Refutation of the Hilbert Grand Hotel paradox

We assume the method and apparatus of Meth8/VŁ4 with \( \top \)autology as the designated proof value, \( \bot \) as contradiction, \( \mathbb{N} \) as truthity (non-contingency), and \( \mathbb{C} \) as falsity (contingency). The 16-valued truth table fragment is row-major and horizontal.

From: en.wikipedia.org/wiki/Hilbert's_paradox_of_the_Grand_Hotel

LET \( p, q \): rooms, guests;
\( \sim \) Not; \( \& \) And; \( + \) Or; \( - \) Not Or; \( > \) Imply, greater than; \( < \) Not Imply, less than;
\( @ \) Not Equivalent; \# necessity, for all; \% possibility, for one or some;
\( (@p>@p) \) 1, one.

"It is demonstrated that a fully occupied hotel with infinitely many rooms may still accommodate additional guests, even infinitely many of them, and this process may be repeated infinitely often."

(1.1)

We take the expression "a fully occupied hotel with infinitely many rooms may still accommodate additional guests" as rooms are greater than guests.

We also take the expression "and this process may be repeated infinitely often" to mean the possibility that both the rooms outnumber the guests and the guests outnumber the rooms.

\[
((@p>@q)\&-(p-q)<(@p>@p)))>
(((p-(@p>@p))&(q-(@p>@p)))>((p+(@p>@p))&(q-(@p>@p))))>
(((p-(@p>@p))&(q+(@p>@p)))>((p+(@p>@p))&(q+(@p>@p)))))) > \%((p>q)&~(p>q)) ;
\]

Eq. 1.2 as rendered is not contradictory but rather falsity. Hence this refutes the Hilbert Grand Hotel paradox.

**Remark:** We could not reduce this paradox to one variable because rooms and guests are distinctly counted.