Abstract: The prime numbers greater than 5 have 4 terminations in their unit to infinity (1,3,7,9) and the composite numbers divisible by numbers greater than 3 have 5 terminations in their unit to infinity, these are (1,3,5,7,9). This paper develops an expression to calculate the prime numbers and composite numbers with ending 7.

Keywords: Prime numbers, composite numbers.

Introduction

The study of the prime numbers is wonderful, But to understand them, first study the composite numbers, in the absence of an expression that involves all of them I have investigated and I have discovered a brilliant expression that contains all the prime numbers greater than 3 and all composite number that are not divisible by 2 and by 3. This expression comes from investigating first how they are distributed the composite numbers with termination 7, this allowed me to explore its order and understand its mechanism. The expression of the prime numbers with termination 7 is its result. This paper has 8 demonstrations.

Theorem 1

Numbers with termination 7. These numbers are interleaved between prime numbers greater than 3 and composite numbers divisible by numbers greater than 3. These are distributed in two well-known sequences.

Numbers with termination 7 within the sequence $\beta$

$$\beta = (6 \times n \pm 1)$$

$\beta_a = (6 \times n + 1) = 7,13,19,25,31,37,43,49,55,61,67,73,79,85,\ldots$  

$\beta_b = (6 \times n - 1) = 5,11,17,23,29,35,41,47,53,59,65,71,77,83,89 \ldots$

Within the beta sequence we find composite numbers and prime numbers. To be able to locate only the numbers that end with 7 we will go to the next point (Theorem 2)
Theorem 2

At point A we will look for numbers with ending 7 within the sequence $\beta_b = (6 \cdot n - 1)$

At point B we will look for composite numbers with ending 7 within the sequence $\beta_b = (6 \cdot n - 1)$

$\beta_b = (6 \cdot n - 1) = 5, 11, 17, 23, 29, 35, 41, 47, 53, 59, 65, 71, 77, 83, 89 \ldots \ldots$

$n > 0$

Reference A007528 (The On-line Encyclopedia of Integer Sequences)

A) Formula for numbers with termination 7 within the sequence $\beta_b$

\[ N_{(b)\tau7} = (30 \cdot n + 17) \]

$N_{(b)\tau7}$ = numbers with termination 7 within the sequence $\beta_b$

$N_{(b)\tau7} = 17, 47, 77, 107, 137, 167, 197, 227, 257, 287, 317, 347, 377, 407, \ldots \ldots$

$n \geq 0$

Reference A128468 (The On-line Encyclopedia of Integer Sequences)

Demonstration 1

B) Formula for composite numbers with termination 7 within the sequence $\beta_b$

Composite numbers congruent to 17 (mod 30) within the sequence $\beta_b = (6 \cdot n - 1)$

\[ N_{C(b)\tau3} = (30 \cdot n + 17) = \beta \cdot (\delta + 30 \cdot z) \]

$n \geq 0$

$z \geq 0$

$\beta$ has infinite values

Formed by the sequence $\beta = (6 \cdot n \pm 1) = 5, 7, 11, 13, 17, 19, 23, 25, 29, 31, \ldots \ldots$

$\beta_1 = 5, \beta_2 = 7, \beta_3 = 11, \beta_4 = 13 \ldots \ldots$

$\delta$ has 10 variants

These 10 variables are always ordered in the same way and are repeated simultaneously until infinity as we add more values of $\beta$.

5 Nothing 11 7 29 31 23 19 25 Nothing 13 17

The multiples of 5 are green and carry the word (nothing), these are not calculated since no multiples of 5 generate numbers with termination 7.
**$NC_{(b)7}$ = Composite numbers termination 7**

