Classical physics derived from the space-time entropy of quantum fields

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We show that the statistical physics treatment of an arbitrary space-time entropy exactly reproduces most of the classical physics missing from quantum field theory; inertia, general relativity, dark energy and the arrow of time. The laws are recovered in the thermodynamic limit (as entropic laws) instead of in the usual classical limit ($\hbar \to 0$).

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I. ARBITRARY SPACE-TIME ENTROPY

We invent a generic micro-state $q \in \mathcal{Q}$ to represent the arbitrary space-time entropy of a system $\mathcal{Q}$. The micro-states will be described by two thermodynamic quantities: time $\tau(q)$ and space $x(q)$. In each case, the quantity must be paired with a conjugate having the appropriate units such that their product yields an energy. As a convention, we will prefix the name of the thermodynamic quantity with "thermal" and its conjugate with "entropic".

A. Thermodynamic quantities

The quantities and their conjugates are:

Symbol	*Name*	*Units*	
$\tau(q)$	thermal time	s	(1)
$x(q)$	thermal space	m	(2)
P	entropic power	J/s	(3)
F	entropic force	J/m	(4)

Using the relation $\tau = 1/\omega$ connecting time to angular-frequency, we can also introduce:

$\omega(q)$	thermal angular-frequency	s^{-1}	(5)
S	entropic action	Js	(6)

As $\tau(q)$ is related to $\omega(q)$, we can select either but not both. Selecting x and $\omega(q)$, the partition function over the ensemble of micro-states $\mathcal{Q}$ is:

$$Z = \sum_{q \in \mathcal{Q}} e^{-\beta[S\omega(q) + Fx(q)]} \tag{7}$$

and its uncountable reciprocal (we replace $q \in \mathcal{Q} \to l \in \mathcal{L}$ where $\mathcal{Q} \in \mathbb{N}$ and $\mathcal{L} \in \mathbb{R}$) is:

$$Z = \frac{1}{h^2} \int_{l \in \mathcal{L}} e^{-\beta[S\omega(l) + Fx(l)]} d\omega dx \tag{8}$$

The equation of state of Z is

$$TdS = Sd\omega(l) + Fx(l) \tag{9}$$

B. Taylor series expansion

Taking the Taylor series expansion of x(l), then its derivative $dx(l)$, we obtain

$$dx(l) = x'(0)dl + x''(0)ldl + \frac{1}{2}x'''(0)l^2dl + 4O(l^3)dl \tag{10}$$

For empirical reasons (the observable universe is a sphere), we will make the following replacements: l^2dl will be connected to the volume of the sphere ($dV = 4\pi l^2 dl$), ldl to the area of its surface ($dA = 8\pi l dl$) and dl to the circumference of a circle ($dL = 2\pi dl$). Second, we absorb the multiplication constant of $4O(l^3)dl$ into $O(l^3)$. Third, as the units of $x''(0)$ are m^{-1} and $x'''(0)$ are m^{-2}, we re-brand them as $1/L_0$ and $1/A_0$, respectively. Fourth, $x'(0)$ is dimensionless, we use it as the baseline and normalize it to 1. Fifth, we inject the Taylor series expansion back into the equation of state of Z. Finally, we obtain:

$$TdS = Sd\omega + \frac{1}{2\pi}FdL + \frac{1}{8\pi L_0}FdA + \frac{1}{8\pi A_0}FdV + O(l^3)dl \tag{11}$$

This equation will be the focus of this letter.

C. Quantum fields

All geometry described by (11) applies to a circle or sphere (periodic), and thus, all energy levels will be quantified. In addition, the construction of Z assigns an angular frequency $\omega(q)$ (oscillator) to every point in space. As a result, we have the beginnings of a quantum field theory.

II. CLASSICAL PHYSICS

What distinguishes classical physics from quantum physics? We put forward the thesis that the parts of classical physics that are missing from quantum field theory

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(inertia, general relativity, dark energy and the arrow of time) are emergent from the entropy of quantum fields (thermodynamic limits) instead of when $\hbar \to 0$ (classical limit). To prove the thesis, we will study the equation of states (11) in different regimes and under different limits.

A. Regimes and cycles

As is usual with any equation of states of statistical physics, we can draw thermodynamic cycles (by posing $dS = 0$, or $d\tau = 0$, etc). Furthermore, the Taylor series expansion allows us to study different limits by varying the intensity of the approximation. In each of the following sections, we will take a special case of the equation of states and link it to a law of classical physics.

