Qubit as a Polarization Division Multiplexed Quadrature Amplitude Modulated Symbol of Light

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Abstract: With optical communication technology today, it is practical to communicate with polarization division multiplexed (PDM) quadrature amplitude modulated (QAM) symbols, which are quantum superposition of horizontally and vertically polarized photons, which are, so called, qubits. As the number of bits encoded by a PDM QAM symbol is limited, according to Shannon-Hartley theorem, by signal to noise ratio, the degree of parallelism of quantum computers is limited. The result is consistent with quantum threshold theorem. Quantum entanglement between qubits only makes the number of bits encoded by the qubits smaller, because entanglement means correlation between the qubits. Thus, quantum computers are not more powerful than classical ones. Finally, it is shown that purely classical computers can be arbitrarily fast and ideal, that is, noiseless, quantum computers are classical.

I. Introduction

Quantum physicists have been considering quantum superposition, which was believed to make quantum computers more powerful than classical ones, beyond any classical intuition. For example, in [1], it is stated: “A left circularly polarized photon can encode a 0, for example, while a right-circularly polarized photon can encode a 1. Quantum systems can also register information in ways that classical digital systems cannot: a transversely polarized photon is in a quantum superposition of left and right polarization, and in some sense encodes both 0 and 1 at the same time.”.

However, for us communication engineers/scientists, it is intuitively obvious that transversely polarized classical/quantum light is obtained by multiplexing left-circularly and right-circularly polarized classical/quantum light with proper relative phase, which means state of so called “quantum” superposition is just a polarization state and can even be classical.

Fundamental misunderstanding of most, if not all, quantum physicists on quantum state is, seemingly, that they thought phase unique to quantum state, even though classical radio waves do have phase. For example, in [1], it is asserted that "the channel that randomizes the phases of input states" "transmits classical information perfectly, but transmits no quantum information at all". Note that, as [1] is, as shown by the second URL of [1], a classical paper in "Physical Review Journals" cited 247 times (as of May 22, 2018), recognitions on quantum state by the author of [1] is commonly shared by most, if not all, quantum physicists.

As qubit, in this case, is superposition of left-circularly and right-circularly (or vertically and horizontally) polarized photons with various relative amplitude and relative phase, which are analog, and we can use QAM (Quadrature Amplitude Modulation) to modulate amplitude and phase of photons, a qubit is a PDM (Polarization Division Multiplexed) QAM symbol of light. While quantum physicists often state "the unit of quantum information is the quantum bit,or qubit" [1], it is as metaphysical as stating "the unit of quantum energy is quantum Joule or quJ". Just as quantum energy is energy, quantum information is information, unit for which is "bit", and a qubit actually is an analog symbol.

Now, as "quantum" superposition is not necessarily quantum, it is almost obvious that quantum computers are not more powerful than classical ones. Rest of the paper provides supplementary information to confirm so. In section II, actual number of bits carried by a qubit is explained, which is mostly a tutorial on Shannon-Hartley theorem [2] on capacity of noisy analog channels. In addition, an explanation is given on why quantum threshold theorem [3], which was believed to make quantum computers enjoy arbitral parallelism through quantum error correction, is not helpful. In section III, effect of quantum entanglement is discussed. In section IV, it is shown that purely classical computers can be arbitrarily fast and that ideal quantum computers are classical. Section V concludes the paper

II. The number of bits carried by a qubit

To perform N parallel quantum computation, N bits of information must be represented by quantum state, which means 2^N quantum state must be reliably distinguished against noise or error. As quantum state of a quantum computer is represented by qubits, to make degree of parallelism of quantum computers N, N bits must be carried by a qubit and it was believed that N can be arbitrary large.

However, according to Shannon-Hartley theorem, which gives upper limit on capacity of noisy analog channel as $B\log_2(SNR+1)$ (B is bandwidth, SNR is signal to noise ratio and noise is white Gaussian), a qubit, as a PDM QAM symbol, can carry only finite number of bits limited by SNR. A rough explanation of the theorem is that, if average power of signal and noise are S and No, respectively, signal is typically blurred by No, only $S/No+1=SNR+1$ states can be reliably distinguished (1 is added because, even with very large No, one state is distinguished as is), even with ideal error corrections.

Thus, degree of parallelism of quantum computers is limited by SNR and is finite, which means quantum computers are not more powerful than classical ones. Applicability of Shannon-Hartley theorem is same regardless of whether a cubit is represented by photon polarization modes or, for example, by electron spin.

