ie} \) suits the conjecture, in addition the operational route
Let us set to prove the Collatz conjecture by the mathematical induction:

1. From 2→1; 3→10→5→16→8→4→2→1; 4→2→1; 5→16→8→4→2→1;
   6→3→10→5→16→8→4→2→1; 7→22→11→34→17→52→26→13→40→20→10
   →5→16→8→4→2→1; 8→4→2→1; 9→28→14→7→22→11→34→17→52→26
   →13→40→20→10→5→16→8→4→2→1; 10→5→16→8→4→2→1; 11→34→17
   →52→26→13→40→20→10→5→16→8→4→2→1; 12→6→3→10→5→16→8→
   4→2→1; 13→40→20→10→5→16→8→4→2→1; 14→7→22→11→34→17→52
   →26→13→40→20→10→5→16→8→4→2→1; 15→46→23→70→35→106→53→
160→80→40→20→10→5→16→8→4→2→1 ; 16→8→4→2→1 ; 17→52→26→13→40→20→10→5→16→8→4→2→1 ; 18→9→28→14→7→22→11→34→17→52→26→13→40→20→10→5→16→8→4→2→1 and 19→58→29→88→44→22→11→34→17→52→26→13→40→20→10→5→16→8→4→2→1, reach the conclusion that each of positive integers \( \leq 19 \) suits the conjecture.

2. Suppose that positive integer \( n \) suits the conjecture, where \( n \geq 19 \).

3. Prove that positive integer \( n+1 \) suits the conjecture.

**Proof** * According to two theorems prepared in advance, to prove that \( n+1 \) suits the conjecture, so long as we can find out a direct relation between \( n+1 \) and a smaller integer \( \leq n \) with \( n \geq 19 \), or an indirect relation between \( n+1 \) and an integer’s expression which suits the conjecture in accordance with the operational rule, then the problem is solved by us.

It is well known that all positive integers are divided into even numbers and odd numbers.

Firstly, for any even number \( 2k \) with \( k \geq 1 \), from \( 2k \rightarrow k \leq 2k \), conclude that \( 2k \) suits the conjecture according to the theorem 1. So, if \( n+1 \) belongs within \( 2k \), then \( n+1 \) suits the conjecture because \( 2k \) suits the conjecture.

Secondly, for all unproved odd numbers, first divide them into two genera, i.e. \( 5+4k \) and \( 7+4k \), where \( k \geq 4 \).

For \( 5+4k \), from \( 5+4k \rightarrow 16+12k \rightarrow 8+6k \rightarrow 4+3k \leq 5+4k \), conclude that \( 5+4k \) suits the conjecture according to the theorem 1. So, if \( n+1 \) belongs within \( 5+4k \), then \( n+1 \) suits the conjecture because \( 5+4k \) suits the conjecture.
For 7+4k, divide it into 3 sorts: 15+12c, 19+12c and 23+12c where c≥0.

For 23+12c, from 15+8c→46+24c→23+12c<15+8c, conclude that 23+12c suits the conjecture according to the theorem 1. So, if n+1 belongs within 23+12c, then n+1 suits the conjecture because 23+12c suits the conjecture.

For 15+12c and 19+12c, when c=0, they have been proved in advance, so merely need us to prove 15+12c and 19+12c where c≥1.

For 15+12c and 19+12c where c≥1, let us operate them right along, so that expound the relation that they act in accordance with either theorem.

Firstly, let us operate 15+12c by the operational rule, as listed below.

\[ 15+12c \rightarrow 46+36c \rightarrow 23+18c \rightarrow 70+54c \rightarrow 35+27c \]

\[ d=2e+1: 29+27c (1) \quad e=2f: 142+486f \rightarrow 71+243f \]

\[ \checkmark 35+27c \downarrow \rightarrow c=2d+1: 31+27d \uparrow \rightarrow \quad d=2e: 94+162e \rightarrow 47+81e \uparrow \rightarrow e=2f+1: 64+81f (2) \]

\[ c=2d: 106+162d \rightarrow 53+81d \downarrow \rightarrow \quad d=2e+1: 67+81e \downarrow \rightarrow e=2f+1: 74+81f (3) \]

\[ d=2e: 160+486e \quad e=2f: 202+486f \rightarrow 101+243f \]

\[ g=2h+1: 200+243h (4) \quad \ldots \]

\[ \checkmark 71+243f \downarrow \rightarrow f=2g+1: 157+243g \uparrow \rightarrow \quad \checkmark g=2h: 472+1458h \rightarrow 236+729h \uparrow \rightarrow \ldots \]

