Introduction and Some Results on the Graph Theory for Interval Valued Neutrosophic Sets

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Abstract—In this paper, motivated by the notion of generalized single-valued neutrosophicgraphs of the first type, we define a new type of neutrosophic graph called the generalized interval- valued neutrosophic graph of first type (GIVNG1) and presented a matrix representation for this graph. Some of the fundamental properties and characteristics of this new concept is also studied. The concept of GIVNG1 is an extension of generalized fuzzy graphs (GFG1) and generalized single-valued neutrosophic graphs of the first type (GSVNG1).

Keywords: interval-valued neutrosophic graph; generalized interval valued neutrosophic graph of first type; matrix representation; neutrosophic graph

1.Introduction

(Smarandache, 1998) grounded the concept of neutrosophic set theory (NS) from a philosophical point of view by incorporating the degree of indeterminacy or neutrality as an independent component to deal with problems involving imprecise, indeterminate and inconsistent information. The concept of neutrosophic set theory is a generalization of the theory of fuzzy sets (Zadeh, 1965), intuitionistic fuzzy sets (Atanassov, 1986;1989) interval-valued fuzzy sets (Turksen, 1986) and interval-valued intuitionistic fuzzy sets (Atanassov, and Gargov, 1989). In neutrosophic set, every element has three membership degrees including a true membership degree $T,$ an indeterminacy membership degree I and a falsity membership degree$F,$ all of which are considered independently. These membership functions assume values within the real standard or nonstandard unit interval $]{-0}, 1{+}[$. Therefore, if their range is restrained within the real standard unit interval [0, 1], Nevertheless, NSs are hard to be apply in practical problems since the values of the functions of truth, indeterminacy and falsity lie in$]{-0}, 1{+}[$.The single valued neutrosophic set was introduced for the first time by Smarandache in his book (Smarandache, 1998). Later on, (Wang, Smarandache, Zhang and Sunderraman, 2010) studied some properties related to single-valued neutrosophic sets (SVNSs). In fact, sometimes the degree of truth-membership, indeterminacy-membership and falsity-membership for a certain statement cannot be defined exactly in the real situations, but may be expressed by several possible interval values. So the interval neutrosophic set (INS) was required. For this purpose, (Wang, Smarandache, Zhang and Sunderraman, 2005)introduced the concept of interval neutrosophic set (abbr. INS), which is more precise and more flexible than SVNSs. The INS model is a generalization of theSVNS, in which the three membership functions $(T, I, F)$are independent, and they assume values in the standard unit interval of [0, 1]. We refer the readers to (http://fs.gallup.unm.edu/NSS/; Broumi et al. 2016e; Broumi et al. 2016g) for further information about neutrosophic sets, INSs, SVNSs, and their applications.

Graphs are the most powerful and handful tool used in representing information involving relationship between objects and concepts. In a crisp graphs two vertices are either related or not related to each other, mathematically, the degree of relationship is either 0 or 1.While in fuzzy graphs, the degree of relationship takes values from [0, 1].The concept fuzzy graphs,intuitionistic fuzzy graphs and their extensions such as interval valued fuzzy graphs (Shannon and Atanassov, 1994 ; Mohideen, 2015; Akram and Dudek, 2011; Rashmanlou and Jun, 2013) andinterval-valued intuitionistic fuzzy graphs (Mishra and Pal, 2013). have been studied extensively in over one hundred research papers. All of these types ofgraphs have a common property that each edge must have a membership value of less than or equal to the minimum membership of the nodes it connects.

(Samanta, Sarkar, Shin and Pal, 2016) proposed a new concept called the generalized fuzzy graphs (GFG) and studied some major properties such as completeness and regularity with proved results. The authors classified the GFG into two type. The first type is called generalized fuzzy graphs of first type (GFG1). The second is called generalized fuzzy graphs of second type (GFG2). Each type of GFG is represented by matrices similar to fuzzy graphs. The authors have claimed that fuzzy graphs defined by several researches are limited to represent for some systems such as social network.

In the case where the description of the object or their relations or both is indeterminate and inconsistent, it cannot be handled by fuzzy, intuitionistic fuzzy, interval-valued fuzzy and interval-valued intuitionistic fuzzy graphs. So, for this purpose, (Smarandache, 2015 b) proposed the concept of neutrosophic graphs based on literal indeterminacy (I) to deal with such situations. Many book on neutrosophic graphs based on literal indeterminacy (I) was completed by (Vasantha Kandasamy, Ilanthenral, and Smarandache, 2015). Later on, (Smarandache, 2015; 2015a). gave another definition for neutrosophic graph theory using the neutrosophic truth-values (T, I, F) and constructed three structures of neutrosophic graphs, namely theneutrosophic edge graphs, neutrosophic vertex graphs and neutrosophic vertex-edge graphs. Later on (Smarandache, 2016)proposed several version of neutrosophic graphs such as neutrosophic off graph, neutrosophic bipolar graphs, neutrosophic tripolar graphs and neutrosophic multipolar graphs. Shortly after it was introduced, several authors have focused deeply on the study of neutrosophic vertex-edge graphs and have explored diverse types of neutrosophic graphs.