For us communication engineers/scientists, applicability of Shannon-Hartley theorem on a qubit is obvious, because information in a qubit is encoded as analog value of relative amplitude and phase between two orthogonal polarization modes (though we know we can enjoy twice more capacity by using absolute amplitude and phase if channel characteristics are known or measured by sacrificing small amount of capacity for training sequences, let's not discuss it in this letter). However, seemingly because of improper terminology of "qubit" suggests it's digital, quantum physicists have been evaluating channel error and capacity by noisy-channel coding theorem of Shannon on erroneous digital channels [4] and constructed additional theories around it. One such theory is quantum threshold theorem. According to the theorem, by recursively applying quantum error corrections, which requires exponential (w.r.t. recursion level) amount of hardware, error probability can be reduced doubly exponentially, which was considered good enough to make degree of quantum parallelism practically large. However, because of logarithm to (SNR+1) in Shannon-Hartley theorem, doubly exponential reduction of error, which is equivalent to doubly exponential reduction of noise, only means singly exponential increase of capacity. That is, quantum threshold theorem achieves exponential parallelism by exponential amount of hardware, which is classically possible and no good.

III. Can quantum entanglement help?

While "quantum" superposition is not really quantum, as is stated in [1] "Even more surprising from the classical perspective are so-called entangled states, in which two or more quantum systems are in superpositions of correlated states, so that two photons can encode, for example, 00 and 11 at once. Such entangled states behave in ways that apparently violate classical intuitions about locality and causality (without, of course, actually violating physical laws).", correlated states of quantum entanglement may be used to make quantum computers more powerful than classical ones. However, as correlation between symbols reduces the number of bits encoded by the symbols, quantum entanglement only reduces the degree of parallelism. With regard to amount of information represented, that "two photons can encode, for example, 00 and 11 at once" [1] is only as good as one photon encoding 0 and 1 at once.

IV. Classical computers are better than quantum ones

As purely classical computers are not annoyed by quantum effects such as size of atoms, which limits machining accuracy, or a unit of electric charge, which limits minimum signal current through shot noise, there is no limitation applying Moore's law [5] and Denard's scaling law [6]. So, if there is a problem of size S requiring, say, $\exp(S)$ time to solve by a purely classical computer and

S is given, by reducing size, voltage and current of the computer 1/exp(S) times, clock can be made exp(S) times faster [6] and the problem is solved in O(1) time. On such computers, all the classical algorithms work as are.

On the other hand, if we can reduce noise of a quantum computer zero, which may not be impossible, it can enjoy unlimited parallelism. However, if the average number of (Bose-Einstein condensed) quanta consisting a qubit (Q) is limited, capacity is limited [7] by quantum noise (as [7] assumes lossless channel and use number state, effective SNR=1/Q, smaller than 1/sqrt(Q) of shot noise with coherent state). To enjoy unlimited parallelism, we must make Q infinitely large, which means it is impossible and, even if it were possible, the computer is purely classical. It should also be noted that a qubit represented by a single quantum is very noisy.

V. Conclusions

It is shown that, as a qubit represented by "quantum" superposition of polarization modes of photons is a PDM QAM symbol of light, the number of bits carried by a qubit is limited by Shannon-Hartley theorem on noisy analog channels, which limits degree of parallelism of quantum computers. As correlation of quantum entanglement between symbols merely reduces the number of bits carried by the symbols, quantum computers can't be more powerful than classical ones.

It is also shown that purely classical computers can be arbitrarily fast and that noiseless quantum computers are classical.

As "qubit" is an improper and metaphysical terminology, because it is an analog symbol, it should be banned. The unit of quantum information is bit.

As for the other metaphysics of "quantum" superposition, which occurs even with classical light or radio waves, the Author is working on a separate paper to explain not-really-quantum superposition in an even more classically intuitive way than using classical light [8].

References

[1] S. Lloyd, "The capacity of the noisy quantum channel", Phys. Rev. A55 (1997) 1613, arXiv:quant-ph/9604015.

[2] C. E. Shannon, "Communication in the presence of noise", Proc. Institute of Radio Engineers, vol. 37, no.1, pp. 10-21, 1949.

[3] D. Aharonov, M. Ben-Or, "Fault-Tolerant Quantum Computation With Constant Error Rate", arXiv:quant-ph/9906129.

[4] C. E. Shannon, "A mathematical theory of communication", Bell System Technical Journal, 27(4), 1948.

[5] G. E. Moore, "Cramming more components onto integrated circuits". Electronics,1965.

[6] R. H. Dennard, F. Gaensslen, H. Yu, L. Rideout, E, Bassous, A, LeBlanc, "Design of ion-implanted

MOSFET's with very small physical dimensions", IEEE Journal of Solid State Circuits, SC–9 (5),1974.
[7] H. P. Yuen, M. Ozawa, "Ultimate Information Carrying Limit of Quantum Systems", Physical Review Letters, Vol. 70, No. 4, 1993.
[8] M. Ohta, "Pendulum represents superposition of binary quantum state of oscillations", (work in progress) http://viXra.org/abs/1805.0393.