A SIMPLE, DIRECT PROOF, USING SET-THEORY, OF FERMAT'S LAST THEOREM (FLT)

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Abstract. There is no confirmed, *simple* proof of FLT for each integral $n > 2$. Our simple proof of FLT is based on our algebraic identity, a function of two variables, denoted as $r^n + s^n = t^n$ for convenience. For $n \geq 1$ we relate (r, s, t) for which $r^n + s^n = t^n$ holds, with (x, y, z) for which $x^n + y^n = z^n$ holds. From these *true equations* we infer by *direct argument* (not by way of contradiction) that $\{(r, s, t)|r, s, t \in \mathbb{Z}, r^n + s^n = t^n\} = \{(x, y, z)|x, y, z \in \mathbb{Z}, x^n + y^n = z^n\}$ for any given n such that these sets are nonempty. Also, we show, for $n > 2$, that $\{(r, s, t)|r, s, t \in \mathbb{Z}\} = \varnothing$. Hence, for $n > 2 :$ $\{(x, y, z)|x, y, z \in \mathbb{Z}\} = \varnothing$.

1. Introduction

FLT states, for integral $n > 2$, that $x^n + y^n = z^n$ does not hold for (x, y, z) with integers $x, y, z \geq 1$. No accepted *simple* proof of FLT exists for integral $n > 2$.

This proposed *direct proof*, by definition, is *not by way of contradiction.*

2. Our Algebraic Equation : The Basis of our Direct Proof

Our argument begins, for integral $n \geq 1$, with a function of two variables, a *true statement* that is shown, below, to be sufficient for this proof, our equation (1) :

$$\left((4q^n)^{\frac{1}{n}}\right)^n + \left((p - 2q^n)^{\frac{1}{n}}\right)^n = \left((p + 2q^n)^{\frac{1}{n}}\right)^n. \tag{1}$$

For all integral values of $n \geq 1$: With *algebraic identity* (1), term q has all positive rational values, and term p has all positive real values such that $p > 2q^n$.

The values of rational q are *sufficient* for our argument, per Prop 3.1, below.

Denote $4q^n$, $p - 2q^n$, and $p + 2q^n$, respectively, by $r^n, s^n, t^n \in \mathbb{R}$, for convenience.

For $n \geq 1$, at minimum $n = 1, 2$, triple (r, s, t) is such that $r, s, t \in \mathbb{Z}, r, s, t \geq 1$ for which $r^n + s^n = t^n$ *holds*, i.e., such that $r^n + s^n = t^n$ is a *true statement.*

3. The Direct Argument Using Elementary Set-Theory

For $n \geq 1$, at minimum $n = 1, 2$, triple (x, y, z) is such that $x, y, z \in \mathbb{Z}, x, y, z \geq 1$ for which $x^n + y^n = z^n$ *holds*, i.e., such that $x^n + y^n = z^n$ is a *true statement.*

Some equations of the form $\alpha^n + \beta^n = \gamma^n$, with $n = 2$, do not hold for (α, β, γ) for which $\alpha, \beta, \gamma \in \mathbb{Z}$; thus, such equations would be *false statements* in our proof.

We intend to infer, for any given $n \geq 1$ such that the following sets are nonempty: $\{(r, s, t)|r, s, t \in \mathbb{Z}, r^n + s^n = t^n\} = \{(x, y, z)|x, y, z \in \mathbb{Z}, x^n + y^n = z^n\}$.

Should we confirm this equality it would show with $n = 3$, as the prime example for values of $n > 2$, that $\{(x, y, z)|x, y, z \in \mathbb{Z}, x^n + y^n = z^n\} = \varnothing$ - - - because, for $n > 2$, it is shown in Sect. 4, below, that $\{(r, s, t)|r, s, t \in \mathbb{Z}, r^n + s^n = t^n\} = \varnothing$.

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3.1. **For Integral $n \geq 1$: Definitions of Formal Sets. . . .**

Let A be $\{(r,s,t)|r,s,t \in \mathbb{R}, r,s,t > 0, r^n + s^n = t^n\}$.

Let B be $\{(r,s,t)|(rs),t$ are coprime, $r,s \in \mathbb{R}, r,s > 0, t \geq 1, r^n + s^n, s = t^n\}$.

Let C be $\{(r,s,t)|r,s,t$ are coprime, $r,s,t \geq 1, r^n + s^n = t^n\}$.