In 2016, using the concepts of SVNSs, (Broumi, Talea, Bakali and Smarandache, 2016) introduced the concept of single-valued neutrosophic graphs, and introduced certain types of single-valued neutrosophic graphs (SVNG) such as strong single-valued neutrosophic graph, constant single-valued neutrosophic graph, complete single-valued neutrosophic graph and subsequently investigated some of their properties with proofs. Later on, (Broumi, Talea, M., Smarandache and Bakali, 2016a) also introduced neighbourhood degree of a vertex and closed neighborhood degree of vertex in single-valued neutrosophic graph as a generalization of neighborhood degree of a vertex and closed neighborhood degree of vertex in fuzzy graph and intuitionistic fuzzy graph. In addition, (Broumi, Bakali, Talea and Smarandache, 2016b) proved a necessary and sufficient condition for a single-valued neutrosophic graph to be an isolated single-valued neutrosophic graph.The same authors (Samanta, Sarkar, Shin and Pal, 2016) defined the concept of bipolar single neutrosophic graphs as the generalization of bipolar fuzzy graphs, N-graphs, intuitionistic fuzzy graphs, single-valuedneutrosophic graphs and bipolar intuitionistic fuzzy graphs. In addition, the same authors (Broumi, Smarandache, Talea and Bakali, 2016 i) proposed different types of bipolar single-valued neutrosophic graphs such as bipolar single -valued neutrosophic graphs, complete bipolar single-valued neutrosophic graphs, regular bipolar single-valued neutrosophic graphs, studied some of their related properties. In (Broumi, Talea, Bakali, Smarandache, 2016c) ;Broumi, Smarandache, Talea, and Bakali, 2016d; Broumi, Smarandache, Talea and Bakali, 2016 k) , the authors initiated the idea of interval-valued neutrosophic graphs andthe concept of strong interval-valued neutrosophic graph, where different operations such as union, join, intersection and complement have been investigated.(Shah, 2016 ; 2016 a)

proposed a new type of graph called neutrosophic soft graphs and have established a link between graphs and neutrosophic soft sets. The authors also, defined some basic operations of neutrosophic soft graphs such as union, intersection and complement. (Akram and Shahzadi, 2017)proposed a new typeof single-valued neutrosophic graphs that are different from those proposed in (Shah, 2016; Broumi, Talea, Bakali and Smarandache, 2016), and presented some fundamental operations on this single-valued neutrosophic graphs. Also, theauthors presented some interesting properties of single-valued neutrosophic graphs by level graphs. Also, in (Dhavaseelan, Vikramaprasad, Krishnaraj, 2015)introduced the concept of strong neutrosophic graphs and studied some interesting properties of strong neutrosophic graphs. In (Singh, 2016) has discussed adequate analysis of uncertainty and vagueness in medical data set using the properties of three-way fuzzy concept lattice and neutrosophic graph introduced by (Broumi, Talea, Bakali and Smarandache, 2016). In (Fathi, Elchawal by and Salama, 2016) computed the dissimilarity between two neutrosophic graphs based on the concept of Haussdorff distance. (Ashraf, Naz, Rashmanlou and Malik, 2017) proposed some novels concepts of edge regular, partially edge regular and full edge regular single-valued neutrosophic graphs and investigated some of their properties. Also the authors introduced the notion of single-valued neutrosophic digraphs (SVNDGs) and presented an application of SVNDG in multi-attribute decision making.

(Mehra and Singh, 2017) introduced the concept of single valued neutrosophic signed graphs and examined the properties of this concept with examples. (Ulucay et al, 2016) introduced the concept of neutrosophic soft expert graphs and have established a link between graphs and neutrosophic soft expert sets (Sahin, Alkhazaleh and Ulucay, 2015) . They also studied some basic operations of neutrosophic soft experts graphs such as union, intersection and complement. Recently, (Naz, Rashmanlou and Malik, 2017) defined basic operations on SVNGs such as direct product, Cartesian product, semi-strong product, strong product, lexicographic product, union, ring sum and join and provided an application of single-valued neutrosophic digraph (SVNDG) in travel time.

Similar to the interval valued fuzzy graphs and interval valued intuitionistic fuzzy graphs, which have a common property that each edge must have a membership value that is less than or equal to the minimum membership of the nodes it connects. Also, the interval-valued neutrosophic graphs presented in the literature (Broumi et al., 2016c ;2016d) have a common property: that edge membership value is less than the minimum of it's end vertex values, whereas the edge indeterminacy-membership value is less than the maximum of it's end vertex values or is greater than the maximum of its's end vertex values. And the edge non-membership value is less than the minimum of it's end vertex values or is greater than the maximum of it's end vertex values.

(Broumi, Bakali, Talea, Hassan and Smarandache;2017) have discussed the removal of the edge degree restriction of single-valued neutrosophic graphs and presented a new class of single-valued neutrosophic graph called generalized single-valued neutrosophic graph of type1, which is an extension of generalized fuzzy graph of type1 (Samanta, Sarkar, Shin and Pal, 2016), that is based on generalized single-valued neutrosophic graphof type1(abbr. GSVNG1) introduced in (Broumi, Bakali, Talea, Hassan and Smarandache;2017). The main objective of this paper is to extend the concept of generalized single-valued neutrosophic graph of first type to interval-valued neutrosophic graphs of first type (GIVNG1) to model systems having indeterminate information and introduced a matrix representation of GIVNG1.

This paper has been arranged as the following: In Section 2, some fundamental concepts about neutrosophic sets, single- valued neutrosophic sets, interval valued neutrosophic graph and generalized single-valued neutrosophic graphs of type 1are presented, all of which will employed in the later sections. In Section 3, the concept of generalized interval-valued neutrosophic graphs of type 1 is presented with an illustrative example. In Section 4, the matrix representation of generalized interval-valued neutrosophic graphs of type 1is introduced. Concluding remarks and the list of references are given at the end of Section 5.

2. Preliminaries

This section contains some basic definitions from references(Smarandache, 1998; Wang, Smarandache, Zhang and Sunderraman, 2010; Broumi, Bakali, Talea, Hassan and Smarandache;2017;Wang, Smarandache, Zhang and Sunderraman, 2010) pertaining to neutrosophic sets, SVNSs, interval- valued neutrosophic graphs and generalized single-valued neutrosophic graphs of type 1, all of which will form the background of this study.