\[ f=2g: 214+1458g \rightarrow 107+729g \downarrow \rightarrow \quad \checkmark g=2h+1: 418+729h \downarrow \rightarrow \ldots \]

\[ g=2h: 322+4374h \rightarrow \ldots \]

\[ \checkmark 101+243f \downarrow \rightarrow f=2g+1: 172+243g \uparrow \rightarrow \quad \checkmark f=2g: 304+1458g \rightarrow 152+729g \downarrow \rightarrow \ldots \]

\[ \ldots \]

\[ \checkmark 160+486e \rightarrow 80+243e \downarrow \rightarrow e=2f+1: 970+1458f \rightarrow 485+729f \uparrow \rightarrow \ldots \]

\[ e=2f: 40+243f \downarrow \rightarrow f=2g+1: 850+1458g \rightarrow 425+729g \uparrow \rightarrow \ldots \]

\[ f=2g: 20+243g \downarrow \rightarrow g=2h: 10+243g (6) \quad \ldots \]

\[ g=2h+1: 880+1458h \rightarrow 440+729h \uparrow \rightarrow \ldots \]

**Annotation:**

(1) Each of letters c, d, e, f, g, h etc at listed above operational routes expresses each of natural numbers plus 0.

(2) Also, there are \[ \checkmark \leftrightarrow \checkmark, \quad \checkmark \leftrightarrow \checkmark, \quad \checkmark \leftrightarrow \checkmark, \quad \text{and} \quad \checkmark \leftrightarrow \checkmark. \]

(3) Aforesaid two points are suitable to latter operational routes of 19+12c similarly.
In the course of operation for $15+12c/19+12c$ by the operational rule, if an operational result is smaller than a kind of $15+12c/19+12c$, and that it first appears at an operational route of $15+12c/19+12c$, then, let us term the operational result “№1 satisfactory operational result”.

Since each kind of $15+12c/19+12c$ derives from a №1 satisfactory operational result, and that the former is greater than the latter always, in addition, both of them coexist at an operational route of $15+12c/19+12c$.

Thereupon, can conclude six kinds of $15+12c$ derived from six №1 satisfactory operational results at the bunch of operational routes of $15+12c$ monogamously to suit the conjecture, ut infra.

From $c=2d+1$ and $d=2e+1$, get $c=2d+1=2(2e+1)+1=4e+3$, then $15+12c=51+48e=51+3\times2^4e\rightarrow154+3^2\times2^4e\rightarrow77+3^2\times2^3e\rightarrow232+3^3\times2^3e\rightarrow116+3^3\times2^2e\rightarrow58+3^3\times2e\rightarrow29+27e$ where mark (1), and $29+27e < 51+48e$, so we get that $51+48e$ suits the conjecture according to the theorem 1.

From $c=2d+1$, $d=2e$ and $e=2f+1$, get $c=2d+1=4e+1=4(2f+1)+1=8f+5$, then $15+12c=75+96f=75+3\times2^5f\rightarrow226+3^2\times2^5f\rightarrow113+3^2\times2^4f\rightarrow340+3^3\times2^4f\rightarrow170+3^3\times2^3f\rightarrow85+3^3\times2^2f\rightarrow256+3^4\times2^2f\rightarrow128+3^4\times2^1f\rightarrow64+81f$ where mark (2), and $64+81f < 75+96f$, so we get that $75+96f$ suits the conjecture according to the theorem 1.

From $c=2d$, $d=2e+1$ and $e=2f+1$, get $c=2d=4e+2=4(2f+1)+2=8f+6$, then $15+12c=87+96f=87+3\times2^5f\rightarrow262+3^2\times2^5f\rightarrow131+3^2\times2^4f\rightarrow394+3^3\times2^4f\rightarrow197+3^3\times2^3f\rightarrow592+3^4\times2^3f\rightarrow296+3^4\times2^2f\rightarrow148+3^4\times2^1f\rightarrow74+81f$ where
mark (3), and $74+81f < 87+96f$, so we get that $87+96f$ suits the conjecture according to the theorem 1.

From $c=2d+1$, $d=2e$, $e=2f$, $f=2g+1$ and $g=2h+1$, get $c=2d+1=4e+1=8f+1=8(2g+1)+1=16g+9=16(2h+1)+9=32h+25$, then $15+12c=315+384h=315+3\times 2^7h \rightarrow 946+3^2\times 2^7h \rightarrow 473+3^2\times 2^6h \rightarrow 1420+3^3\times 2^6h \rightarrow 710+3^3\times 2^5h \rightarrow 355+3^3\times 2^4h \rightarrow 1066+3^4\times 2^4h \rightarrow 533+3^4\times 2^3h \rightarrow 1600+3^5\times 2^3h \rightarrow 800+3^5\times 2^2h \rightarrow 400+3^5\times 2^1h \rightarrow 200+243h$ where mark (4), and $200+243h < 315+384h$, so we get that $315+384h$ suits the conjecture according to the theorem 1.