$NC_{(b)7} = (30 \times n + 17)$

- $\beta_1$ Nothing = 5 Nothing
- $\beta_2$ *(11 + 30\(z\)) = 7 *(11 + 30\(z\))
- $\beta_3$ *(7 + 30\(z\)) = 11 *(7 + 30\(z\))
- $\beta_4$ *(29 + 30\(z\)) = 13 *(29 + 30\(z\))
- $\beta_5$ *(31 + 30\(z\)) = 17 *(31 + 30\(z\))
- $\beta_6$ *(23 + 30\(z\)) = 19 *(23 + 30\(z\))
- $\beta_7$ *(19 + 30\(z\)) = 23 *(19 + 30\(z\))
- $\beta_3$ Nothing = 25 Nothing
- $\beta_9$ *(13 + 30\(z\)) = 29 *(13 + 30\(z\))
- $\beta_{10}$ *(17 + 30\(z\)) = 31 *(17 + 30\(z\))
- $\beta_{15}$ Nothing = 35 Nothing
- $\beta_{12}$ *(11 + 30\(z\)) = 37 *(11 + 30\(z\))
- $\beta_{13}$ *(7 + 30\(z\)) = 41 *(7 + 30\(z\))
- $\beta_{14}$ *(29 + 30\(z\)) = 43 *(29 + 30\(z\))
- $\beta_{15}$ *(31 + 30\(z\)) = 47 *(31 + 30\(z\))
- $\beta_{16}$ *(23 + 30\(z\)) = 49 *(23 + 30\(z\))
- $\beta_{17}$ *(19 + 30\(z\)) = 53 *(19 + 30\(z\))
- $\beta_{18}$ Nothing = 55 Nothing
- $\beta_{19}$ *(13 + 30\(z\)) = 59 *(13 + 30\(z\))
- $\beta_{20}$ *(17 + 30\(z\)) = 61 *(17 + 30\(z\))

The series is repeated every 10 blocks, (nothing, 11, 7, 29, 31, 23, 19, nothing, 13, 17).

We can add more $\beta$ numbers and expand the formula infinitely.

**Demonstration 2**

We solve when $z = 0$, $z = 1$, $z = 2$, .......

$NC_{(b)7} = (30 \times n + 17)$

- $\beta_1$ Nothing = (30 * $n$ + 17)
- $\beta_{12}$ *(11 + 30\(z\)) = 77,287,497, .......
- $\beta_{13}$ *(7 + 30\(z\)) = 77,407,737, .......
- $\beta_{14}$ *(29 + 30\(z\)) = 377,767,1157, .......
- $\beta_{15}$ *(31 + 30\(z\)) = 437,1007,1577, .......
- $\beta_{16}$ *(23 + 30\(z\)) = 437,1127,1817, .......
- $\beta_{17}$ *(19 + 30\(z\)) = 377,1247,2117, .......
- $\beta_{18}$ Nothing = 527,1457,2387, .......

continue infinitely

**Demonstration 3**

C) Distances between composite numbers with termination 7 in column $b$.

The distance between composite numbers with termination 7 when we use the same value for $\beta$ is equal to:

**Distance between composite number $D_7 = 30 \times \beta$**

$D_7 =$ Distance between composite number (Termination 7).

**Example**

- $\beta = 7; \quad D_7 = 30 \times 7 = 210$
- $\beta = 11; \quad D_7 = 30 \times 11 = 330$
- $\beta = 13; \quad D_7 = 30 \times 13 = 390$
Theorem 3

At point A we will look for numbers with ending 7 within the sequence \( \beta_a = (6 \cdot n + 1) \)
At point B we will look for composite numbers with ending 7 within the sequence \( \beta_a = (6 \cdot n + 1) \)

\( \beta_a = (6 \cdot n + 1) = 7,13,19,25,31,37,43,49,55,61,67,73,79,85,…….. \)
\( n > 0 \)

Reference A016921 (The On-line Enciclopedia of integers sequences)

A) Formula for numbers with termination 7 within the sequence \( \beta_a \)

\[ N(a)_t7 = (30 \cdot n + 7) \]

\( N(a)_t7 = 7,37,67,97,127,157,187,217,247,277,307,337,367,397,…….. \)
\( n \geq 0 \)
\( z \geq 0 \)

Reference A128471 (The On-line Enciclopedia of integers sequences)

Demonstration 4

B) Formula for composite numbers with termination 7 within the sequence \( \beta_a \)

Composite numbers congruent to 7 (mod 30) within the sequence \( \beta_a = (6 \cdot n + 1) \)

\[ Nc(a)_t7 = (30 \cdot n + 7) = \beta \cdot (\delta + 30 \cdot z) \]

\( n \geq 0 \)
\( z \geq 0 \)

\( \beta \) has infinite values

Formed by the sequence \( \beta = (6 \cdot n \pm 1) = 5,7,11,13,17,19,23,25,29,31,….. \)
\( \beta_1 = 5, \beta_2 = 7, \beta_3 = 11, \beta_4 = 13….. \)

