Revisiting the Derivation of Heisenberg's Uncertainty Principle: The Collapse of Uncertainty at the Planck Scale

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Abstract

In this paper we will revisit the derivation of Heisenberg's uncertainty principle. We will see how the Heisenberg principle collapses at the Planck scale by introducing a minor modification. The beauty of our suggested modification is that it does not change the main equations in quantum mechanics; it only gives them a Planck scale limit where uncertainty collapses. We suspect that Einstein could have been right after all, when he stated, "God does not throw dice." His now-famous saying was an expression of his skepticism towards the concept that quantum randomness could be the ruling force, even at the deepest levels of reality. Here we will explore the quantum realm with a fresh perspective, by re-deriving the Heisenberg principle in relation to the Planck scale.

Our modified theory indicates that renormalization is no longer needed. Further, Bell's Inequality no longer holds, as the breakdown of Heisenberg's uncertainty principle at the Planck scale opens up the possibility for hidden variable theories. The theory also suggests that the superposition principle collapses at the Planck scale. Further, we show how this idea leads to an upper boundary on uncertainty, in addition to the lower boundary. These upper and lower boundaries are identical for the Planck mass particle; in fact, they are zero, and this highlights the truly unique nature of the Planck mass particle.

1 Introduction to the Momentum and Energy operator

A commonly used wave function[1] in quantum mechanics is

$$\Psi(x,t) = e^{i(kx-\omega t)} \tag{1}$$

where $\omega = \frac{E}{\hbar}$, and

$$k = \frac{2\pi}{\lambda} \tag{2}$$

From the de Broglie matter wave, we know that

$$\lambda = \frac{h}{p} \tag{3}$$

This means we have

$$k = \frac{p}{\hbar} \tag{4}$$

and this means we can write the wave equation also as (well known)

$$\Psi = e^{i\left(\frac{p}{\hbar}x - \frac{E}{\hbar}t\right)} \tag{5}$$

Next we take the partial derivative with respect to x and get

$$\frac{\partial \Psi}{\partial x} = \frac{ip}{\hbar}\Psi \tag{6}$$

Multiplying each side with $\frac{\hbar}{i}$ we get

[1] The plane wave solution to the Klein–Gordon equation and the Schröedinger equation.

$$
\begin{aligned}
\frac{\hbar}{i}\frac{\partial \Psi}{\partial x} &= p\Psi \\
-i\hbar\frac{\partial \Psi}{\partial x} &= p\Psi
\end{aligned} \tag{7}
$$

and from this we have the well-known momentum operator

$$\hat{p} = -i\hbar\frac{\partial}{\partial x} \tag{8}$$

and the partial derivative of the wave equation with respect to time is

$$
\begin{aligned}
\frac{\hbar}{i}\frac{\partial \Psi}{\partial t} &= E\Psi \\
-i\hbar\frac{\partial \Psi}{\partial t} &= E\Psi
\end{aligned} \tag{9}
$$

which means the energy operator must be

$$\hat{E} = -i\hbar\frac{\partial}{\partial t} \tag{10}$$

Next, we will use this information to derive Heisenberg's uncertainty principle.

2 Introduction to Commutators, Operators, and Heisenberg's Uncertainty Principle

We will first introduce the standard approach to obtain the Heisenberg uncertainty relation. A standard commutator is given by

$$[\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A} \tag{11}$$

If $[\hat{A}, \hat{B}] \neq 0$, then $\hat{A}$ and $\hat{B}$ do not commute. If $[\hat{A}, \hat{B}] = 0$, then $\hat{A}$ and $\hat{B}$ do commute. Based on this, we have the following uncertainty

$$\sigma_A\sigma_B = \frac{1}{2}|\langle\hat{A}, \hat{B}\rangle| = \frac{1}{2}|\int \Psi^*[\hat{A}, \hat{B}]\Psi dt| \tag{12}$$

and we see from the expression above that if $\hat{A}$ and $\hat{B}$ commute there is no uncertainty. The Heisenberg uncertainty principle [1, 2] can be derived from the following commutator