Definition 2.1 (Smarandache, 1998). Let X be a space of points (objects) with generic elements in X denoted by x; then the neutrosophic set A (NS A) is an object having the form A = {< x: $T_A(x)$, $I_A(x)$, $F_A(x)$>, x ∈ X}, where the functions T, I, F: X→]$^-$0,1$^+$[define respectively the truth-membership function, indeterminacy-membership function, and falsity-membership function of the element x ∈ X to the set A with the condition:

$$^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+. \quad (1)$$

The functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of]$^-$0,1$^+$[.

Since it is difficult to apply NSs to practical problems, Smarandache [7] introduced the concept of a SVNS, which is an instance of a NS and can be used in real scientific and engineering applications.

Definition 2.2 (Wang, Smarandache, Zhang and Sunderraman, 2010)**.** Let X be a space of points (objects) with generic elements in X denoted by x. A single valued neutrosophic set A (SVNS A) is characterized by truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. For each point x in X, $T_A(x)$, $I_A(x)$, $F_A(x)$ ∈ [0, 1]. A SVNS A can be written as

A={< x: $T_A(x)$, $I_A(x)$, $F_A(x)$>, x ∈ X} (2)

Definition 2.3(adopted from(Broumi, Bakali, Talea, Hassan and Smarandache;2017)). Let the following statements hold:

a) V is a non-void set.

b)ρ_T, ρ_I, ρ_F are three functions, eachfrom V to [0,1].

c) A= $\{(\rho_T(u),\rho_T(v)) \mid u, v \in$ V $\}$,

B = $\{(\rho_I(u),\rho_I(v)) \mid u, v \in$ V $\}$,

C= $\{(\rho_F(u),\rho_F(v)) \mid u, v \in$ V $\}$.

d)α:A→ [0,1] , β:B→ [0,1],δ:C→ [0,1] are three

functions.

e) ρ=(ρ_T, ρ_I, ρ_F) ; and

ω=(ω_T, ω_I, ω_F) with

$\omega_T(u, v) = \alpha((\rho_T(x),\rho_T(v)))$,

$\omega_I(u, v)= \beta((\rho_I(x),\rho_I(v)))$,

$\omega_F(u, v) = \delta((\rho_F(x),\rho_F(v)))$,

for all $u, v \in$ V .

Then:

i) the structure$\xi = <V, \rho, \omega>$is said to be a *generalized single valued neutrosophic graph of type* 1 (GSVNG1) .

Remark: ρ depends on ρ_T, ρ_I, ρ_F. And ω depends onα, β, δ . Hence there are 7 mutually independent parameters in total that make up a CNG1: V, ρ_T, ρ_I, ρ_F, α, β, δ.

ii) For each $\in$ V , x is said to be a *vertex* of ξ . The entire set Vis thus called the *vertex set* of ξ .

iii) For each $u, v \in$ V, (u, v) is said to be a *directed edge* of ξ .

In particular, (v, v) is said to be a *loop* of ξ .

iv) For each vertex $:\rho_T(v)$, $\rho_I(v)$, $\rho_F(v)$ are said to be the *truth-membership value*, *indeterminate-membership value*, and *false-membership value*, respectively of that vertex$v$. Moreover, if$\rho_T(v)$ =$\rho_I(v)$ =$\rho_F(v)$ = 0, then $v$is said to be a *void* vertex.

v) Likewise, for each edge (u,v) : $\omega_T(u,v)$, $\omega_I(u,v)$,$\omega_F(u,v)$ are respectively said to be the *truth-membership value*, *indeterminate-membership value*, and *false-membership value*, of that directed edge(u,v). Moreover, if $\omega_T(u,v)$ = $\omega_I(u,v)$ = $\omega_F(u,v)$ = 0,then (u,v) is said to be a *void* directed edge.

Remark: It follows that:$\mathrm{V} \times \mathrm{V} \rightarrow [0,1]$.

3.Generalized Interval Valued Neutrosophic Graph of First Type

In the modelling of real life scenarios with neutrosophic system (i.e. neutrosophic sets, neutrosophic graphs, etc), the truth-membership value, indeterminate-membership value, and false-membership value are often taken to mean the *ratio out of a population* who find reasons to "agree", "be neutral" and "disagree". It can also by any 3 analogous descriptions, such as "seek excitement" "loft around" and "relax". However, there are real life situations where even such ratio out of the population are subject to conditions. One typical example will be having the highest and the lowest value. For example "It is expected that 20% to 30% of the population of country X will disagree with the Prime Minister's decision".

To model such an event, therefore, we generalize Definition 2.3 so that the truth-membership value, indeterminate-membership value, and false-membership value can be any closed subinterval of [0,1], instead of a single number from [0,1].

Remark For all the remaining parts of this paper, we shall denote:

$$\Delta_1 = \{[x,y]: 0 \leq x \leq y \leq 1\}$$

Definition 3.1(adopted from(Broumi, Bakali, Talea, Hassan and Smarandache;2017)). Let the following statements hold:

a) V is a non-void set.

b)ρ_T, ρ_I, ρ_F are three functions, eachfrom V to Δ_1.

c) A= $\{(\rho_T(u),\rho_T(v)) \mid u,v \in$ V $\}$,

B = $\{(\rho_I(u),\rho_I(v)) \mid u,v \in$ V $\}$,

C= $\{(\rho_F(u),\rho_F(v)) \mid u,v \in$ V $\}$.

d)α:A→ Δ_1 , β:B→ Δ_1,δ:C→ $\Delta_1$are three functions.

e) ρ=(ρ_T, ρ_I, ρ_F) ; and

ω=(ω_T, ω_I, ω_F) with

$\omega_T(u,v) = \alpha((\rho_T(x),\rho_T(v)))$,

$\omega_I(u,v)= \beta((\rho_I(x),\rho_I(v)))$,

$\omega_F(u,v) = \delta((\rho_F(x),\rho_F(v)))$,

for all $u,v \in V$.