\( \delta \) has 10 variants

These 10 variables are always ordered in the same way and are repeated simultaneously until infinity as we add more values of \( \beta \).

\[
\begin{array}{cccccccccccc}
5 & \text{Nothing} & 31 & 17 & 19 & 11 & 13 & 29 & 25 & \text{Nothing} & 23 & 7
\end{array}
\]

The multiples of 5 are green and carry the word (nothing), these are not calculated since no multiples of 5 generate numbers with termination 7.
**NC\(_{(a)}\) t\(_{7}\)= Composite numbers termination 7**

\[
NC_{(a)}t_7 = (30 \ast n + 7) = (30 \ast n + 7)
\]

The series is repeated every 10 blocks (nothing, 31,17,19,11,13,29,nothing, 23,7) to infinity. We can add more β numbers and expand the formula infinitely.

**Demonstration 5**
We solve when \(z = 0, z = 1, z = 2, \ldots\)

\[
NC_{(a)}t_7 = (30 \ast n + 7) = (30 \ast n + 7)
\]

**Demonstration 6**

A) **Distances between composite numbers with termination 7 in column a**

The distance between composite numbers with termination 7 when we use the same value for \(\beta\) is equal to:

Distance between composite number \(D_7 = 30 \ast \beta\)

\(D_7= Distance \ between \ composite \ number \ (Termination \ 7)\)

**Example**

A. \(\beta = 7; \quad D_7 = 30 \ast 7 = 210\)
B. \(\beta = 11; \quad D_7 = 30 \ast 11 = 330\)
C. \(\beta = 13; \quad D_7 = 30 \ast 13 = 390\)
**Theorem 4** We will use the same information that we obtained to calculate the numbers composed in the theorem 2, but in this occasion we will use the inequality to obtain only results of prime numbers.

At point A we will look for prime numbers with ending 3 within the sequence \( \beta_b = (6 \cdot n - 1) \)

\[
\beta_b = (6 \cdot n - 1) = 5, 11, 17, 23, 29, 35, 41, 47, 53, 59, 65, 71, 77, 83, 89 \ldots
\]

\( n > 0 \)

Reference A007528 (The On-line Encyclopedia of Integer Sequences)

**Demonstration 7**

A) Formula for Prime numbers with termination 7 within the sequence

\[
\beta_b = (6 \cdot n - 1)
\]

\[
P_{(b)7} = (30 \cdot n + 17) \neq \beta (\delta + 30 \cdot z)
\]

\( n \geq 0 \)

\( z \geq 0 \)

\( \beta \) has infinite values

Formed by the sequence \( \beta = (6 \cdot n \pm 1) = 5, 7, 11, 13, 17, 19, 23, 25, 29, 31, \ldots \)

\( \beta_1 = 5, \beta_2 = 7, \beta_3 = 11, \beta_4 = 13 \ldots \)

\( \delta \) has 10 variants

These 10 variables are always ordered in the same way and are repeated simultaneously until infinity as we add more values of \( \beta \).

<table>
<thead>
<tr>
<th>5 Nothing</th>
<th>11</th>
<th>7</th>
<th>29</th>
<th>31</th>
<th>19</th>
<th>25 Nothing</th>
<th>13</th>
<th>17</th>
</tr>
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</table>

The multiples of 5 are green and carry the word (nothing), these are not calculated since no multiples of 5 generate numbers with termination 7.

Primes congruent to 17 (mod 30) within the sequence \( \beta_b = (6 \cdot n - 1) \)

\[
P_{(b)7} = \text{Prime numbers, termination 7}
\]

\[
P_{(b)7} = (30 \cdot n + 17) \neq (30 \cdot n + 17)
\]

\[
\#5 \text{ Nothing}
\]

\[
\#7 \cdot (11 + 30 \cdot z)
\]

\[
\#11 \cdot (7 + 30 \cdot z)
\]

\[
\#13 \cdot (29 + 30 \cdot z)
\]

\[
\#17 \cdot (31 + 30 \cdot z)
\]

\[
\#19 \cdot (23 + 30 \cdot z)
\]

\[
\#23 \cdot (19 + 30 \cdot z)
\]

\[
\#25 \text{ Nothing}
\]

\[
\#29 \cdot (13 + 30 \cdot z)
\]

\[
\#31 \cdot (17 + 30 \cdot z)
\]

continue infinitely

continue infinitely
\(P_{(b)17} = 17, 47, 107, 137, 167, 197, 227, 257, 317, 347, 467, 557, 617, 647, 677, 797, 827, 857, 887, 947, 977, 1097, 1187, 1217, 1277, 1307, 1367, 1427, 1487, 1607, 1637, 1667, 1697, 1787, 1847, 1877, 1907, 1997, 2027, 2087, 2207, 2237, 2267, 2297, 2357, 2417, \ldots\)