$$[\hat{p}, \hat{x}] = \hat{p}\hat{x} - \hat{x}\hat{p} \tag{13}$$

where the $\hat{p}$ is the momentum operator and $\hat{x}$ is the position operator. Again, the momentum operator is given by

$$\hat{p} = -i\hbar\frac{\partial}{\partial x} \tag{14}$$

and the position operator is given by

$$\hat{x} = x \tag{15}$$

From this we have

$$
\begin{aligned}
[\hat{p}, \hat{x}]\Psi &= [\hat{p}\hat{x} - \hat{x}\hat{p}]\Psi \\
&= \left(-i\hbar\frac{\partial}{\partial x}\right)(x)\Psi - (x)\left(-i\hbar\frac{\partial}{\partial x}\right)\Psi \\
&= -i\hbar\left(\Psi + x\frac{\partial \Psi}{\partial (x)}\right) + i\hbar x\frac{\partial \Psi}{\partial (x)} \\
&= -i\hbar\left(\Psi + x\frac{\partial \Psi}{\partial (x)} - \frac{\partial \Psi}{\partial (x)}\right) \\
&= -i\hbar\Psi
\end{aligned} \tag{16}
$$

And we have the following uncertainty

$$\begin{aligned}\sigma_p\sigma_x &\geq \frac{1}{2}\left|\int \Psi^*[\hat{p},\hat{x}]\Psi dt\right| \\ &\geq \frac{1}{2}\left|\int \Psi^*(-i\hbar)\Psi dt\right| \\ &\geq \frac{1}{2}\left|-i\hbar\int \Psi^*\Psi dt\right| \end{aligned} \tag{17}$$

and since $\int \Psi^*\Psi dt$ must sum to 1 (there must be 100% probability for the particle to be somewhere), we are left with

$$\begin{aligned}\sigma_p\sigma_x &\geq \frac{1}{2}|-i\hbar| \\ \sigma_p\sigma_x &\geq \frac{\hbar}{2}\end{aligned} \tag{18}$$

that is as expected, we arrive at the Kennard version of Heisenberg's uncertainty principle. The Heisenberg uncertainty principle is the foundation of many of the results and interpretations of quantum mechanics. If we derive the uncertainty principle from the energy and time operator instead, we get

$$\sigma_E\sigma_t \geq \frac{\hbar}{2} \tag{19}$$

This is shown in detail in the Appendix.

3 The Planck Scale and Haug's Maximum Velocity for Matter

In 1899, Max Planck [3, 4] introduced what he called the 'natural units': the Planck mass, the Planck length, the Planck time, and the Planck energy. He derived these units using dimensional analysis, assuming that the Newton gravitational constant, the Planck constant and the speed of light where the most important universal constants. Lloyd Motz, while working at the Rutherford Laboratory in 1962, [5, 6, 7] suggested that there was probably a very fundamental particle with a mass equal to the Planck mass that he called the "Uniton." Motz acknowledged that his Unitons (Planck mass particles) had far too much mass compared to known subatomic masses. He tried to address this issue by claiming that the Unitons had radiated most of their energy away:

> According to this point of view, electrons and nucleons are the lowest bound states of two or more Unitons that have collapsed down to the appropriate dimensions gravitationally and radiated away most of their energy in the process. – Lloyd Motz

Others have suggested that there were plenty of Planck mass particles around just after the Big Bang; see [8], but that most of the mass of these super-heavy particles has radiated away. Modern physics has also explored the concept of a hypothetical Planck particle that has $\sqrt{\pi}$ more mass than the Uniton originally suggested by Motz. Some physicists, including Motz and Hawking, have suggested such particles could be micro-black holes [9, 10, 11]. Planck mass particles have even been proposed as candidates for cosmological dark matter, [12, 13]. [2]

We will suggest that the Planck mass particle only lasts for one Planck second and that its mass should be seen as approximately 1.17×10^{-51} kg. The Planck mass particle is, in our view, the mass-gap. It is a time-dependent mass. We suspect that all other masses are time-dependent as well, but this will first be noticeable when one is trying to measure their mass below their reduced Compton time. The electron's mass can be found from the electron's reduced Compton length, for example.