From $c=2d$, $d=2e+1$, $e=2f$, $f=2g+1$ and $g=2h$, get $c=2d=2(2e+1)=4e+2=8f+2=8(2g+1)+2=16g+10=32h+10$, then $15+12c=135+384h=135+3\times 2^7h \rightarrow 406+3^2\times 2^7h \rightarrow 203+3^2\times 2^6h \rightarrow 610+3^3\times 2^6h \rightarrow 305+3^3\times 2^5h \rightarrow 916+3^4\times 2^5h \rightarrow 458+3^4\times 2^4h \rightarrow 229+3^4\times 2^3h \rightarrow 688+3^5\times 2^3h \rightarrow 344+3^5\times 2^2h \rightarrow 86+243h$ where mark (5), and $86+243h < 135+384h$, so we get that $135+384h$ suits the conjecture according to the theorem 1.

From $c=2d$, $d=2e$, $e=2f$, $f=2g$ and $g=2h$, get $c=2d=32h$, then $15+12c=15+3\times 2^7h \rightarrow 46+3^2\times 2^7h \rightarrow 23+3^2\times 2^6h \rightarrow 70+3^3\times 2^6h \rightarrow 35+3^3\times 2^5h \rightarrow 106+3^4\times 2^5h \rightarrow 53+3^4\times 2^4h \rightarrow 160+3^5\times 2^4h \rightarrow 80+3^5\times 2^3h \rightarrow 40+3^5\times 2^2h \rightarrow 10+243h$ where mark (6), and $10+243h < 15+384h$, so we get that $15+384h$ suits the conjecture according to the theorem 1.

Secondly, let us continues to operate $19+12c$ by the operational rule, as listed below.
Like that, we can conclude six kinds of $19+12c$ derived from six $\mathbb{N}$ satisfactory operational results at the bunch of operational routes of $19+12c$ monogamously to suit the conjecture, ut infra.

From $c=2d$ and $d=2e$, get $c=2d=4e$, then $19+12c=19+48e=19+3\times2^4e\rightarrow58+3^2\times2^4e\rightarrow29+3^2\times2^3e\rightarrow88+3^3\times2^2e\rightarrow44+3^3\times2^2e\rightarrow22+3^3\times2e\rightarrow11+27e$ where mark $(\alpha)$, and $11+27e < 19+48e$, so we get that $19+48e$ suits the conjecture according to the theorem 1.

From $c=2d$, $d=2e+1$ and $e=2f$, get $c=2d = 2(2e+1) = 4e+2=8f+2$, then $19+12c= 43+96f= 43+3\times2^5f\rightarrow130+3^2\times2^5f\rightarrow65+3^3\times2^4f\rightarrow196+3^3\times2^4f\rightarrow98+3^3\times2^3f\rightarrow49+3^3\times2^2f\rightarrow148+3^4\times2^2f\rightarrow74+3^3\times2^1f\rightarrow37+81f$ where mark $(\beta)$, and $37+81f < 43+96f$, so we get that $43+96f$ suits the conjecture according to the theorem 1.
From $c=2d+1$, $d=2e+1$ and $e=2f$, get $c=2d+1=4e+3=8f+3$, then $19+12c = 55+96f=55+3\cdot2^3f\to 166+3^2\cdot2^5f\to 83+3^3\cdot2^4f\to 250+3^3\cdot2^4f\to 125+3^3\cdot2^3f \to 376+3^4\cdot2^3f\to 188+3^4\cdot2^2f\to 94+3^4\cdot2^1f\to 47+81f$ where mark ($\gamma$), and $47+81f < 55+96f$, so we get that $55+96f$ suits the conjecture according to the theorem 1.

From $c=2d$, $d=2e+1$, $e=2f+1$, $f=2g+1$ and $g=2h$, get $c=2d=2(2e+1)=4e+2 =4(2f+1)+2=8f+6=8(2g+1)+6=16g+14=32h+14$, then $19+12c=187+384h = 187+3\cdot2^7h\to 562+3^2\cdot2^7h \to 281+3^3\cdot2^6h \to 844+3^3\cdot2^6h \to 422+3^3\cdot2^5h \to 211+3^3\cdot2^4h \to 634+3^4\cdot2^4h \to 317+3^4\cdot2^3h \to 952+3^5\cdot2^3h \to 476+3^5\cdot2^2h \to 238+3^5\cdot2^1h \to 119+243h$ where mark ($\delta$), and $119+243h < 187+384h$, so we get that $187+384h$ suits the conjecture according to the theorem 1.

