The Golden Section in Physics

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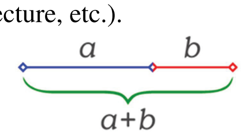
The physical constants play important role in physics. It is fact that the accuracy of the physical constants grows year by year. Special attention is paying to the dimensionless constants; the most familiars among them are the fine structure constant, the electron/proton and electron/muon mass-ratios, the ratio of the gravitational/electromagnetic interaction, the Weinberg angle in the electro-weak interaction theory, etc. The one of the most important questions is for a long time: are there any physical and/or mathematical relations between the fundamental physical constants. The paper gives a recently explored simple math relation between them. The precise theoretical explanation of this amazing finding need more detailed investigations related to the physical background.

Keywords: exponential relations between physical constants, Titius-Bode rule, new atomic mass formula.

1. Introduction

The mathematical notion of the "golden section" or "golden ratio" was first published in the works of Pythagoras and Euclid, but it was fashionable in the Middle Ages, but not only in mathematics, but also in the arts (painting, sculpture, architecture, etc.).



The gold section follows the above figure and corresponds to the next section ratio

$$\frac{a+b}{a} = \frac{a}{b} = \Phi = \frac{1+\sqrt{5}}{2} = 1.618... \qquad (1.1)$$

where Φ is a dimensionless figure, the ratio of the gold section. Is there a dimensionless figure in nature, specifically in physics, that is prominent? The subject of the present work is the finding of this supposed universal ratio what is playing important role in nature, exactly in the fundamentals of physics. The long-term study has shown that there is such ratio having central role in physics, but it cannot be considered as exactly value as the mathematics golden section is defined. In the nature, the exponential dependence between the observable quantities frequently occurs, without causing any surprise for us. Typical examples are the radioactive decay in physics, or the bacteria propagation in the biology. The speed distribution of molecules shows exponential function in the Maxwell-Boltzmann kinetic theory of the gases. Are any exponential relation between the fundamental physical constants, this was the first question which initialized to publish the present paper. The elaborated statistic studies led to an amazing result; the fundamental physical constants are connecting to each-other with a simple exponential form. For the illustration here is an example for the dimensionless fine structure constant

$$\alpha \approx 3Q^4, \qquad (1.2)$$

where the accentuated number is

$$Q = 2/9 = 0.222... \qquad (1.3)$$

A similar finding is

$$m_e/M_p \approx Q^5, \qquad (1.4)$$

where m_e is the electron mass, M_p is the proton mass. The third example shows exponential relation between the electron mass and muon mass

$$m_e/2m_\mu \approx Q^4. \qquad (1.5)$$

In this paper it will be clearly shown that the number Q = *2 / 9* has a central significance in the mathematical connection between the fundamental physical constants. The explored demonstrative results led us to ensure, that behind of these exponential connections of fundamental physical constants must be an important physical background. Nevertheless, the exact physical background is missing at now.

2. Exponential Forms of the Fundamental Physical Constants

Certainly accidentally, the above introduced exponential form is valid approximately for many of dimensioned fundamental physical constants, which are *expressed in the internationally accepted and applied SI units*. Generally, a physical constant signed with X can be written into a simple mathematical expression

$$\lambda X \approx Q^S; \;\; Q = 2/9, \qquad (2.1)$$

where S is integer number and λ is a 'simple' constant. The SI values of physical constants for the calculation are obtained from the database of *National Institute of Standards and Technology* [1]. Surprisingly, the most important physical constants in the SI system can also be expressed by the integer powers of the distinguished number Q