B. Special relativity

We consider the regime $dS = 0$ and the approximation $O(l)dl \to 0$ (first Taylor term of the derivative). The equation of states is thus approximately:

$$0 = Sd\omega + \frac{1}{2\pi}FdL \tag{12}$$

We convert ω to τ using the relation $\tau = 1/\omega$ and then τ to t using $t = 2\pi\tau$. We define $P := S\tau^{-2}$. We obtain:

$$0 = -\frac{1}{2\pi}Pdt + \frac{1}{2\pi}FdL \tag{13}$$

$$\implies dL = \frac{P}{F}dt \tag{14}$$

Recall that as per standard statistical physics, the Lagrange multipliers are constant throughout the system; thus, the entropic power divided by the entropic force produces a constant ratio. The units of this ratio are meters per second. This ratio will be adopted as our definition for the speed of light $c := P/F$. We prove that the Lagrange multipliers of Z are in fact the Planck constants in Annex A. Indeed when P is the Planck power and F the Planck force, we do obtain the speed of light: $(c^5/G)/(c^4/G) = c$. This equation $dL = cdt$ is the fundamental relation of special relativity and is enough to recover the Lorentz factor.

C. Law of inertia

To derive the law of inertia, we use the same approximation as we did in the case of special relativity ($O(l)dl \to 0$) but instead of the $dS = 0$ regime we use the $d\omega = 0$ regime. To link the temperature to the acceleration, we will need the Unruh temperature[1–4] $T_u = \hbar a/(2\pi ck_B)$, an exact result of special relativity. The Unruh temperature is a temperature perceived by any accelerated object. This derivation has been done before by Erik Verldinde[4] under various entropy assumption ($\Delta S = 2\pi k_B$ and $\Delta x = \hbar/(mc)$). but, here we get equivalents to these assumptions for free from (11). In the current regime, the equation of states is thus approximately:

$$\frac{1}{2\pi}FdL = TdS \tag{15}$$

Solving for F and injecting $T := T_u$ consistent with the thermodynamic limit (constant temperature), we obtain

$$F = \left(\frac{\hbar}{ck_B}\frac{dS}{dL}\right)a \tag{16}$$

This equation has the same form as $F = ma$ provided that $m := \frac{\hbar}{ck_B}\frac{dS}{dL}$. Let's see how reasonable that is. First, it has the units of the kilogram (that's good!). Second, lets solve for $\frac{dS}{dL}$ assuming the definition of m.

$$\frac{dS}{dL} = k_B\frac{mc}{\hbar} \tag{17}$$

This amazing result tells us what the Compton wavelength ($\lambda = \hbar/(mc)$) really is! It scales the growth in entropy required to describe an object away from an origin. A bigger object (small Compton wavelength) must be more finely located that a smaller object (big Compton wavelength), and thus more entropy is required to describe its position.

D. General relativity

It is well-known that in General relativity, local space is Euclidean and cosmological space obeys the Einstein field equation (e.i. it is a manifold). We can understand why that is in the context of (11). Indeed, on local scales ($O(l)dl \to 0$) the entropy grows linearly $TdS = \frac{1}{2\pi}Fdl$ and we recover the "Euclidean" result of $F = ma$. It is this first Taylor term that imposes manifolds for the solutions of higher order terms. We will extend the approximated equation of states to the second term of the Taylor series ($O(l^2)dl \to 0$), and we will show that it reproduces General relativity. We pose regime $dS = 0$ and $d\omega = 0$, and we consider the case where $dA \gg dL$. The equation of states is thus approximately:

$$TdS = \frac{1}{8\pi L_0}FdA \tag{18}$$

Deriving the Einstein field equation from an area law for entropy has been done before by Jacobson[5], then later (and differently) by Erik Verlinde[4]. Furthermore, key insights were provided by Christoph Schiller[6]. Our strategy here will be injecting the Planck force ($F := c^4/G$) applied to a circular motion at the speed of light ($a = c^2/L_0$). We obtain:

$$TdS = \frac{c^2}{8\pi G}adA \tag{19}$$

We can think of it as a massive accretion disk at the cosmological horizon whose elements are orbiting at the speed of light (e.i. a sort of holographic screen). Additional elements that fall back in or out of the horizon can be modeled geometrically. To show how we produce a sketch of the proof by Jacobson.