In a series of recent publications, Haug [14, 15, 16, 17] has suggested that there is a maximum velocity for anything with rest-mass given by

$$v_{max} = c\sqrt{1 - \frac{l_p^2}{\bar{\lambda}^2}} \tag{20}$$

where l_p is the Planck length, and $\bar{\lambda}$ is the reduced Compton wavelength of the elementary particle in question. For any observed particle, the maximum velocity will be very close to that of the speed of

[2]We are quite skeptical towards the interpretation of dark matter, but that is beyond the scope of this paper.

light, but considerably above the speed achieved in the Large Hadron Collider. An electron has a reduced Compton wavelength of $\bar{\lambda}_e \approx 3.86159 \times 10^{-13}$ m and can never be accelerated to a velocity faster than

$$v = c\sqrt{1 - \frac{l_p^2}{\bar{\lambda}_e^2}} = c \times 0.9999999999999999999999999999999999999999999999912416 \tag{21}$$

In the above calculation, we have assumed a Planck length of 1.616199×10^{-35}. As there is considerable uncertainty about the exact value for the Planck length, there is also some uncertainty about the theoretical value for the maximum speed limit of the electron. In our framework, the Planck length and the Planck mass can be measured independent of any prior knowledge of Newtonian gravity or the gravitational constant, as recently shown by [17, 18]. For additional context, Haug's maximum velocity was first derived in [14], and has later been derived from Heisenberg's uncertainty principle when assuming the minimum uncertainty in position is $x = l_p$. Here we will show that the Heisenberg uncertainty principle breaks down at the Planck scale if the maximum velocity for matter follows this expression, which can be derived from special relativity equations by assuming that the reduced Compton wavelength can never can undergo more length contraction than the Planck length. Alternatively, we can assume that the maximum frequency is the Planck frequency, or that the maximum relativistic mass of an elementary particle is the Planck mass.

This also means there is a maximum limit on the relativistic momentum of

$$P_{max} = \frac{mv_{max}}{\sqrt{1 - \frac{v_{max}^2}{c^2}}} = \frac{mc\sqrt{1 - \frac{l_p^2}{\bar{\lambda}^2}}}{\sqrt{1 - \frac{\left(c\sqrt{1 - \frac{l_p^2}{\bar{\lambda}^2}}\right)^2}{c^2}}} = m_p c\sqrt{1 - \frac{l_p^2}{\bar{\lambda}^2}} \tag{22}$$

Further, the maximum kinetic energy is given by

$$\begin{aligned}
E_k &= \frac{mc^2}{\sqrt{1 - \frac{v_{max}^2}{c^2}}} - mc^2 \\
&= \frac{mc^2}{\sqrt{1 - \frac{\left(c\sqrt{1 - \frac{l_p^2}{\bar{\lambda}^2}}\right)^2}{c^2}}} - mc^2 \\
&= \frac{mc^2}{\sqrt{1 - 1 + \frac{l_p^2}{\bar{\lambda}^2}}} - mc^2 \\
&= m_p c^2 - mc^2 \\
&= \frac{\hbar}{l_p}\frac{1}{c}c^2 - \frac{\hbar}{\bar{\lambda}}\frac{1}{c}c^2 \\
&= \hbar c\left(\frac{1}{l_p} - \frac{1}{\bar{\lambda}}\right)
\end{aligned} \tag{23}$$