Let D be $\{(x,y,z)|x,y,z \in \mathbb{R}, x,y,z > 0, x^n + y^n = z^n\}$.

Let E be $\{(x,y,z)|(xy),z$ are coprime, $x,y \in \mathbb{R}, x,y > 0, z \geq 1, x^n + y^n = z^n\}$.

Let F be $\{(x,y,z)|x,y,z$ are coprime, $x,y,z \geq 1, x^n + y^n = z^n\}$.

Let G be $\{\frac{rs}{t}|\frac{rs}{t} \in \mathbb{R}, \frac{rs}{t} > 0, (r,s,t) \in A\}$.

Let H be $\{\frac{rs}{t}|\frac{rs}{t} \in \mathbb{Q}, \frac{rs}{t} > 0, (r,s,t) \in A\}$.

Let J be $\{\frac{rs}{t}|\frac{rs}{t} \in \mathbb{Q}, \frac{rs}{t} > 0, (r,s,t) \in B\}$.

Let K be $\{\frac{xy}{z}|\frac{xy}{z} \in \mathbb{R}, \frac{xy}{z} > 0, (x,y,z) \in D\}$.

Let L be $\{\frac{xy}{z}|\frac{xy}{z} \in \mathbb{Q}, \frac{xy}{z} > 0, (x,y,z) \in D\}$.

Let M be $\{\frac{xy}{z}|\frac{xy}{z} \in \mathbb{Q}, \frac{xy}{z} > 0, (x,y,z) \in E\}$.

3.2. **Formal Propositions.**

Proposition 3.1. *For any given n such that H, L are nonempty sets, $H = L$.*

Proof. Note that $G \supset H$, and that $K \supset L$. *For any given value of* n : Due *solely* to varying unrestricted real p, expression $\frac{(4q^n)^{\frac{1}{n}}(p-2q^n)^{\frac{1}{n}}}{(p+2q^n)^{\frac{1}{n}}} \in G$, or $\frac{rs}{t} \in G$, *takes every value* of $\frac{xy}{z} \in K$ - - which is a *key point* since $A \neq D$, per Sect. 4, below.

Term $\frac{xy}{z} \in K$ takes every value of $\frac{rs}{t} \in G$ because $x^n + y^n = z^n$ is the most general triple-nth power formulation. Hence, for any given n, $\frac{rs}{t} \in G = \frac{xy}{z} \in K$.

So, for any given n such that H, L are nonempty, $\frac{rs}{t} \in H \subset G = \frac{xy}{z} \in L \subset K$. Note : We consider solely 'initial' values of $\frac{rs}{t}$ and 'initial' values of $\frac{xy}{z}$, not multiples of $\frac{rs}{t}$ or of $\frac{xy}{z}$, nor reductions to lower or lowest terms of $\frac{rs}{t}$ or of $\frac{xy}{z}$. □

Values of rational q are sufficient for Prop. 3.1 to be true, as follows :

Irrational values of q are irrelevant because values taken by p, q, with q being independent of determining Prop. 3.1, are *sufficient* for our proof of Prop. 3.1.

Proposition 3.2. *For any given n such that B, E are nonempty sets, $B = E$.*

Proof. Per prop. 3.1 : For any given value of n such that sets H, L are nonempty, $\frac{rs}{t} \in H = \frac{xy}{z} \in L$. This implies, for any given n for which B, E are nonempty, that $\frac{rs}{t} \in B \subset H = \frac{xy}{z} \in E \subset L$. Again, note that we consider solely 'initial' values of $\frac{rs}{t}$ and 'initial' values of $\frac{xy}{z}$, not multiples of $\frac{rs}{t}$ or of $\frac{xy}{z}$, nor reductions to lower or lowest terms of $\frac{rs}{t}$ or of $\frac{xy}{z}$ (thereby, avoiding multiplication by real constants). □

Proposition 3.3. *For any given value of n such that set B is nonempty, the elements of B are :* $r = \left(\frac{w^n \pm \sqrt{w^{2n}-4v^n}}{2}\right)^{\frac{1}{n}}$, $s = \left(\frac{w^n \mp \sqrt{w^{2n}-4v^n}}{2}\right)^{\frac{1}{n}}$, *and* $t = w$.