We first connect TdS to an arbitrary coordinate system and energy flow rates:

$$TdS = \int T_{ab} k^a d\Sigma^b \tag{20}$$

Here T_{ab} is an energy-momentum tensor, k is a killing vector field, and $d\Sigma$ the infinitesimal elements of the coordinate system. Jacobson then shows that the area part can be rewritten as follows:

$$a d\overline{A} = c^2 \int R_{ab} k^a d\Sigma^b \tag{21}$$

where R_{ab} is the Ricci tensor describing the space-time curvature. This relation is obtained via the Raychaud-Huri equation, giving it a geometric justification. Combining the two with a local law of conservation of energy, he obtains

$$\int T_{ab} k^a d\Sigma^b = \frac{c^4}{8\pi G} \int R_{ab} k^a d\Sigma^b \tag{22}$$

which can only be satisfied if

$$T_{ab} = \frac{c^4}{8\pi G} \left[R_{ab} - \left(\frac{R}{2} + \Lambda \right) g_{ab} \right] \tag{23}$$

Here, the full field equations of general relativity are recovered, including the cosmological constant (as an integration constant), and the numerical term $1/(8\pi)$ (which usually can only be recovered by manually inserting it, or by imposing that General relativity to must connect to (independently-derived) Euclidean gravitation in the non-relativistic limit.) Furthermore, the manifold requirement is also taken empirically, whereas here we get it for free from the first Taylor series term. As a result, this is perhaps one of the most "complete" derivations of General relativity.

E. Dark energy

Finally, we show that the last term of the Taylor series expansion connects to the negative pressure associated with dark energy. Here, we pose the $d\omega = 0$ regime, $dV \gg dA \gg dL$ and $O(l^2)dl \to 0$. The approximated equation of states is:

$$TdS = \frac{1}{8\pi A_0} F dV \tag{24}$$

We define the pressure as a force divided by an area; $p := -F/A$ where $A := 8\pi A_0$. The negative sign, introduced by definition, is to illustrate that the pressure

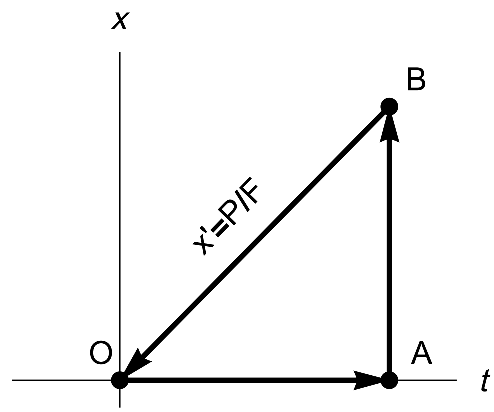

FIG. 1. A thermodynamic cycle in involving space and time is a light cone.

points towards the region of increasing entropy (outside the volume of the sphere). This type of derivation has been done by before by Easson[7,8], but again a lot of assumptions are required to get there —whereas here we pretty much get everything for free.

The radius of the observable universe is a sphere with the Hubble radius ($R = c/H$). Thus, its area is $4\pi c^2/H^2$. The pressure becomes:

$$p = -\frac{c^2 H^2}{4\pi G} \tag{25}$$

As argued by Easson et al., this is close to the current measured value for the negative pressure associated with dark energy. Expressed in terms of critical density, Easson rewrites p as

$$p = -\frac{2}{3} c^2 \rho_{\text{critical}} \tag{26}$$

As the dark energy is empirically measured to constitute approximately $\approx$ 70% of the observable universe, this result is quite remarkable. As we can see, the suggested entropic derivation of dark energy applies to the third term of the Taylor expansion.

F. Arrow of time

Now for what is perhaps the main result of (11); a partition function of statistical physics which includes a time quantity $\tau(q)$ is aware of past, present and future entropy. By studying thermodynamic cycles in space-time, we were able to draw conclusions that pertain to the arrow of time. In this model, the universe is an entropy sink used to power a forward translation in time.

Let us see, as an example (Figure 1), a simple thermodynamic cycle in space-time applicable to special relativity ($dS = 0$ and $O(l)dl \to 0$). The cycle can also be interpreted as a light cone but is able to make additional entropy claims.