This means the wave function at the suggested maximum velocity for the anything with rest- mass is given by

$$\begin{aligned}
\Psi &= e^{i\left(\frac{p_{max}}{\hbar}x - 2\pi\frac{E_{max}}{\hbar}\right)} \\
&= e^{i\left(\frac{m_p c\sqrt{1 - \frac{l_p^2}{\bar{\lambda}^2}}}{\hbar}x - \frac{\hbar c\left(\frac{1}{l_p} - \frac{1}{\bar{\lambda}}\right)}{\hbar}t\right)}
\end{aligned} \tag{24}$$

from this we have

$$\begin{aligned}
\frac{\partial \Psi}{\partial x} &= \frac{i m_p c\sqrt{1 - \frac{l_p^2}{\bar{\lambda}^2}}}{\hbar}\Psi \\
\frac{\hbar}{i}\frac{\partial \Psi}{\partial x} &= m_p c\sqrt{1 - \frac{l_p^2}{\bar{\lambda}^2}}\Psi \\
-i\hbar\frac{\partial \Psi}{\partial x} &= p\Psi
\end{aligned} \tag{25}$$

so the momentum operator is

$$\hat{p} = -i\hbar \frac{\partial}{\partial x} \tag{26}$$

and the energy operator must be

$$\hat{E} = -i\hbar \frac{\partial}{\partial t} \tag{27}$$

That is, the same momentum and energy operator are just as before, so this will not change Heisenberg's uncertainty principle. However, there is one exception to the rule, namely for a Planck mass particle where the reduced Compton wavelength is $\bar{\lambda} = l_p$. Inserted into the wave equation, we get

$$\begin{aligned} \Psi &= e^{i\left(\frac{m_p c\sqrt{1-\frac{l_p^2}{l_p^2}}}{\hbar}x - \frac{\hbar c\left(\frac{1}{l_p}-\frac{1}{l_p}\right)}{\hbar}t\right)} \\ &= e^{i\left(\frac{m_p c\sqrt{1-1}}{\hbar}x - \frac{\hbar c(1-1)}{\hbar}t\right)} \\ &= e^{i\left(\frac{m_p c\times 0}{\hbar}x - \frac{\hbar c\times 0}{\hbar}t\right)} = 0 \end{aligned} \tag{28}$$

This means we have

$$\frac{\partial \Psi}{\partial x} = 0 \tag{29}$$

and

$$\frac{\partial \Psi}{\partial t} = 0 \tag{30}$$

That is the momentum operator and the energy operator must be zero for the Planck mass particle. This means we must have

$$\begin{aligned} [\hat{p}, \hat{x}]\Psi &= [\hat{p}\hat{x} - \hat{x}\hat{p}]\Psi \\ &= \left(-0 \times \frac{\partial}{\partial x}\right)(x)\Psi - (x)\left(-0 \times \frac{\partial}{\partial x}\right)\Psi \\ &= 0 \end{aligned} \tag{31}$$

That is $\hat{p}$ and $\hat{x}$ commute for the Planck particle, but do not commute for any other particle. For formality's sake, the uncertainty in the special case of the Planck particle must be

$$\begin{aligned} \sigma_p\sigma_x &\geq \frac{1}{2}\left|\int \Psi^*[\hat{p}, \hat{x}]\Psi dt\right| \\ &\geq \frac{1}{2}\left|\int \Psi^*(0)\Psi dt\right| \\ &\geq \frac{1}{2}\left|-0 \times \int \Psi^*\Psi dt\right| = 0 \end{aligned} \tag{32}$$

and also

$$\begin{aligned} \sigma_E\sigma_t &\geq \frac{1}{2}\left|\int \Psi^*[\hat{E}, \hat{t}]\Psi dt\right| \\ &\geq \frac{1}{2}\left|\int \Psi^*(0)\Psi dt\right| \\ &\geq \frac{1}{2}\left|-0 \times \int \Psi^*\Psi dt\right| = 0 \end{aligned} \tag{33}$$

In the special case of the Planck mass particle, the uncertainty principle collapses to zero. In more technical terms this implies that the quantum state of a Planck mass particle can simultaneously be a position and a momentum eigenstate. That is, for the special case of the Planck mass particle we have certainty.

