FLT PROVEN SIMPLY AND DIRECTLY

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ABSTRACT. No simple proof of FLT has been established for $\{n > 2 \in \mathbb{Z}\}$. We devise, for $n \geq 1 \in \mathbb{Z}$, a detailed algebraic identity, $r^n + s^n = t^n$, that holds for $(r, s, t)|r, s, t \geq 1 \in \mathbb{Z}$, which we relate to $(x, y, z)|x, y, z \geq 1 \in \mathbb{Z}$ for which $x^n + y^n = z^n$ holds. For $r, s, t, x, y, z \in Z$ we infer that $\{(r, s, t)\} = \{(x, y, z)\}$ by using the unrestricted variable in our identity. For $n > 2$, we show there exists no $(r, s, t)|r, s, t \in \mathbb{Z}$. So, for $n > 2$, there exists no $(x, y, z)|x, y, z \in \mathbb{Z}$.

1. Introduction

Fermat's last theorem (FLT) states, for $n > 2 \in \mathbb{Z}$, that $x^n + y^n = z^n$ does not hold for $(x, y, z)|x, y, z \geq 1 \in \mathbb{Z}$. There is no *simple* proof of FLT for $\{n > 2\}$.

We argue, for $n > 2$, as if $\{(x, y, z)|x, y, z \in \mathbb{Z}\} = \varnothing$ is not yet at all proven.

2. Our Algebraic Identity : The Basis of our Direct Argument

We start a *direct argument* (not BWOC) with an *algebraic identity* we designed to be *sufficient* for implying FLT, though, not necessarily unique, our equation (1):

$$\left((4q^n)^{\frac{1}{n}}\right)^n + \left((m - 2q^n)^{\frac{1}{n}}\right)^n = \left((m + 2q^n)^{\frac{1}{n}}\right)^n. \tag{1}$$

For all integral values of $n \geq 1$: Term q has all positive rational values, and term m has all positive real values such that $m > 2q^n$.

Use $r, s, t \in \mathbb{R}$, respectively, to *denote* $(4q^n)^{\frac{1}{n}}$, $(m - 2q^n)^{\frac{1}{n}}$, and $(m + 2q^n)^{\frac{1}{n}}$.

With $r^n, s^n, t^n \geq 1$, existing values of $r, s, t \in \mathbb{Z}$ each is a unique n-th root.

Rational q is *legitimate*, being *sufficient* for our argument, per Prop. 3.1, below.

3. The Direct Argument

We want to relate $r^n + s^n = t^n$, which holds for $\{(r, s, t)|r, s, t \geq 1 \in \mathbb{Z}\} \subset \mathbb{R}$, with the Fermat equation, $x^n + y^n = z^n$, which holds for $\{(x, y, z)|x, y, z \geq 1 \in \mathbb{Z}\}$.

We intend to show for these equations that $\{(r, s, t) \in \mathbb{Z}\} = \{(x, y, z) \in \mathbb{Z}\}$.

Establishing this equality would confirm our belief, with $x^n + y^n = z^n$, for $n = 3$ as an example, that $\{(x, y, z)|x, y, z \in \mathbb{Z}\} = \varnothing$ - - - since we have additionally established with $r^n + s^n = t^n$, for $n > 2$, that $\{(r, s, t)|r, s, t \in \mathbb{Z}\} = \varnothing$ (in Sect. 4).

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3.1. **Formal Sets Defined for Any Given Value of n.** :

Let A be $\{(r,s,t)|r,s,t>0\in\mathbb{R}\}$ for which $r^n+s^n=t^n$ holds.

Let B be $\{(r,s,t)\in A|rs,t$ are coprime$\}$ for which $r^n+s^n=t^n$ holds.

Let C be $\{(r,s,t)\in B|r,s,t$ are coprime$\}$ for which $r^n+s^n=t^n$ holds.

Let D be $\{(x,y,z)|x,y,x>0\in\mathbb{R}\}\supset\mathbb{Z}$ for which $x^n+y^n=z^n$ holds.

Let E be $\{(x,y,z)\in D|xy,z$ are coprime$\}$ for which $x^n+y^n=z^n$ holds.

Let F be $\{(x,y,z)\in E|x,y,z\geq 1$ are coprime$\}$ for which $x^n+y^n=z^n$ holds.

Let G be $\{\frac{rs}{t}\in\mathbb{R}|(r,s,t)\in A\}$.

Let H be $\{\frac{rs}{t}\in G|rs,t\in\mathbb{Q}\}$.

Let I be $\{\frac{rs}{t}\in H|(r,s,t)\in B\}$.

Let J be $\{\frac{xy}{z}|(x,y,z)\in D\}$.

Let K be $\{\frac{xy}{z}\in J|xy,z\in\mathbb{Q}\}$.

Let L be $\{\frac{xy}{z}\in K|(x,y,z)\in E\}$.

3.2. **Formal Propositions.**

Proposition 3.1. *For any given n, with sets H,K nonempty, $H=K$.*

Proof. For any given n : Due *solely* to varying unrestricted real m, term $\frac{rs}{t}\in G$ or, alternate expression $\frac{(4q^n)^{\frac{1}{n}}(m-2q^n)^{\frac{1}{n}}}{(m+2q^n)^{\frac{1}{n}}}$, takes every value of $\frac{xy}{z}\in J$.

