Barut´s lepton mass formula, its correction, and the deduction from it of a "proton mass".

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Abstract

In a PRL published in 1979 A.O.Barut proposed a lepton mass formula of the form $m(n)= 3/(2\ \alpha)n^4\ M_e$, where M_e is the electron mass, α is the fine-structure constant and n is an integer, with increasing leptons masses obtained from the values for $m(n)$ added in sequence of n to M_e . Such model assumes the leptons excess mass $m(n)$ comes from kinetic-magnetic energies and arises from a coupling between the electron magnetic moment and the resulting magnetic field. The formula is good for the muon, with n=1. However, we show that the n-dependence in this formula should be n^2 rather than n^4(the proposed fourth power is incorrect !). Such correction makes Barut´s model formula consistent with the energies obtained for the physically analogous superconducting loop case, treated theoretically by Byers and Yang, which scales as n^2. We apply the corrected formula and reobtain the mass for the tau-lepton, now corresponding to n=4 and not 2, and for n=3 a "proton" with $m \approx$ 945 Mev/c^2 mass.

Introduction.

In 1961 Deaver and Fairbank [1]carried out an experiment in which they demostrated that magnetic flux can be trapped into a superconducting ring only in quantized amounts. The theoretical analysis of such experiment was carried out by Byers and Yang [2]in a letter published in the same edition of PRL. The arguments used were based on the imposition of the continuity of the phase of the wave functions of electron pairs around the ring. Continuity of phase is describable by the Bohr-Sommerfeld (BS) formula which implies action quantization in a closed path. The phase includes the magnetic vector potential action term, to keep gauge covariance in the presence of trapped magnetic fields inside the ring. Applying the BS condition to the quasiparticles momentum one introduces an integer n for the number of turns around the closed path. It is then straightforward to show that the kinetic energy of the particles is proportional to n^2, and in addition it should be periodic in the trapped magnetic flux inside the ring. A review paper by R. Parks [3] discusses Byers and Yang´s analysis and also the related experiment of Little and Parks[4], which displays measurable evidence for the n^2 dependence of the energies, from the effect of the magnetic flux upon the superconductor transition temperature of the ring material.

Current loops have been used as models for the “point” electron intrinsic trajectories in view of the zitterbewegung motion obtainable from Dirac´s equation [5](the loop area would be associated with uncertainty in position and not with the size of the electron itself).

In their model for leptons, Barut and collaborators considered [6,7]also the additional introduction into Dirac´s equation of a convective term with the purpose of accounting for self-energy effects, following previous work by Rosen[8]. The results consistently produced a correct prediction of the muon mass.

Would it be possible to extend the method to other particles besides the muon? Barut noted the following[9]. In the absence of a detailed field-theoretical treatment, an heuristic treatment based upon the semi-classical quantization of self-energy effects in the BS theoretical lines would allow the extrapolation of the method to predict the masses of heavier leptons like the Tau and Delta leptons. They would correspond to higher values of the principal quantum number *n* mentioned earlier.

Barut´s model and necessary corrections.

Barut and collaborators considered self-field effects upon the rest energy of leptons in two ways. One of them[6,7] through an altered version of Dirac´s equation. Such equation would include a convective-like term and its solution produces two possible values for mass. One is the mass of the parent lepton (the one which produces the field), and the other a dressed mass affected by the self-interaction. The parent lepton would be the electron and the dressed lepton the muon(μ). The following formula is obtained(notation explained in the Abstract):

$$M_{\mu}= M_{e}(1+(3/2\ \alpha)) \qquad (1)$$

Barut considered also a second, considerably simpler way. Aware of the possibility of introducing quantum conditions into periodic particles motion without solving Schroedingers equation(the "old" quantum theory), Barut imposed the BS restrictions on action integrals, which should produce integer numbers of the Planck constant. It is well known that BS ignores the wavelike properties and therefore does not impose boundary conditions at the periodic motion turning points, so that details like the ½ extra factor in the harmonic oscillator energy are left aside in the BS solution. However, if interference of waves is negligible the BS solution for the energies should be correct away from the ground state energy. Barut then considered the motion of a particle in a circular orbit, subject to the dipolar magnetic force produced by its own magnetic moment ([9]; cf. ref. 2 of the Letter, which is actually a footnote). In this case the particle producing the moment is an electron , and the moment is the Bohr magneton $\mu_B= e\hbar/2M_e c$ (CGS units).

Newton´s law results in the expression:

$$Mv^2/R= ev\mu_B/ R^3\, c \qquad (2)$$

Here M is the dressed particle mass to be calculated. The BS quantization of action around the circular orbit of radius R results in $(2\pi R)Mv = nh$, and thus:

$$R = n\hbar/Mv \qquad (3)$$

which eliminates R from (2). In the following steps of [9]there is a mistake. Barut argues that since v^2 is proportional to n^4 such n^4 dependence would remain in the mass expression. However, he had not yet introduced the Bohr magneton expression for the magnetic moment in (2), and furthermore the M in the denominator of his final formula should actually be M^2. After correction one obtains:

$$(Mv)^2 = 4c^4\hbar^2 n^4 M_e^2/e^4 \qquad (4)$$

Using $\alpha = e^2/\hbar c$, the fine-structure constant, and neglecting differences between v and c in this intrinsic orbital motion, one immediately obtains:

$$M = (\, 2\, n^2/\, \alpha)\, M_e \qquad (5)$$

which is proportional to n squared and not to the fourth power, and is inversely proportional to α. The actual factor should be 3/2 rather than 2 from Dirac´s equation solution. Such mass ($m(n)$ in the Abstract) should be added to the electron mass as (a large) additional term. One would then recover (1) for the case of the muon, n=1.

A "proton" mass.

We consider each higher order lepton as resulting from such self-energy effects acting upon a bare electron, although Barut´s proposal of accumulating the effects of successive members of the sequence of n cannot be discarded. In this way, for n=4 one obtains

$M = M_e + (3/2 \times 16) \times 137\, M_e = 3289\, M_e = 1680$ Mev/c^2.

This is about 90 Mev/c^2 smaller than the observed mass for the Tau lepton. If the muon mass is included the agreement becomes perfect[9]. For n=2 one obtains the exact kaon mass if a muon is included.

A very interesting result is obtained for n=3. In this case:

$M = M_e + (3/2 \times 9) \times 137\, M_e = 1851\, M_e = 945$ Mev/c^2.

This is essentially the proton mass(we discard the possibility of a stable proton enclosing an unstable muon). The correct sign of its charge might be obtained by considering a positron as the source particle. The n=3 might be associated with structure details in the proton composition.

Such later result was not accessible to Barut in view of the mentioned error in the formula. It must be mentioned that Barut´s approach is essentially the same adopted by E. Post [10] with his description of the electron mass as the energy of a loop of current trapping a quantum of magnetic flux *hc/e* . As discussed in the Introduction, similar arguments had been adopted in [2-4] for the energy associated with a ring trapping magnetic flux, and accordingly resulted in an n^2 dependence of energy, now considered the rest energy of a particle.

There is a wealth of experimental data demonstrating the general proportionality of mass of particles with the inverse of the alpha constant. Leptons, mesons, baryons, follow such behavior[11]. Obtaining a proton mass from a self-field effect acting upon a lepton in some way indicates the perspective that a unified theoretical approach applicable to all particles (which is actually evident in the experimental data) is possible and should be sought for in a more incisive fashion.

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