From $c=2d+1$, $d=2e$, $e=2f+1$, $f=2g$ and $g=2h+1$, get $c=2d+1=4e+1=4(2f+1)+1=8f+5=16g+5=16(2h+1)+5=32h+21$, then $19+12c = 271+384h = 271+3\cdot2^7h\to 814+3^2\cdot2^7h \to 407+3^3\cdot2^6h \to 1222+3^3\cdot2^6h \to 611+3^3\cdot2^5h \to 1834+3^4\cdot2^5h \to 917+3^4\cdot2^4h \to 2752+3^5\cdot2^4h \to 1376+3^5\cdot2^3h \to 688+3^5\cdot2^2h \to 344+3^5\cdot2^1h \to 172+243h$ where mark ($\epsilon$), and $172+243h < 271+384h$, so we get that $271+384h$ suits the conjecture according to the theorem 1.

From $c=2d+1$, $d=2e+1$, $e=2f+1$, $f=2g+1$ and $g=2h+1$, get $c=2d+1=2(2e+1)=4e+3=4(2f+1)+3=8f+7=8(2g+1)+7=16(2h+1)+15=32h+31$, then $19+12c = 391+384h = 391+3\cdot2^7h \to 1174+3^2\cdot2^7h \to 587+3^2\cdot2^6h \to 1762+3^3\cdot2^6h \to 881+3^3\cdot2^5h \to 2644+3^4\cdot2^5h \to 1322+3^4\cdot2^4h \to 661+3^4\cdot2^3h \to 1984+3^5\cdot2^3h \to 992+3^5\cdot2^2h \to 496+3^5\cdot2^1h \to 248+243h$ where mark ($\zeta$), and
$248 + 243h < 391 + 384h$, so we get that $391 + 384h$ suits the conjecture according to the theorem 1.

It is obvious that if $n+1$ belongs within any kind of $15 + 12c/19 + 12c$ derived from a №1 satisfactory operational result, such as preceding $51 + 48e$, $75 + 96f$, $87 + 96f$, $315 + 384h$, $135 + 384h$, $15 + 384h$, $19 + 48e$, $43 + 96f$, $55 + 96f$, $187 + 384h$, $271 + 384h$ or $391 + 384h$, then $n+1$ suits the conjecture according to the theorem 1.

Thus it can be seen, that following two laws accord with objective reality.

Firstly, each kind of $15 + 12c/19 + 12c$ derived from a №1 satisfactory operational result and the №1 satisfactory operational result coexist at an operational route of $15 + 12c/19 + 12c$, and that both of them share a variable.

Secondly, the greatest exponent of factor 2 of coefficient of variable within each kind of $15 + 12c/19 + 12c$ is exactly the number of times that divided by 2 in the operational course from the kind of $15 + 12c/19 + 12c$ to №1 satisfactory operational result.

Thereinafter, let us further explain the relation of mutual dependence of №1 satisfactory operational results and kinds of $15 + 12c/19 + 12c$ at the bunch of operational routes of $15 + 12c/19 + 12c$.

It is observed that variables $d$, $e$, $f$, $g$, $h$ … etc. of integer’s expressions appear at the bunch of operational routes of $15 + 12c/19 + 12c$, in fact, the purpose which substitutes c by them is that in order to avoid the confusion and for convenience. Thus, let $\chi$ represent intensively variables
d, e, f, g, h … etc., but $\chi$ can not represent the variable c directly.

Evidently, the kind of $15+12c/19+12c$ which contains n+1 can be written into $(n+1)+3\times2^y\chi$ in accordance with the above discussions, where $y$ is a exponent of 2, and $y > 4$.

After substitute variables d, e, f, g, h … etc by variable $\chi$, the odevity of part integer’s expressions that contain the variable $\chi$ at operational routes of $15+12c/19+12c$ is still indeterminate. Or rather, for every such integer’s expression, both regard it as an odd number to operate, and regard it as an even number to operate. Thus, we label such integer’s expressions “odd-even expressions”.

For any odd-even expression at operational routes of $15+12c/19+12c$, two kinds of operations synchronize at itself. After regard an odd-even expression as an odd number to operate, get a greater operational result>, itself. Yet, after regard it as an even number to operate, get a smaller operational result< itself. Moreover, after pass operations for each odd-even expression, would add to an operational route inevitably.