Reference
A039949 (The On-line Encyclopaedia of integers sequences)

**Theorem 5** We will use the same information that we obtained to calculate the composite numbers in the theorem 3, but in this occasion we will use the inequality to obtain only results of prime numbers.

At point A we will look for prime numbers with ending 7 within the sequence \(\beta_a = (6 \cdot n + 1)\)

\[
\beta_a = (6 \cdot n + 1) = 7, 13, 19, 25, 31, 37, 43, 49, 55, 61, 67, 73, 79, 85, \ldots
\]

\(n > 0\)

Reference A016921 (The On-line Encyclopaedia of integers sequences)

**Demonstration 8**

A) Formula for Prime numbers with termination 7 within the sequence

\[
P_{(a)7} = (30 \cdot n + 7) \neq \beta \cdot (\delta + 30 \cdot z)
\]

\(n \geq 0\)

\(z \geq 0\)

\(\beta\) has infinite values

Formed by the sequence \(\beta = (6 \cdot n \pm 1) = 5, 7, 11, 13, 17, 19, 23, 25, 29, 31, \ldots\)

\(\beta_1 = 5, \beta_2 = 7, \beta_3 = 11, \beta_4 = 13\ldots\)

\(\delta\) has 10 variants

These 10 variables are always ordered in the same way and are repeated simultaneously until infinity as we add more values of \(\beta\).

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<th>5 Nothing</th>
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<th>17</th>
<th>19</th>
<th>11</th>
<th>13</th>
<th>29</th>
<th>25 Nothing</th>
<th>23</th>
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The multiples of 5 are green and carry the word (nothing), these are not calculated since no multiples of 5 generate numbers with termination 7.

Primes congruent to 7 (mod 30) within the sequence \(\beta_a = (6 \cdot n + 1)\)

\[
P_{(a)7} = \text{Prime numbers termination 7}
\]
\[ P(a)_7 = (30 \times n + 7) \]

Reference A132231 (The On-line Encyclopedia of integers sequences)

**Theorem 6** Graphic table 1, with termination 7.

<table>
<thead>
<tr>
<th>A</th>
<th>( \beta_a )</th>
<th>B</th>
<th>( \beta_b )</th>
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</table>

In yellow the prime numbers

Composite number in red

Prime numbers with termination 7 in green

In red the composite numbers

Prime number in Yellow

Composite numbers with termination 7 in light blue
Conclusion

The numbers with ending 7 are ordered every 30 numbers interspersed between composite numbers and prime numbers. These have two variables. The first variable shows that the numbers of the formula \( N_{(b)c} = (30 \cdot n + 17) \) are located in the column (B). The sum of their digits always generates the sequence 2,5,8. The second variable shows that the numbers of the formula \( N_{(a)c} = (30 \cdot n + 7) \) are located in another column (A). The sum of its digits always generates the sequence 1,4,7.

By means of equalities and inequalities we can condition these formulas to obtain all prime numbers greater than 3 and all composite numbers divisible by numbers greater than 3 by means of a simple, unique and infinite expression.

By equalities we obtain composite numbers divisible by numbers greater than 3 with termination 7. By inequalities we obtain the prime numbers greater than 3 with termination 7.

The formula developed with the 10 variables of the delta letter allows you to obtain them infinitely. These 10 variables are the key for the formula to work.

Thanks to this expression we can understand how the prime numbers and the composite numbers with ending 7 are distributed.

The multiples of 5 in beta are excluded since they do not generate numbers with ending 7.

This formula demonstrates that it is possible to calculate and obtain the sequence of prime numbers with ending 7 and also that of the composite numbers.

This model is applied to the other three terminations (1,3,9) although the locations of the delta numbers vary, since these are the same but they are located differently.

Acknowledgements

To Santino, Lisandro and Natalia for their contribution, patience and support.

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