Then:

i) The structureξ = <V, ρ,ω>is said to be a *generalized interval-valued neutrosophic graph of type* 1 (GIVNG1) .

ii) For each ∈ V , x is said to be a *vertex* of ξ . The entire set

Vis thus called the *vertex set* of ξ .

iii) For each $u,v \in$ V, (u,v) is said to be a *directed edge* of ξ. In particular, (v,v) is said to be a *loop* of ξ .

iv) For each vertex : $\rho_T(v)$, $\rho_I(v)$, $\rho_F(v)$ are said to be the *truth-membership value*, *indeterminate-membership value*, and *false-membership value*, respectively, of that vertex$v$.Moreover, if$\rho_T(v)$ =$\rho_I(v)$ =$\rho_F(v)$ = [0,0], then v is said to be a *void* vertex.

v) Likewise, for each edge (u,v) : $\omega_T(u,v)$, $\omega_I(u,v)$,$\omega_F(u,v)$ are said to be the *truth-membership value*, *indeterminate-membership value*, and *false-membership value*, respectively of that directed edge(u,v). Moreover, if$\omega_T(u,v)$ = $\omega_I(u,v)$ = $\omega_F(u,v)$= [0,0],then (u,v) issaid to be a *void* directed edge.

Remark: It follows that :V × V → Δ_1.

Note that each vertex v in a GIVNG1 possess a single, undirected loop, whether void or not. And each two distinct vertices u, v in a GIVNG1posssses*two* directed edges, resulting from $(u, v)$and (v, u) , whether void or not.

Recall that in classical graph theory, we often deal with ordinary (or undirected) graphs, and also simple graphs. To further relate our GIVNG1 with it, we now proceed with the following definition.

Definition 3.2. Let ξ = <V, ρ,ω> be a GIVNG1.

a) If $\omega_T(a, b)$= $\omega_T(b, a)$,$\omega_I(a, b)$ = $\omega_I(b, a)$ and $\omega_F(a, b) = \omega_F(b, a)$, then

$\{a, b\}$ = $\{(a, b)$, $(b, a)\}$

is said to be an (*ordinary*) *edge* of ξ . Moreover, $\{a, b\}$ is said to be a *void* (ordinary) edge if both (a, b) and $(b, a)$are void.

b) If $\omega_T(u, v)$= $\omega_T(v, u)$,$\omega_I(u, v)$ = $\omega_I(v, u)$ and $\omega_F(u, v) = \omega_F(v, u)$ holds for all $v \in$ V , then ξ is said to be *ordinary* (or *undirected*), otherwise it is said to be *directed.*

c) If all the loops of ξ are void, then ξ is said to be *simple*.

In the following section, we discuss a real life scenario, for which GSVNG1 is insufficient to model it - it can only be done by using GIVNG1.

Example 3.3. Part 3.3.1 The scenario

Country X has 4 cities $\{a, b, c, d\}$. The cities are connected with each other by some roads, there are villages along the four roads (all of them are two way) $\{a, b\}$, $\{c, b\}$, $\{a, c\}$ and $\{d, b\}$. As for the other roads, such as $\{c, d\}$, they are either non-exitstant, or there are no population living along them (e.g. industrial area, national park, or simply forest).

The legal driving age of Country X is 18.The prime minister of Country X would like to suggest anamendment of the legal driving age from 18 to 16. Before conducting a country wide survey involving all the citizens, the prime minister discuss with all members of the parliament about the expected outcomes.

The culture and living standard of all the cities and villages differ from one another. In particular:

The public transport in c is so developed that few will have to drive. The people are rich enough to buy even air tickets.

People in d tend to be more open minded in culture.Moreover, sports car exhibitions and shows are commonly held there.

A fatal road accident just happened along {c,b}, claiming the lives of five unlicensed teenagers racing at 200km/h.

{a,c} is governed by an opposition leader who is notoriousfor being very uncooperative in all parliament affairs.

Eventually the parliament meeting was concluded with the following predictions:

		Expected percentage of citizens that will -					
		support		be neutral		Against	
		at least	at most	at least	at most	at least	at most
cities	*a*	0.1	0.4	0.2	0.6	0.3	0.7
	b	0.3	0.5	0.2	0.5	0.2	0.5
	c	0.1	0.2	0.0	0.3	0.1	0.2
	d	0.5	0.7	0.2	0.4	0.1	0.2
Villages along the roads	{*a*,*b*}	0.2	0.3	0.1	0.4	0.4	0.7
	{*c*,*b*}	0.1	0.2	0.1	0.2	0.5	0.8
	{*a*,*c*}	0.1	0.7	0.1	0.8	0.1	0.7
	{*d*,*b*}	0.2	0.3	0.3	0.6	0.2	0.5

Without loss of generality: It is either {*c*,*d*} does not exist, or there are no people living there, so all the six values – support(least, most), neutral(least, most), against(least, most), are all zero.

Part 3.3.2Representing with GIVNG1

We now follow all the steps from a) to e) in Definition 3.1, to represent the scenario with a particular

GIVNG1.

a) TakeV_0 = {*a*, *b*, *c*, *d*}

b) In accordance with the scenario, define the three functions

ρ_T, ρ_I, ρ_F, as illustrated in the following table.