Begin with any odd-even expression to operate continuously by the operational rule, every such operational route via consecutive greater operational results will be getting longer and longer, and that in general, the sum of the constant term plus the coefficient of $\chi$ of integer’s expression is getting greater and greater along continuation of operations.

On the other, for a smaller operational result in synchronism with a
greater operational result, if it can be divided by $2^\mu$ with $\mu \geq 2$ to get an even smaller integer’s expression, once the even smaller integer’s expression is first smaller than a kind of $15+12c/19+12c$, then the even smaller integer’s expression is exactly $\#1$ satisfactory operational result. Accordingly, it can derive a kind of $15+12c/19+12c$ from the $\#1$ satisfactory operational result, so operations at the operational route may stop at here.

If the even smaller integer’s expression is greater than any kind of $15+12c/19+12c$ still, or the smaller operational result itself is an odd expression, then it needs us to operate continuously.

By this token, on the one hand, the number of operational routes at the bunch of operational routes of $15+12c/19+12c$ increases continually; on the other hand, the number of operational routes at the bunch of operational routes of $15+12c/19+12c$ reduces continually.

Since every $\#1$ satisfactory operational result can only appear at an operational route of $15+12c/19+12c$, and every operational route of $15+12c/19+12c$ is an embranchment of the bunch of operational routes of $15+12c/19+12c$, this shows that every $\#1$ satisfactory operational result must exist at the bunch of operational routes of $15+12c/19+12c$.

Yet every kind of $15+12c/19+12c$ can only derive from a $\#1$ satisfactory operational result, so every $\#1$ satisfactory operational result determines a kind of $15+12c/19+12c$. 
It is observed that every operational route from a kind of 15+12c/ 19+12c to the №1 satisfactory operational result contains a part of an operational route therein, at the bunch of operational routes of 15+12c/19+12c.

After start with 15+12c/ 19+12c to operate continuously, every gotten two operational routes intersect from each other undoubtedly. Namely every two operational routes at the bunch of operational routes of 15+12c/ 19+12c intersect from each other.

Therefore, an operational route gotten after operate each of kinds of 15+12c/ 19+12c, it intersects with any operational route except for itself at the bunch of operational routes of 15+12c/ 19+12c surely, including each of operational routes which contain №1 satisfactory operational results monogamously. Or else, the operational route exists not at the bunch of operational routes of 15+12c/19+12c, of course can’t get a №1 satisfactory operational result, also derive not a kind of 15+12c/ 19+12c.

Since c≥1, then there are infinitely many kinds of 15+12c/19+12c, accordingly there are infinitely many operational routes of 15+12c/ 19+12c monogamously. Well then, there should be a №1 satisfactory operational result at every such operational route of 15+12c/19+12c.

Start with any confirmable kind of 15+12c/19+12c to operate by the operational rule, in a manner of speaking, operations in regard to it are possessed of finite length surely, because either there are some odd-even expressions at the operational route in regard to it, to lead to come into
being smaller and smaller integer’s expressions, up to $\mathbb{N} \neq 1$ satisfactory operational result; or it is operated to intersect another operational route which contains smaller operational result $< \text{the confirmable kind itself.}$

Now that $n+1$ belongs within a kind of $15+12c/19+12c$, of course, not excepting the kind of $15+12c/19+12c$ which contains $n+1$, either it is operated into $\mathbb{N} \neq 1$ satisfactory operational result like each of listed above twelve kinds of $15+12c/19+12c$ to use the theorem 1; or it is operated to intersect another operational route which has an operational result $< n+1$ to use the theorem 2, like as $19+12(5+2^3j) = 79+2^5 \times 3j \rightarrow 238+2^5 \times 3^2j \rightarrow 119+2^4 \times 3^3j \rightarrow 358+2^4 \times 3^3j \rightarrow 179+2^3 \times 3^3j \rightarrow 538+2^3 \times 3^4j \rightarrow 269+2^2 \times 3^4j \rightarrow 808+2^2 \times 3^5j \rightarrow 404+2 \times 3^5j \rightarrow 202+3^5j \leftarrow 67+3^4j \leftarrow \ldots$

Therefore, if $n+1$ belongs within a kind of $15+12c/19+12c$, then $n+1$ suits the conjecture, because the kind of $15+12c/19+12c$ has been proved to suit the conjecture.

Hereto, $n+1$ has been proved to suit the conjecture, no matter which genus, which sort or which kind of odd numbers that contains $n+1$, or $n+1$ is exactly an even number.

Likewise, we can prove positive integers $n+2$, $n+3$, $n+4$ and otherwise up to every positive integer to suit the conjecture in the light of the old way of doing things.

The proof was thus brought to a close. As a consequence, the Collatz conjecture is tenable.
References


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