4 Maximum Uncertainty in Addition to Minimum Uncertainty

Next let us look at the maximum kinetic energy multiplied by the relativistic reduced Compton time of the particle in question

$$
\begin{aligned}
E_k t &= \left(\frac{mc^2}{\sqrt{1-\frac{v_{max}^2}{c^2}}} - mc^2\right)\frac{\bar{\lambda}}{c}\sqrt{1-\frac{v^2}{c^2}} \\
&= \left(\frac{mc^2}{\sqrt{1-\frac{\left(c\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}\right)^2}{c^2}}} - mc^2\right)\frac{\bar{\lambda}}{c}\sqrt{1-\frac{\left(c\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}\right)^2}{c^2}} \\
&= \left(\frac{mc^2}{\sqrt{1-1+\frac{l_p^2}{\bar{\lambda}^2}}} - mc^2\right)\frac{\bar{\lambda}}{c}\sqrt{1-1+\frac{l_p^2}{\bar{\lambda}^2}} \\
&= (m_p c^2 - mc^2)\frac{l_p}{c} \\
&= \left(\frac{\hbar}{l_p}\frac{1}{c}c^2 - \frac{\hbar}{\bar{\lambda}}\frac{1}{c}c^2\right)\frac{l_p}{c} \\
&= \hbar - \hbar\frac{l_p}{\bar{\lambda}} \\
&= \hbar\left(1-\frac{l_p}{\bar{\lambda}}\right)
\end{aligned} \tag{34}
$$

we will suggest this is the maximum uncertainty for an elementary particle, so that we must have

$$
\frac{\hbar}{2} \le \sigma_E \sigma_t \le \hbar\left(1-\frac{l_p}{\bar{\lambda}}\right) \tag{35}
$$

This means we have an extended uncertainty principle with lower boundary, similar to that of Heisenberg, and an upper boundary. However, in the special case of a Planck mass particle the lower and upper boundaries on uncertainty are zero. The correct interpretation here is that for the Planck mass particle we have a certainty principle. The energy times time for a Planck mass particle is always

$$
E_p t_p = m_p c^2 \frac{l_p}{c} = \hbar \tag{36}
$$

Basically, this means if we detect a Planck mass particle we know it is at rest and it has a reduced Compton wavelength of l_p that cannot undergo any length contraction, which is why it is at rest. This is also why its reduced Compton wavelength is certain. Other particles have a velocity that can vary from zero to almost c; this means great uncertainty in their position, their relativistic reduced Compton wavelength, and their relativistic mass. This interpretation is not the standard one, but we find it to be more logical.

The Planck mass particle, in our view, is also linked to photon-photon collisions. The velocity of a light particle at the precise moment when it collides with another light particle is the mmeting point of light and matter; see also [19].

Our analysis is fully consistent with our maximum velocity and the relativistic energy momentum relation

$$
\begin{aligned}
E &= \sqrt{p^2c^2 + (mc^2)^2} \\
E &= \sqrt{\left(\frac{mv_{max}}{\sqrt{1-\frac{v_{max}^2}{c^2}}}\right)^2 c^2 + (mc^2)^2} \\
E &= \sqrt{\frac{m^2 v_{max}^2 c^2}{1-\frac{v_{max}^2}{c^2}} + m^2c^4} \\
E &= \sqrt{\frac{m^2 \frac{v_{max}^2}{c^2} c^4}{1-\frac{v_{max}^2}{c^2}} + m^2c^4} \\
E &= \sqrt{\frac{m^2c^4\left(\frac{v_{max}^2}{c^2}-1\right)}{1-\frac{v_{max}^2}{c^2}} + \frac{m^2c^4}{1-\frac{v_{max}^2}{c^2}} + m^2c^4} \\
E &= \sqrt{-m^2c^4 + \frac{m^2c^4}{1-\frac{v_{max}^2}{c^2}} + m^2c^4} \\
E &= \sqrt{\frac{m^2c^4}{1-\frac{v_{max}^2}{c^2}}} \\
E &= \frac{mc^2}{\sqrt{1-\frac{v_{max}^2}{c^2}}}
\end{aligned}
\tag{37}
$$