	A	b	c	d
ρ_T	[0.1,0.4]	[0.3,0.5]	[0.1,0.2]	[0.5,0.7]
ρ_I	[0.2,0.6]	[0.2,0.5]	[0.0,0.3]	[0.2,0.4]
ρ_F	[0.3,0.7]	[0.2,0.5]	[0.1,0.2]	[0.1,0.2]

c) By statement c) from Definition 3.1: Let

$A_0=\{(\rho_T(u),\rho_T(v))\ |u,v \in \{a,b,c,d\}\}$

$B_0=\{(\rho_I(u),\rho_I(v))|\ u,v \in \{a,b,c,d\}\}$

$C_0=\{(\rho_F(u),\rho_F(v))|\ u,v \in \{a,b,c,d\}\}$

d) In accordance with the scenario, define

α:$A_0 \rightarrow \Delta_1$, β:$B_0 \rightarrow \Delta_1$,δ:$C_0 \rightarrow \Delta_1$,

as illustrated in the following tables.

$\alpha((\rho_T(u),\rho_T(v)))$:

v u	a	b	c	d
a	0	[0.2,0.3]	[0.1,0.7]	0
b	[0.2,0.3]	0	[0.1,0.2]	[0.2,0.3]
c	[0.1,0.7]	[0.1,0.2]	0	0
d	0	[0.2,0.3]	0	0

$\beta((\rho_I(u),\rho_I(v)))$:

v u	a	b	c	d

a	0	[0.1,0.4]	[0.1,0.8]	0
b	[0.1,0.4]	0	[0.1,0.2]	[0.3,0.6]
c	[0.1,0.8]	[0.1,0.2]	0	0
d	0	[0.3,0.6]	0	0

$\delta((\rho_F(u),\rho_F(v)))$:

v u	a	b	c	d
a	0	[0.4,0.7]	[0.1,0.7]	0
b	[0.4,0.7]	0	[0.5,0.8]	[0.2,0.5]
c	[0.1,0.7]	[0.5,0.8]	0	0
d	0	[0.2,0.5]	0	0

e) By statement e) from Definition 3.1, let

$\rho_0=(\rho_T, \rho_I, \rho_F)$; and

$\omega_0=(\omega_T, \omega_I, \omega_F)$ with

$\omega_T(u,v) = \alpha((\rho_T(u),\rho_T(v)))$,

$\omega_I(u,v)= \beta((\rho_I(u),\rho_I(v)))$,

$\omega_F(u,v) = \delta((\rho_F(u),\rho_F(v)))$,

for all $u, v \in V_0$. We now have formed $<V_0, \rho_0, \omega_0>$, which is a GIVNG1.

One of the way of representing the entire $<V_0, \rho_0, \omega_0>$ is by using a diagram that is analogous with graphs as in classical graph theory, as shown in the figure 1 below

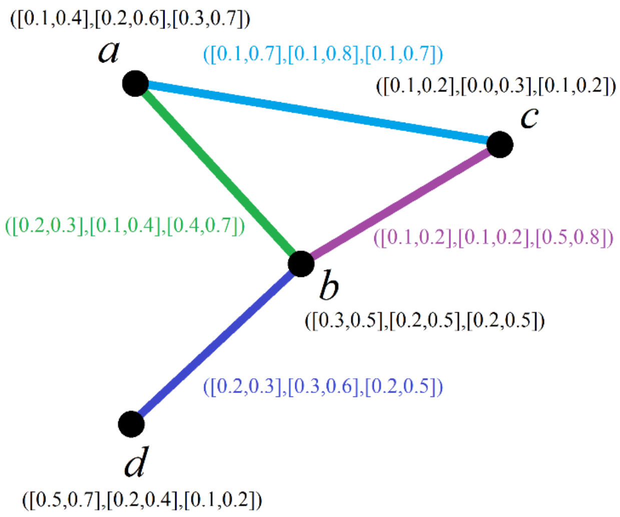

Fig.1.

In other words, only the non-void edges (whether directed or ordinary) and vertices are to be drawn in such a diagram.

Also recall that, in classical graph theory, a graph can be represented by anadjacency matrix, for which the entries are either a positive integer (connected) or 0 (not connected).

This motivates us to represent GIVNG1, by a matrix as well, in such a similar manner. Nonetheless, instead of a single value that is either 0 or 1, we have *three* values to deal with: ω_T, ω_I, ω_F , with each of them being elements of Δ_1. Moreover, each of the vertices themselves also contains ρ_T, ρ_I, ρ_F, which must be taken into account as well.

4. Representation of interval valued Neutrosophic Graph of Type 1 BY adjacency matrix

Section 4.1 The two methods of representation

In this section, we discuss the representation of GIVNG1 by two ways which are both analogous to the one encountered in classical literature.

Letξ = <V, ρ,ω> be a GIVNG1 where vertex set V={v_1,v_2,…,v_n}(i.e. GIVNG1 has finite vertices). Recall that GIVNG1has its edge membership values (T,I,F) depends on the membership values (T,I,F) of adjacent vertices, in accordance with the functions α, β, δ. Furthermore:

$\omega_T(u,v)$= $\alpha((\rho_T(u),\rho_T(v)))$ for allu,v ∈ V, where

α :A→ Δ_1 , and A= {($\rho_T(u)$,$\rho_T(v)$) |u,v ∈ V},

$\omega_I(u,v)$= $\beta((\rho_I(u),\rho_I(v)))$ for allu,v ∈ V, where

β :B→ Δ_1, and B= {($\rho_I(u)$,$\rho_I(v)$) |u,v ∈ V},

$\omega_F(u,v)$= $\delta((\rho_F(u),\rho_F(v)))$ for all$u,v \in$ V , where

δ :C→ Δ_1, and C = {($\rho_F(u)$,$\rho_F(v)$) |$u,v \in$ V}.