- While transiting from O to A and by keeping the distance constant $dx = 0$, the system decreases its entropy : $dS/d\tau = -P/T$.

- While transiting from A to B and by keeping the time constant $d\tau = 0$, the system increases its entropy : $dS/dx = F/(2\pi T)$.

- While transiting from B to O and by keeping the entropy constant $dS = 0$, the system has speed c : $dL/dt = P/F$.

The decrease in entropy during the forward time translation is produced by the negative power and it must be offset by a proportional increase in the space quantities (to save the second law of thermodynamics). The result pits the expansion of the universe against the forward translation in time. To move backward, one "simply" needs to shrink the observable universe. We can see this more precisely in these equations:

$$F\frac{dx}{d\tau} < P \implies \frac{dS}{d\tau} < 0 \quad \text{decreasing} \tag{27}$$
$$F\frac{dx}{d\tau} = P \implies \frac{dS}{d\tau} = 0 \quad \text{constant} \tag{28}$$
$$F\frac{dx}{d\tau} > P \implies \frac{dS}{d\tau} > 0 \quad \text{increasing} \tag{29}$$

At (28), we have an inflection point and a shift occurs in the direction of the production of entropy over time. It is the point at which the production of entropy caused by the space quantities overtake and exceed the reduction in entropy caused by the time quantity. The second law of thermodynamics states that $dS/d\tau \geq 0$ and will hold for (28) and (29), but will be violated for (27). As an observer at rest lives in (27) he must additionally perceive an increase in entropy provided by the degrees of freedom of other physical laws (e.g. standard model, particles, and quantum fields), whereas no such requirement exists outside the universe.

III. CONCLUSION

We conclude that an arbitrary space-time entropy of a spherical observable universe implies both a quantum field (all points in space x are represented by an oscillator ω and the dimensions are periodic) and is able to recover the missing parts of classical physics as a thermodynamics limit instead of the usual classical limit ($\hbar \to 0$). The simple construction of Z and the always applicable Taylor series are likely to be the most general derivation of such.

Appendix A: Planck units

To derive the value of Lagrange multipliers of Z, we must link each one of them to known laws of physics which contains them. As we have done in the section of special relativity, we can link $c := P/F$. Here, we will further link $S := \hbar$ and $G := L^2c^3/\hbar$.

First, we find S; we assume a pool of energy then we ask how much energy must be taken from the pool to raise the angular-frequency of a micro-state at constant entropy. We pose $\omega := 2\pi f$ obtain:

$$dE = -2\pi S df \tag{A1}$$

Then, integrating, we obtain:

$$E = -2\pi S f + C \tag{A2}$$

Which, with $C := 0$, implies that potential energy of a micro-state with frequency f (we reverse the sign) is $E = 2\pi S f$. Recall that a constant entropy ($dS = 0$) implies that the system has a speed of c. Thus, we recover the energy of the photon $E = hf$ iff $S := \hbar$. Thus, S is the Planck action.

Second, we find G; to derive an expression for the gravitational constant, we will recover the law of gravitation of Newton. To do so, we assume an area entropy, then we obtain a corresponding temperature and finally, we solve for an entropic force as shown in Verlinde[4]. From the equipartition theorem ($E = \frac{1}{2}k_BTN$) and at thermodynamic equilibrium, we can expect the temperature at the surface of an entropy-bearing sphere (where each entropy bearer occupies an area of L^2, thus $N = 4\pi R^2/L^2$) to be:

$$T = \frac{L^2}{2\pi k_B}\frac{E}{x^2} \tag{A3}$$

Then injecting this temperature into (11) approximated with the same assumptions that produced the law of inertia ($d\omega = 0$ and $O(l)dl \to 0$), namely into : $\frac{1}{2\pi}FdL = TdS$. We solve F and obtain:

$$F = \left(\frac{L^2}{k_B}\frac{E}{x^2}\right)\frac{dS}{dL} \tag{A4}$$

Recall dS/dL? It is the Compton wavelength (17). Injecting (17) and posing $E := mc^2$, we recover the law of gravitation, and thus the definition of the gravitational constant

$$F = \left(\frac{L^2c^3}{\hbar}\right)\frac{Mm}{x^2} \implies G := \frac{L^2c^3}{\hbar} \tag{A5}$$

With S, c and G, it is trivial to recover all the Planck constants.

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