Speed of light $c = 2.997925 \times 10^8\ m/s \approx$

$$\approx Q^{-13};\ Q = 0.222811... \tag{2.2}$$

Gravitational constant $G = 6.674080 \times 10^{-11}\ SI \approx$

$$\approx 2Q^{16};\ Q = 0.221417... \tag{2.3}$$

Planck constant $\hbar = 1.0545717261 \times 10^{-34}\ Js \approx$

$$\approx Q^{52};\ Q = 0.222125... \tag{2.4}$$

Boltzmann constant $k_B = 1.380650 \times 10^{-23}\ J/K \approx$

$$\approx Q^{35};\ Q = 0.222259... \tag{2.5}$$

Coulomb constant $k_C = 8.987552 \times 10^9 SI \approx$

$$\approx Q^{-16}/\pi;\ Q = 0.222242... \tag{2.6}$$

Elementary charge $e = 1.602176 \times 10^{-19}\ C \approx$

$$\approx \sqrt{2} \times Q^{29};\ Q = 0.222175... \tag{2.7}$$

Rydberg constant $R_y = 2.179872 \times 10^{-18}\ J \approx$

$$\approx Q^{27};\ Q = 0.221752... \tag{2.8}$$

Bohr radius $R_B = 5.2917721092 \times 10^{-11}\ m \approx$

$$\approx Q^{15}/3;\ Q = 0.222185... \tag{2.9}$$

Electron mass $m_e = 9.109382 \times 10^{-31}\ kg \approx$

$$\approx Q^{46};\ Q = 0.222303... \tag{2.10}$$

Muon mass $m_\mu = 1.883531 \times 10^{-28}\ kg \approx$

$$\approx Q^{42}/2;\ Q = 0.222303... \tag{2.11}$$

Tau mass $m_\tau = 1.883531 \times 10^{-27}\ kg \approx$

$$\approx 2Q^{41};\ Q = 0.221990... \tag{2.12}$$

Proton mass $m_p = 1.672621 \times 10^{-27}\ kg \approx$

$$\approx Q^{41};\ Q = 0.222286... \tag{2.13}$$

1./By the above, for the values are fulfilled

$$\left\{\hbar^2\right\} = \left\{G m_e m_\mu\right\}, \tag{2.14}$$

from which

$$\{G\} = \left\{\hbar^2/m_e m_\mu\right\} = 6.4817221... \times 10^{-11} SI; \tag{2.15a}$$

$$G(CODATA) = 6.674\,08 \times 10^{-11} SI. \tag{2.15b}$$

2./By the above, for the value is fulfilled

$$\left\{\hbar c^4\right\} \approx 1. \tag{2.16}$$

The calculation gives an approximate result

$$\left\{\hbar c^4\right\} = 0.851... \tag{2.17}$$

An important question arises as to whether the re-defining of the SI system units can be used to fine-tune the above values.

3. The Weak Mixing Angle

The weak mixing angle or Weinberg angle is a parameter in the *Weinberg-Salam theory* of the *electroweak force*. It gives a relationship between the charged W and neutral Z boson masses [2]. The experimentally best estimated value of the Weinberg parameter is

$$\sin^2\Theta_W = 0.2223(21) \approx Q. \tag{3.1}$$

4. Mass Formula of the Leptons

In the literature there are two empirical relations for the three lepton masses. The one of them is the famous *Koide formula* [3]

$$\frac{m_e + m_m + m_t}{\left(\sqrt{m_e} + \sqrt{m_m} + \sqrt{m_t}\right)^2} = 0.666659(10) \approx \frac{2}{3}, \tag{4.1}$$

where m_e = electron mass, m_m= muon mass and finally m_t = tau mass. There is a more important but a less well-known formula for the calculation of lepton masses

$$m_k \approx C_0 \left[1 + \sqrt{2}\cos\left(2k\pi/3 + Q\right)\right]^2, (k = 1, 2, 3). \tag{4.2}$$

where m_k = electron, muon and tau mass for k = *1*, k = *2* and k = *3*, respectively. This formula was published by *Gerald Rosen* [4]. It can be easily proved that the formula of Rosen can be obtained from the Koide formula with the help of two fitting parameters. The result of the fitting procedure is

$$C_0 = 313.85773\, MeV;\ Q = 2/9. \tag{4.3}$$

The accuracy of the lepton mass formula is very good

$$\begin{aligned} 0.51099650\, MeV &= m_e(1 - 4.7 \times 10^{-6}); \\ 105.65891\, MeV &= m_m(1 + 5.09 \times 10^{-6}); \\ 1776.9764\, MeV &= m_t(1 - 7.63 \times 10^{-6}). \end{aligned} \tag{4.4}$$

5. Exponential Interpretation of the Titius-Bode Law

The Bode's law, better called the Titius-Bode Rule, was first published by *Johann Daniel Titius*, but did not become well known until it was republished by *Johann*