We first form an n×n matrix as shown

$$\mathbf{M} = [\mathbf{a}_{i,j}]_n = \begin{pmatrix} \mathbf{a}_{1,1} & \mathbf{a}_{1,2} & \cdots & \mathbf{a}_{1,n} \\ \mathbf{a}_{2,1} & \mathbf{a}_{2,2} & & \mathbf{a}_{2,n} \\ \vdots & & \ddots & \vdots \\ \mathbf{a}_{n,1} & \mathbf{a}_{n,2} & \cdots & \mathbf{a}_{n,n} \end{pmatrix},$$

Where$\mathbf{a}_{i,j} = (\omega_T(v_i,v_j), \omega_I(v_i,v_j), \omega_F(v_i,v_j))$ for all i , j .

In other words, each element of the matrix **M** is itself an ordered set of 3 closed subintervals of [0,1], instead of just a number of either 0 or 1 in classical literature.

Remark:

Since ξ can only possess undirected loops, we decided *not* to multiply the main diagonal elements of **M** by 2, as seen in adjacency matrices for graphs classical literature (2 for undirected, 1 for directed, 0 for void).

Meanwhile, also recall that each of the vertices in ξcontain ρ_T, ρ_I, ρ_F, which must be taken into account as well.

So we form another matrix **K** as shown

$$\mathbf{K} = [\mathbf{k}_i]_{n,1} = \begin{pmatrix} \mathbf{k}_1 \\ \mathbf{k}_2 \\ \vdots \\ \mathbf{k}_n \end{pmatrix},$$

Where$\mathbf{k}_i = (\rho_T(v_i), \rho_I(v_i), \rho_F(v_i))$ for all i .

To accomplish <u>one of our way</u> of representing the entire ξ, we therefore augment the matrix **K** with **M**, forming the *adjacency matrix o f*GIVNG1, [**K|M**], as shown:

$$[\mathbf{K}|\mathbf{M}] = \begin{pmatrix} \mathbf{k}_1 & \mathbf{a}_{1,1} & \mathbf{a}_{1,2} & \cdots & \mathbf{a}_{1,n} \\ \mathbf{k}_2 & \mathbf{a}_{2,1} & \mathbf{a}_{2,2} & & \mathbf{a}_{2,n} \\ & \vdots & & \ddots & \vdots \\ \mathbf{k}_n & \mathbf{a}_{n,1} & \mathbf{a}_{n,2} & \cdots & \mathbf{a}_{n,n} \end{pmatrix},$$

where $\mathbf{a}_{i,j} = (\omega_T(v_i,v_j), \omega_I(v_i,v_j), \omega_F(v_i,v_j))$, and$\mathbf{k}_i = (\rho_T(v_i), \rho_I(v_i), \rho_F(v_i))$, for all i and j.

Although [**K**|**M**] is an$n\times(n+1)$ matrix and therefore not a square, this representation will save us another *separate* ordered set to represent the ρ_T, ρ_I, ρ_F values of the vertices themselves.

Sometimes it is more convenient to separately deal witheach of the three kinds of membership values for both edges and vertices. As a result, here we provide another way of representing the entire ξ: using three $n\times(n+1)$ matrices, $[\mathbf{K}|\mathbf{M}]_T$, $[\mathbf{K}|\mathbf{M}]_I$ and $[\mathbf{K}|\mathbf{M}]_F$,each derived from $[\mathbf{K}|\mathbf{M}]$ by taking only one kind of the membership values from its elements:

$$[\mathbf{K}|\mathbf{M}]_T = [\mathbf{K}_T|\mathbf{M}_T] = \begin{pmatrix} \rho_T(v_1) & \omega_T(v_1,v_1) & \omega_T(v_1,v_2) & \cdots & \omega_T(v_1,v_n) \\ \rho_T(v_2) & \omega_T(v_2,v_1) & \omega_T(v_2,v_2) & & \omega_T(v_2,v_n) \\ & \vdots & & \ddots & \vdots \\ \rho_T(v_n) & \omega_T(v_n,v_1) & \omega_T(v_n,v_2) & \cdots & \omega_T(v_n,v_n) \end{pmatrix},$$

$$[\mathbf{K}|\mathbf{M}]_I = [\mathbf{K}_I|\mathbf{M}_I] = \begin{pmatrix} \rho_I(v_1) & \omega_I(v_1,v_1) & \omega_I(v_1,v_2) & \cdots & \omega_I(v_1,v_n) \\ \rho_I(v_2) & \omega_I(v_2,v_1) & \omega_I(v_2,v_2) & & \omega_I(v_2,v_n) \\ & \vdots & & \ddots & \vdots \\ \rho_I(v_n) & \omega_I(v_n,v_1) & \omega_I(v_n,v_2) & \cdots & \omega_I(v_n,v_n) \end{pmatrix},$$

$$[\mathbf{K}|\mathbf{M}]_F = [\mathbf{K}_F|\mathbf{M}_F] = \begin{pmatrix} \rho_F(v_1) & \omega_F(v_1,v_1) & \omega_F(v_1,v_2) & \cdots & \omega_F(v_1,v_n) \\ \rho_F(v_2) & \omega_F(v_2,v_1) & \omega_F(v_2,v_2) & & \omega_F(v_2,v_n) \\ & \vdots & & \ddots & \vdots \\ \rho_F(v_n) & \omega_F(v_n,v_1) & \omega_F(v_n,v_2) & \cdots & \omega_F(v_n,v_n) \end{pmatrix}.$$

$[\mathbf{K}|\mathbf{M}]_T$, $[\mathbf{K}|\mathbf{M}]_Iand[\mathbf{K}|\mathbf{M}]_F$shall thus be called respectively the *truth-adjacency matrix*, the *indeterminate-adjacency matrix*, and the *false-adjacency matrix*ofξ.