But for a Planck mass particle it is

$$
\begin{aligned}
E &= \sqrt{p^2c^2 + (m_pc^2)^2} \\
E &= \sqrt{\left(\frac{m_p \times 0}{\sqrt{1-\frac{0^2}{c^2}}}\right)^2 c^2 + (m_pc^2)^2} \\
E &= m_pc^2
\end{aligned}
\tag{38}
$$

The uncertainty principle is, in this new perspective, actually an uncertainty about the velocity of the particle in question, that again is linked to the uncertainty in the relativistic reduced Compton wavelength of the particle. The uncertainty in the reduced Compton wavelength of a particle with momentum or kinetic energy different from zero must be

$$
\begin{aligned}
l_p \geq & \quad \bar{\lambda}\sqrt{1-\frac{(\Delta v)^2}{c^2}} & \leq \bar{\lambda} \\
l_p \geq & \quad \Delta x & \leq \bar{\lambda}
\end{aligned}
\tag{39}
$$

while for the Planck mass particle we have $\Delta\lambda = 0$ because it is for the Planck mass particle always $\bar{\lambda} = l_p$, this again must mean the Planck mass particle not can move, it is at absolute rest for one Planck second.

5 Implications

Our maximum velocity of matter, which is directly linked to the Planck scale, has a series of important implications for quantum mechanics.

Renormalization

Renormalization should no longer be needed. Even if renormalization has become an accepted method over time, this was not the case originally. One prominent critic of renormalization was Richard Feynman [20]. Clearly, he had a central role in the development of quantum electrodynamics, and yet he claimed

> *The shell game that we play ... is technically called 'renormalization'. But no matter how clever the word, it is still what I would call a dippy process! Having to resort to such hocus-pocus has prevented us from proving that the theory of quantum electrodynamics is mathematically self-consistent. It's surprising that the theory still hasn't been proved self-consistent one way or the other by now; I suspect that renormalization is not mathematically legitimate.* – Richard Feynman, 1985

In 1987, Feynman [21] again commented on renormalization

> *Some twenty years ago one problem we theoretical physicists had was that if we combined the principles of quantum mechanics and those of relativity plus certain tacit assumptions, we seemed only able to produce theories (the quantum field theories), which gave infinity for the answer to certain questions. These infinities are kept in abeyance (and now possibly eliminated altogether) by the awkward process of renormalization.* – Richard Feynman, 1987

Again, our maximum velocity limit provides a clear cut-off point on energy limits in elementary particles and renormalization should no longer be needed.

Bell's Theorem

Several researchers have pointed out that by implicitly assuming all possible Bell measurements occur simultaneously, then all proofs of Bell's Theorem [22] violate Heisenberg's uncertainty principle [23]. We wonder what it could mean for the interpretation of Bell's Theorem if Heisenberg's uncertainty principle breaks down at the Planck scale and we then go from uncertainty to certainty (determinism). Interestingly, Clover states [24]

> *By implicitly assuming that all measurements occur simultaneously, Bell's Theorem only applied to local theories that violated Heisenberg's uncertainty principle.*

If Heisenberg's uncertainty principle breaks down at the Planck scale, this should open up the possibility of hidden variables, as suggested by Einstein, Podolsky, and Rosen in 1935; see [25]. We have shown that, under our theory, Planck mass particles can commute. Further, we claim that the Planck mass particle may be the building block of all other particles. Our theory again opens up the way for hidden variable theories and in this framework, Bell's theorem likely is invalid.