Proof. For any given value of n with nonempty set J, notate taken-as-known values of $\frac{rs}{t} \in J$ by $\frac{v}{w}$ for which v, w are positive coprime values, such that $v \neq w$.
Thus, $\frac{rs}{t} = \frac{v}{w}$. Therefore, $t \in B = w$, and $rs \in B = v$.
The values for r, s are each uniquely determined by the following operations :
Solving $t = w$ and $rs = v$ simultaneously with $r^n + s^n = t^n$ results in
$(r^n)^2 - (r^n)(w^n) + v^n = 0$ and $(s^n)^2 - (s^n)(w^n) + v^n = 0$.
The *solution* in J is $r = \left(\frac{w^n \pm \sqrt{w^{2n}-4v^n}}{2}\right)^{\frac{1}{n}}$, $s = \left(\frac{w^n \mp \sqrt{w^{2n}-4v^n}}{2}\right)^{\frac{1}{n}}$, $t = w$. □

Proposition 3.4. *For any given value of n such that set E is nonempty, the elements of E are :* $x = \left(\frac{w^n \pm \sqrt{w^{2n}-4v^n}}{2}\right)^{\frac{1}{n}}$, $y = \left(\frac{w^n \mp \sqrt{w^{2n}-4v^n}}{2}\right)^{\frac{1}{n}}$, *and* $z = w$.

Proof. For any given value of n with nonempty set M, notate taken-as-known values of $\frac{xy}{z} \in M$ by $\frac{v}{w}$, with coprime v, w, as with Prop. 3.3, per Props. 3.1, 3.2. Thus, $\frac{xy}{z} = \frac{v}{w}$. So, $z \in E = w$, and $xy \in E = v$. The values of x, y are each uniquely determined, as follows : Solving $z = w$ and $xy = v$ simultaneously with $x^n + y^n = z^n$ results in $(x^n)^2 - (x^n)(w^n) + v^n = 0$ and $(y^n)^2 - (y^n)(w^n) + v^n = 0$.
The *solution* in M is $x = \left(\frac{w^n \pm \sqrt{w^{2n}-4v^n}}{2}\right)^{\frac{1}{n}}$, $y = \left(\frac{w^n \mp \sqrt{w^{2n}-4v^n}}{2}\right)^{\frac{1}{n}}$, $z = w$. □

Proposition 3.5. *For any given n for which C, F are each nonempty, $C = F$.*

Proof. Per propositions 3.3-3.4, for any given value of n such that B, E are each nonempty, $(r, s, t) \in B = (x, y, z) \in E$. Consequently, for any given value of n such that B, C, E, F are each nonempty, $(r, s, t) \in C \subset B = (x, y, z) \in F \subset E$. □

4. Results and Conclusion

With the triple $((4q^n)^{\frac{1}{n}}, (p - 2q^n)^{\frac{1}{n}}, (p + 2q^n)^{\frac{1}{n}})$, term $(4q^n)^{\frac{1}{n}}$ reduces to $2^{\frac{2}{n}}q$.

It logically follows, for $n > 2$, with $q \in \mathbb{Q}$, that $\{2^{\frac{2}{n}}q \in \mathbb{Q}, r^n + s^n = t^n\} = \varnothing$.
Thus, for $n > 2$, it is true that its subset, $\{2^{\frac{2}{n}}q \in Z, \subset \mathbb{Q}, r^n + s^n = t^n\} = \varnothing$.

[So, for $n > 2$, sets $A \neq D$. For example, with $n = 3$, equation $x^n + y^n = z^n$ holds for $x = 1$, $y = 2$, and $z = 9^{\frac{1}{3}}$; though, for $n = 3$, equation $r^n + s^n = t^n$ does not hold with $(4q^n)^{\frac{1}{n}} = 1$, $(p - 2q^n)^{\frac{1}{n}} = 2$, and $(p + 2q^n)^{\frac{1}{n}} = 9^{\frac{1}{3}}$. However, $A \neq D$ is irrelevent to this proof since $C = F$ is possible either with $A \neq D$ or $A = D$.]

Thus, for $n > 2$, equation (1) does not hold for (r, s, t) such that $r, s, t \in C$.
Per proposition 3.5, for any given value of n such that sets C, F are each nonempty, $(r, s, t) \in C = (x, y, z) \in F$.

Ergo, by using our simple, direct argument we conclude the following :
For $n > 2 :$ $x^n + y^n = z^n$ does not hold for (x, y, z) such that $x, y, z \in \mathbb{Z}$.

QED