Remark 1: If $[\mathbf{K}|\mathbf{M}]_I$ = $[\mathbf{K}|\mathbf{M}]_F$=$[[0,0]]_{n,n+1}$, $\mathbf{K}_T$ = $[[1,1]]_{n,1}$, all the entries of $\mathbf{M}_T$are either [1,1] or [0,0] , then ξ is reduced to a graph in classical literature. Furthermore, if that $\mathbf{M}_T$is symmetrical and with main diagonal elements being zero, then ξ is further reduced to a simple ordinary graph in classical literature.

Remark2: If $[\mathbf{K}|\mathbf{M}]_I$ = $[\mathbf{K}|\mathbf{M}]_F$=$[[0,0]]_{n,n+1}$, and all the entries of $[\mathbf{K}|\mathbf{M}]_T = [[a_{i,j}, a_{i,j}]]_{n,n+1}$,then ξ is reduced to a generalized fuzzy graph type 1 (GFG1).

Remark 3: If $[\mathbf{K}|\mathbf{M}]_T = [[a_{i,j}, a_{i,j}]]_{n,n+1}$, $[\mathbf{K}|\mathbf{M}]_I = [[b_{i,j}, b_{i,j}]]_{n,n+1}$, $[\mathbf{K}|\mathbf{M}]_F = [[c_{i,j}, c_{i,j}]]_{n,n+1}$, then ξ is reduced to a generalized single valued neutrosophic graphs of type 1 (GSVNG1).

Section 4.2Application on our example.

For the sake of brevity, we now give representation for our example in the scenario by the latter way i.e. by using three matrices: $[\mathbf{K}|\mathbf{M}]_T$, $[\mathbf{K}|\mathbf{M}]_I$and $[\mathbf{K}|\mathbf{M}]_F$:

$$[\mathbf{K}|\mathbf{M}]_T = \begin{pmatrix} [0.1,0.4] & [0,0] & [0.2,0.3] & [0.1,0.7] & [0,0] \\ [0.3,0.5] & [0.2,0.3] & [0,0] & [0.1,0.2] & [0.2,0.3] \\ [0.1,0.2] & [0.1,0.7] & [0.1,0.2] & [0,0] & [0,0] \\ [0.5,0.7] & [0,0] & [0.2,0.3] & [0,0] & [0,0] \end{pmatrix},$$

$$[\mathbf{K}|\mathbf{M}]_I = \begin{pmatrix} [0.2,0.6] & [0,0] & [0.1,0.4] & [0.1,0.8] & [0,0] \\ [0.2,0.5] & [0.1,0.4] & [0,0] & [0.1,0.2] & [0.3,0.6] \\ [0.0,0.3] & [0.1,0.8] & [0.1,0.2] & [0,0] & [0,0] \\ [0.2,0.4] & [0,0] & [0.3,0.6] & [0,0] & [0,0] \end{pmatrix},$$

$$[\mathbf{K}|\mathbf{M}]_F = \begin{pmatrix} [0.3,0.7] & [0,0] & [0.4,0.7] & [0.1,0.7] & [0,0] \\ [0.2,0.5] & [0.4,0.7] & [0,0] & [0.5,0.8] & [0.2,0.5] \\ [0.1,0.2] & [0.1,0.7] & [0.5,0.8] & [0,0] & [0,0] \\ [0.1,0.2] & [0,0] & [0.2,0.5] & [0,0] & [0,0] \end{pmatrix},$$

5. Some theoretical results on ordinary GIVNG1

We now discuss some theoretical results that follows from the definition of ordinary GIVNG1, as well as its representation with adjacency matrix. Since we are concerning about ordinary GIVNG1, all the edges that we will be referring to are ordinary edges.

Definition5.1 The operation + is defined on Δ_1 as follows:

$[p, q]$+$[r, s]$ = $[p + q, r + s]$ for all $p, q, r, s \in [0,1]$.

Definition5.2 Letξ = <V, ρ,ω> bean ordinaryGIVNG1. Let V={v_1,v_2,…,v_n} to be the vertex set of ξ . Then, for each *i*, the *degree*of v_i, denoted as $D(v_i)$, is defined to be the ordered set

$$(D_T(v_i), D_I(v_i), D_F(v_i)),$$

for which

a)$D_T(v_i) = \sum_{r=1}^{n} \omega_T(v_i, v_r)$ $+\omega_T(v_i, v_i)$,

b)$D_I(v_i) = \sum_{r=1}^{n} \omega_I(v_i, v_r)$+ $\omega_I(v_i, v_i)$,

c)$D_F(v_i) = \sum_{r=1}^{n} \omega_F(v_i, v_r)$+ $\omega_F(v_i, v_i)$.

Remark 1: In analogy to classical graph theory, each undirected loop has both its ends connected to the same vertex, so is counted twice.

Remark 2: each values of $D_T(v_i)$, $D_I(v_i)$ and $D_F(v_i)$are elements of Δ_1 instead of a single number.

Definition5.3 Letξ = <V, ρ,ω> bean ordinary GIVNG1. Let V={v_1,v_2,…,v_n} to be the vertex set of ξ. Then, the *amount of edges* of ξ, denoted as E_ξ, is defined to be the ordered set (E_T, E_I, E_F) for which

a)E_T= $\sum_{\{r,s\}\subseteq\{1,2,\ldots,n\}} \omega_T(v_r, v_s)$,

b)E_I= $\sum_{\{r,s\}\subseteq\{1,2,\ldots,n\}} \omega_I(v_r, v_s)$,

c)E_F= $\sum_{\{r,s\}\subseteq\{1,2,\ldots,n\}} \omega_F(v_r, v_s)$.

Remark 1: As in classical graph theory, each edge is counted only once, as shown by $\{r, s\} \subseteq \{1,2, \ldots, n\}$ in the expression.