Elert Bode in the 18th century. It is supposed to predict the distances of the planets from the Sun in astronomical units (Sun-Earth middle distance) by the formula

$$a_n = 0.4 + 0.3 \times 2^n \quad (5.1)$$

but is usually represented by a table as shown here

Planet	***a_n Real***	***a_n Calc.***
Mercury	0.39	0.4 + 0.3 x 0 = 0.4
Venus	0.72	0.4 + 0.3 x 1 = 0.7
Earth	1	0.4 + 0.3 x 2 = 1.0
Mars	1.52	0.4 + 0.3 x 4 = 1.6
Ceres	2.77	0.4 + 0.3 x 8 = 2.8
Jupiter	5.2	0.4 + 0.3 x 16 = 5.2
Saturn	9.54	0.4 + 0.3 x 32 = 10
Uranus	19.19	0.4+0.3 x 64 = 19.6
Neptune	30.07	0.4+0.3 x 128 = 38.8

Table 1. Demonstration of the Bode-Titius Rule

In the table, the second column contains the measured distances of the planets. The third column contains the calculated planet's distances from formula (5.1). Mini planet Ceres was discovered by chance, not by application of the Titius-Bode rule. Nevertheless, its orbit fit the rule so perfectly that there had been active search for a planet at that distance and the discovery was considered to be another vindication. The Titius-Bode rule was used in the calculations that led to the discovery of Neptune. Remarkable that the physical background of this observed rule has remained unclear until this time, which shows at least exponential behavior of the planet distances from the Sun. In the frame of present study the Titius-Bode rule has been fitted to the recognized exponential relation involving the 'special number' Q. The simple expression of the Kepler's third law is

$$P^2/a^3 = const., \quad (5.2)$$

where P is the orbital period and a is the semi major axis of orbit for the planets of Solar System. When certain units are chosen, namely P is measured in sidereal years and a in astronomical units, P^2a^{-3} has the value one for all planets in the Solar System. From this reason Kepler's third law for the planets can be written into simple form

$$\frac{P_n^2}{a_n^3} \equiv \frac{Q^n}{Q^n} \approx 1; \ \ (n = integer); \quad (5.3)$$

where for the Earth $n = 0$ selection is valid. This approximation defines the astronomical distance of each planet from the Sun in exponential form

$$a_n \approx Q^{n/3}; \ \ (n = integer). \quad (5.4)$$

Nevertheless, in this equation the number Q has not a fixed value. The next table shows the calculated Q-values depending on distances from the Sun for each planets

Planet	***a_n Real***	***n***	***Q Calc.***
Mercury	0.39	2	0.243555
Venus	0.72	1	0.373248
Earth	1	0	ooooo
Mars	1.52	-1	0.284754
Ceres	2.77	-2	0.216911
Jupiter	5.2	-3	0.192308
Saturn	9.54	-4	0.184221
Uranus	19.19	-5	0.169885
Neptune	30.07	-6	0.182362

Table 2. The results of the Q-calculations

The average of the calculated Q-values is near to its 'nominal value' 2 / 9

$$\langle Q \rangle = 0.230905... \approx 2/9. \quad (5.5)$$

The standard deviation of the calculated Q-values is

$$\sigma(Q) \approx 11\%. \quad (5.6)$$

This interesting result strengthens the supposed physical significance of the explored special number Q.

6. A New Atomic Mass Formula

A few years ago the author of the present work has published an atomic mass formula in the physical journal *Galilean Electrodynamics* [5]. According to the generally accepted physical model, the synthesis of the heavy elements may happen at a very high temperature in supernova explosions. In consequence of nuclear fusion, the supernova stars emit a very strong electromagnetic (EM) radiation, predominantly in the form of X-rays and gamma rays. The intensive EM radiation drastically decreases the masses of the exploding stars, directly causing mass defects of the nuclei. The general description of black body EM radiation is based on the famous Planck's radiation theory, which supposes the existence of independent quantum oscillators inside the black body. In this model, it is supposed that in exploding supernova stars, the EM radiating oscillators can be identified with the nascent heavy elements losing their specific yields of their own rest masses in the radiation process. The final binding energy of the nuclei is additionally determined by strong neutrino radiation, which also follows the Maxwell-Boltzmann distribution in extremely high temperature. Extending Planck's radiation law for discrete radiation energies, a very simple formula is obtained for the theoretical description of the measured neutral atomic masses. The realized new atomic mass formula is the next

$$M(Z,A) = AM_0 + M_{rad}(A) + M_{as}(Z,A) +$$
$$+ M_p(Z,A), \ (A \geq 2). \quad (6.1.)$$