Therefore, $\{\frac{rs}{t}\in H\subset G\}=\{\frac{xy}{z}\in K\subset J\}$. □

Rational q is legitimate, being sufficient for Prop. 3.1 to be true, as follows :

Irrational values of q are irrelevant because values taken by m,q, with q independent of determining Prop. 3.1, are *sufficient* for our proof of Prop. 3.1.

Proposition 3.2. *For any given n with B,E nonempty, $B=E$.*

Proof. Per Prop. 3.1 : $\frac{(4q^n)(m-2q^n)}{m+2q^n}=\frac{(xy)^n}{z^n}$. With q and $\frac{xy}{z}$ each rational, the solutions for m are solely rational values. So, $(4q^n)$, $(m-2q^n)$, and $(m+2q^{n_1})$ are each rational : The terms r,s,t are rational for which $(4q^n)$; $(m-2q^n)$; $(m+2q^n)$ are existing n-th power rationals; alternatively, r,s,t is rational for which $(m+2q^n)$ and $(4q^n)(m-2^pq^n)$ are each existing n-th power rationals.

Reducing both sides of the equation $\frac{rs}{t}\in H=\frac{xy}{z}\in K$ to lowest terms yields $\frac{rs}{t}\in I\subset H=\frac{xy}{z}\in L\subset K$. Note : $(rs),t$ are coprime, and $(xy),z$ are coprime.

Consequently, $\{rs\in B\}=\{xy\in E\}$ and $\{t\in B\}=\{z\in E\}$. □

Proposition 3.3. *For any given n, we determine $\{(r,s,t) \in B\}$ uniquely.*

Proof. For any given n, with nonempty set H, notate taken-as-known values of $\frac{rs}{t} \in I$ by $\frac{v}{w}$ for which v, w are positive coprime values, $|v \neq w$. Thus, $\{\frac{rs}{t}\} = \{\frac{v}{w}\}$. So, $\{t \in B\} = \{w\}$, and $\{rs \in B\} = \{v\}$, are each determined uniquely, as follows:
Solving $t = w$ and $rs = v$ simultaneously with $r^n + s^n = t^n$ yields
$(r^n)^2 - (r^n)(w^n) + v^n = 0$ and $(s^n)^2 - (s^n)(w^n) + v^n = 0$.
Existing solutions in I are $r = \left(\frac{w^n \pm \sqrt{w^{2n} - 4v^n}}{2}\right)^{\frac{1}{n}}$; $s = \left(\frac{w^n \mp \sqrt{w^{2n} - 4v^n}}{2}\right)^{\frac{1}{n}}$; $t = w$.
Therefore, $\{(r,s,t) \in B\}$ is determined uniquely. □

Proposition 3.4. *For any given n, we determine $\{(x,y,z) \in E\}$ uniquely.*

Proof. For any given n with nonempty set K, we notate taken-as-known values of $\frac{xy}{z} \in L$ by $\frac{v}{w}$, with coprime v, w, as with Prop. 3.3, per Props. 3.1, 3.2. Thus, $\{\frac{xy}{z}\} = \{\frac{v}{w}\}$. So, $\{z \in E\} = \{w\}$, and $\{xy \in E\} = \{v\}$ are determined uniquely : Solving $z = w$ and $xy = v$ simultaneously with $x^n + y^n = z^n$ *gives the same result as in Prop. 3.3*, viz., $(x^n)^2 - (x^n)(w^n) + v^n = 0$ and $(y^n)^2 - (y^n)(w^n) + v^n = 0$.
Existing solutions in L are $x = \left(\frac{w^n \pm \sqrt{w^{2n} - 4v^n}}{2}\right)^{\frac{1}{n}}$; $y = \left(\frac{w^n \mp \sqrt{w^{2n} - 4v^n}}{2}\right)^{\frac{1}{n}}$; $t = w$.
Therefore, $\{(x,y,z) \in E\}$ is determined uniquely. □

Proposition 3.5. *For any given n with set C and set F nonempty, $C = F$.*

Proof. $\{(r,s,t) \in B\} = \{(x,y,z) \in E\}$. Thus, $\{r, s \in \mathbb{R}\} = \{x, y \in \mathbb{R}\}$.
Hence, $\{r, s$ are coprime $\in \mathbb{Z}\} \subset \mathbb{R} = \{x, y$ are coprime $\in \mathbb{Z}\} \subset \mathbb{R}$.
Therefore, $\{(r,s,t) \in C\} = \{(x,y,z) \in F\}$. □

4. Results and Conclusion

The triple $\{((4q^n)^{\frac{1}{n}}, (m - 2q^n)^{\frac{1}{n}}, (m + 2q^n)^{\frac{1}{n}})\}$ is such that $(4q^n)^{\frac{1}{n}} = 2^{\frac{2}{n}}q$.
For $n > 2$, with $q \in \mathbb{Q}$, thus, $\{2^{\frac{2}{n}}q \in \mathbb{Q}\} = \varnothing$, hence, $\{2^{\frac{2}{n}}q \in Z \subset \mathbb{Q}\} = \varnothing$.

Hence, for $n > 2$, equation (1) does not hold for $(r,s,t)|r,s,t \in \mathbb{Z}$.

Consequently, for $n > 2$, the Fermat equation $x^n + y^n = z^n$ does not hold for $(x,y,z)|x,y,z \geq 1 \in \mathbb{Z}$, per proposition 3.5.

QED