For example, if $\omega_T(v_a, v_b)$ is added, we will not add $\omega_T(v_b, v_a)$ again since $\{a, b\} = \{b, a\}$.

Remark 2: each values of E_T, E_I and $E_F$are elements of Δ_1 instead of a single number, and need not be 0 or 1 as in classical graph literature. As a result, we call it the "amount" of edges, instead of the "number" of edges as in the classical literature.

E_T, E_I, E_F are closed subintervals of [0,1], and $D_T(v_i)$, $D_I(v_i)$, $D_F(v_i)$ are also closed subintervals of [0,1] for each vertex v_i. These give rise to the following lemmas

Lemma5.4 Letξ = <V, ρ,ω> bean ordinaryGIVNG1. Let V={v_1,v_2,…,v_n} to be the vertex set of ξ. Denote

a)$\omega_T(v_i, v_j) = [\phi_{T,(i,j)}, \psi_{T,(i,j)}]$,

b)$\omega_I(v_i, v_j) = [\phi_{I,(i,j)}, \psi_{I,(i,j)}]$, c)$\omega_F(v_i, v_j) = [\phi_{F,(i,j)}, \psi_{F,(i,j)}]$, for all i, j.

Then,for each i :

i)$D_T(v_i)$ = [$\sum_{r=1}^{n} \phi_{T,(i,r)}$ +$\phi_{T,(i,i)}$,$\sum_{r=1}^{n} \psi_{T,(i,r)}$ +$\psi_{T,(i,i)}$] ,

ii)$D_I(v_i)$ = [$\sum_{r=1}^{n} \phi_{I,(i,r)}$ + $\phi_{I,(i,i)}$,$\sum_{r=1}^{n} \psi_{I,(i,r)}$ + $\psi_{I,(i,i)}$] ,

iii)$D_F(v_i)$ = [$\sum_{r=1}^{n} \phi_{F,(i,r)}$ + $\phi_{F,(i,i)}$, $\sum_{r=1}^{n} \psi_{F,(i,r)}$ + $\psi_{F,(i,i)}$] .

Furthermore:

iv)E_T = [$\sum_{\{r,s\}\subseteq\{1,2,\ldots,n\}} \phi_{T,(r,s)}$,$\sum_{\{r,s\}\subseteq\{1,2,\ldots,n\}} \psi_{T,(r,s)}$] ,

v)E_I = [$\sum_{\{r,s\}\subseteq\{1,2,\ldots,n\}} \phi_{I,(r,s)}$,$\sum_{\{r,s\}\subseteq\{1,2,\ldots,n\}} \psi_{I,(r,s)}$] ,

vi)E_F = [$\sum_{\{r,s\}\subseteq\{1,2,\ldots,n\}} \phi_{F,(r,s)}$, $\sum_{\{r,s\}\subseteq\{1,2,\ldots,n\}} \psi_{F,(r,s)}$] .

Proof

The proof is straightforward by applying Definition 5.1 to both Definition 5.2 and Definition 5.3. ■

We now proceed with two of our theorems which both serves as generalizations of the well-known theorem in classical literature:

"For an ordinary graph, the sum of the degree of all its vertices is always twice the number of its edges."

Theorem5.5 Letξ = <V, ρ,ω> bean ordinaryGIVNG1. Then

$$\sum_{r=1}^{n} D(v_r) = 2E_\xi$$

Proof. As $D(v_i)$ = ($D_T(v_i), D_I(v_i), D_F(v_i)$) for all i , and E_ξ = (E_T, E_I, E_F) . Without loss of generality, it suffices to prove that $2E_T$= $\sum_{r=1}^{n} D_T(v_r)$:

E_T = $\sum_{\{r,s\}\subseteq\{1,2,\ldots,n\}} \omega_T(v_r, v_s)$

= $\sum_{\substack{\{r,s\}\subseteq\{1,2,\ldots,n\} \\ r\neq s}} \omega_T(v_r, v_s)$ + $\sum_{r=1}^{n} \omega_T(v_r, v_r)$.

Since $\{r, s\} = \{s, r\}$ for all s and r ,

$2E_T$ = 2 $\sum_{\substack{\{r,s\}\subseteq\{1,2,\ldots,n\} \\ r\neq s}} \omega_T(v_r, v_s)$ + 2 $\sum_{r=1}^{n} \omega_T(v_r, v_r)$

$$= \sum_{\substack{r\in\{1,2,\dots,n\}\\ s\in\{1,2,\dots,n\}\\ r\neq s}} \omega_T(v_r, v_s) + 2\sum_{r=1}^{n} \omega_T(v_r, v_r)$$

$$= \sum_{\substack{r\in\{1,2,\dots,n\}\\ s\in\{1,2,\dots,n\}}} \omega_T(v_r, v_s) + \sum_{r=1}^{n} \omega_T(v_r, v_r)$$

$$= \sum_{r=1}^{n}\sum_{s=1}^{n} \omega_T(v_r, v_s) + \sum_{r=1}^{n} \omega_T(v_r, v_r)$$

$$= \sum_{r=1}^{n}\left(\sum_{s=1}^{n} \omega_T(v_r, v_s) + \omega_T(v_r, v_r) \right)$$

$$= \sum_{r=1}^{n} D_T(v_r).$$

This completes the proof. ■

6. CONCLUSION

The concept of generalized single valued neutrosophic graphs of type 1 (GSVNG1) was extended to introduce the concept of generalized interval-valued neutrosophic graph of type 1(GIVNG1). The matrix representation of GIVNG1 was also introduced. The future direction of this research includes the study of completeness, regularity of GIVNG1, and also define the concept of generalized interval-valued neutrosophic graphs of type 2.

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