The first term is the *initial mass* before nuclear fusion

$$M_{in}(A) = AM_0. \qquad (6.2.)$$

The second term is the *thermo-radiation*

$$M_{rad}(A) = -C_{rad}\frac{f^4(A)}{B^f-1} = -C_{rad}\frac{(A-3/2)^2M_0^2}{R(A)}, \qquad (6.3.)$$

where

$$R(A) = B^{\sqrt{AM_0}} - 1 \qquad (6.4)$$

is proportional to the nuclear radius of the atom. The third term of the mass formula is the *asymmetry energy*

$$M_{as}(Z,A) = C_{as}M_0^2\left(\frac{A-2Z}{A+3}\right)^2. \qquad (6.5)$$

The fourth term is the *pair energy*

$$M_p(Z,A) = -\frac{1}{2}C_pM_0^2\frac{(-1)^Z+(-1)^{A-Z}}{R(A)}. \qquad (6.6)$$

This last term connects to observation that the nuclei having even number of protons and even number of neutrons (even-Z, even-N), or, in short even-even nuclei, are most abundant and more stable. The odd-odd nuclei are the least stable, while even-odd and odd-even nuclei are intermediate in stability. Due to the *Pauli exclusion principle* the nucleus would have a lower energy if the number of protons with spin up were equal to the number of protons with spin down. This is also true for neutrons. This term firstly appeared in the nuclear drop model of *von Weizsäcker* [6] in 1935. Remarkable specialty of this new mass formula is that the coefficients of each member depend on only one parameter, namely by the unique Q number

$$C_{rad} = Q^5/2,\ C_{as} = Q,\ C_p = Q^4/2,\ B = 1+Q. \qquad (6.7)$$

The optimal values of the two fit parameters (which were fitted into about 2000 isotope masses [7])

$$M_0 = 934.529...\ MeV;\ Q = 2/9 = 0.222... \qquad (6.8)$$

The relative standard deviation of this neutral atom mass formula (N is equal to about 2000 isotopes)

$$\sigma = \sqrt{\frac{1}{N-1}\sum_{n=1}^{N}\left(\frac{M_{calc}-M_{\exp}}{M_{\exp}}\right)_n^2} = 1.55...\times 10^{-4}, \qquad (6.9)$$

which is comparable to the accuracy of von Weizsäcker's liquid drop model of the atomic mass calculation. However, the presented atomic mass calculation model contains only two parameters (M_0 and Q) versus the liquid drop model, which contains five fit parameters. The obtained radiating nucleon mass M_0 is less about *5 MeV* than the known rest mass of the neutron. Physical explanation of this fact that at a very high fusion temperature, the average value of neutron masses decreases. The missing parts of the neutron masses appear in the energy of the thermal radiation field (what remains constant in the radiation cavity). Taking this into account, the concept of *total binding energy*, regarding to the initial A number neutrons, can be introduced

$$E_B(Z,A) = A\,(M_0 - M_N) + M_{rad}(A) + M_{as}(Z,A) + \\ + M_p(Z,A),\ (A \geq 2);\ (M_N = 939.565413...\ MeV). \qquad (6.10)$$

The next diagram shows the binding energy components

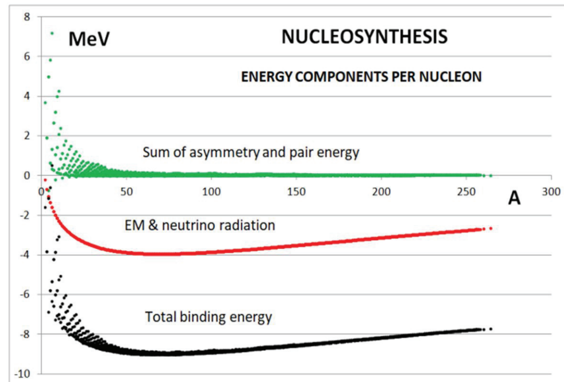


Diagram 1. Binding energy components per nucleon

7. Conclusion

In this paper a dimensionless number $Q = 2/9$ has been introduced, which is suitable to express many important physical constant in similar exponential forms having exclusively integer exponents. From the demonstrative examples one can safely conclude, that all the fundamental physical constants very likely must be quantizied by unique exponential rule. From this statement directly follows, that between all fundamental physical constants must exist simple exponential relationships. The next important question is that this statement is an axiom without any possibility for a deeper physical explanation, or must have an unknown physical background of it. For the answer of this important question needs certainly more detailed research in the future.

8. References

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