Negative Probabilities and Negative Energy: A New Logical Interpretation

In addition to a minimum uncertainty of $\sigma_p\sigma_x \geq \frac{\hbar}{2}$, there is a maximum uncertainty of

$$\sigma_E\sigma_t \leq \hbar c\left(1-\frac{l_p}{\bar{\lambda}}\right) \tag{40}$$

Assume that we now multiply both sides with minus one and we get

$$-\sigma_E\sigma_t \geq -\hbar c\left(1-\frac{l_p}{\bar{\lambda}}\right) \tag{41}$$

In other words, we are basically flipping the sign of the energy operator (and the momentum operator). We speculate that the theoretical negative energy one can mathematically get from the relativistic energy momentum relationship when used in connection to for example the Klein–Gordon equation should be interpreted to mean that there is an upper limit on the relativistic energy level of elementary particles. Negative probabilities could be linked to negative uncertainty, which naturally is impossible, but mathematically it simply means we have flipped the sign of the inequality and that there is a maximum limit on uncertainty, in addition to a lower bound. As we have discussed, in the special case of the Planck mass, the upper and lower bound are zero, and thus there is no uncertainty in that case. There are no negative probabilities per-se, they are just an indication of an also upper boundary condition on the maximum velocity for anything with rest-mass.

6 Conclusion

Based on Haug's recently suggested maximum velocity for matter, we have shown that the momentum and position operators, as well as the energy and time operators, commute at the Planck scale, but not before that. This means that Einstein may have been right, as it opens up the possibility for hidden variable techniques, and also means that Bell's Inequality does not necessarily hold. Further, this means that we get a relativistic quantum mechanics where there should no longer be a need for renormalization, as we get an exact upper limit on energies linked to the Planck scale. Our new theory seems to be consistent in all aspects. It means Lorentz symmetry is broken at the Planck scale, but not before that, something that a series of quantum gravity theories predict could be the case. We think potentially that the so-called negative energies that come out from the relativistic energy momentum relationship and therefore are embedded in the Klein–Gordon equation could be reinterpreted, as there also is an upper energy limit?

Appendix

Here we will derive the Kennard version of Heisenberg's uncertainty principle relation from the energy and time operator (instead of momentum and position operator)

$$[\hat{E},\hat{t}] = \hat{E}\hat{t} - \hat{t}\hat{E} \tag{42}$$

where $\hat{E}$ is the energy operator and $\hat{t}$ is the time operator. The energy operator is given by

$$\hat{E} = -i\hbar\frac{\partial}{\partial t} \tag{43}$$

and the time operator is given by

$$\hat{t} = t \tag{44}$$

From this we have

$$\begin{aligned}
[\hat{E},\hat{t}]\Psi &= [\hat{E}\hat{t} - \hat{t}\hat{E}]\Psi \\
&= \left(-i\hbar\frac{\partial}{\partial t}\right)(t)\Psi - (t)\left(-i\hbar\frac{\partial}{\partial t}\right)\Psi \\
&= -i\hbar\left(\Psi + t\frac{\partial\Psi}{\partial(t)}\right) + i\hbar t\frac{\partial\Psi}{\partial(t)} \\
&= -i\hbar\left(\Psi + t\frac{\partial\Psi}{\partial(t)} - \frac{\partial\Psi}{\partial(t)}\right) \\
&= -i\hbar\Psi
\end{aligned} \tag{45}$$

And we have the following uncertainty

$$\begin{aligned}
\sigma_E\sigma_t &\geq \frac{1}{2}\left|\int \Psi^*[\hat{E},\hat{t}]\Psi dt\right| \\
&\geq \frac{1}{2}\left|\int \Psi^*(-i\hbar)\Psi dt\right| \\
&\geq \frac{1}{2}\left|-i\hbar\int \Psi^*\Psi dt\right|
\end{aligned} \tag{46}$$

and since $\int \Psi^*\Psi dt$ must sum to 1 (there must be 100% probability for the particle to be somewhere), we are left with

$$\begin{aligned}
\sigma_E\sigma_t &\geq \frac{1}{2}|-i\hbar| \\
\sigma_E\sigma_t &\geq \frac{\hbar}{2}
\end{aligned} \tag{47}$$

that is, we get the same uncertainty relation as derived from the momentum and position operators.

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