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Neutrosophic Goal Geometric Programming Problem based on Geometric Mean Method and its Application

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Abstract: This paper describes neutrosophic goal geometric programming method, a new concept to solve multi-objective non-linear optimization problem under uncertainty. The proposed method is described here as an extension of fuzzy and intuitionistic fuzzy goal geometric programming technique in which the degree of acceptance , degree of indeterminacy and degree of rejection is simultaneously considered. A bridge network complex model is presented here to demonstrate the applicability and efficiency of the proposed method. The method is numerically illustrated and the result shows that the neutrosophic goal geometric programming is very efficient to find the best optimal solution than compare to other existing methods.

Keywords: Neutrosophic set, Goal programming, Geometric programming, Bridge network, Reliability optimization.

INTRODUCTION:

In real life situations, most of the time it is unable to find deterministic optimization problems which are well defined because of imprecise information and unknown data. Thus to handle this type of uncertainty and imprecise nature , fuzzy set theory was first introduced by Zadeh [1] in 1965. Fuzzy optimization problems are more realistic and allow to find solutions which are more acceptable to the real problems . In recent time, fuzzy set theory has been widely developed and there are various modification and generalizations has appeared, intuitionistic fuzzy sets (IFS) is one of them. In 1986, Atanassov [2] developed the idea of IFS , which is characterized by the membership degree as well as non-membership degree such that the sum of these two values is less than one. Intuitionistic fuzzy sets can handle the incomplete information but unable to deal with the indeterminate information. Thus further generalization of it is required. To overcome this, neutrosophy [3] was first introduced by Samarandache in 1995, by adding another independent membership function named as indeterminacy membership along with truth membership and falsity membership function.

Goal programming (GP) is one of the most effective and efficient methods among various kinds of existing methods to solve a particular type of non-linear multi-objective decision making problems. In 1977, Charns and Copper [4] first introduced goal programming problem for a linear model. In a standard GP problem, goals and constraints are not always well defined and it is not possible to find the exact value due to vague nature of the coefficients and parameters. Fuzzy and intuitionistic fuzzy approach can handle this type of situations. Many authors use fuzzy goal programming technique to solve various types of multi-objective linear programming problems [7] ,[8] . M.Zangiabadi [18] applied goal programming approach to solve multi-objective transportation problem in fuzzy environment. B.B.Pal [5] described a goal programming procedure for multi-objective linear programming problem. Since geometric programming gives better result to solve non-linear goal programming problem compare to the other non-linear programming methods, P.Ghosh and T.K.Roy [12] ,[13] described the fuzzy goal geometric programming method in intuitionistic environment. Paramanik and Roy [6] introduced intuitionistic fuzzy goal programming approach in vector optimization problem. Sometimes goal of the system and conditions include some vague and undetermined situations. Hence we cannot handle this type of situations by the concept of fuzzy set and intuitionistic fuzzy set theory. Mathematically, to express the decision maker's unclear target levels for the goals and to optimize all goals at the same level, we have to go through a complicated calculations. Here we introduced neutrosophic approach for goal programming to solve this kind of unclear difficulties. Many researchers applied goal programming for solving multi-objective problems in neutrosophic environment [9],[10],[11] . But it is very first when neutrosophic goal geometric programming method is applied to multi-objective non-linear programming problem.

The present study investigates computational algorithm for solving multi-objective goal geometric programming problem by single valued NGGPP technique . The motivation of this paper is to apply an efficient and modified optimization technique to find a pareto optimal solution of the proposed bridge network reliability model to produce highly reliable system with minimum system cost than the other existing methods. An illustrative example is given to show the utility of NGGPP on the reliability model and also the result of the proposed approach is compared with fuzzy goal geometric programming (FGGP) and intuitionistic fuzzy goal geometric programming (IFGGP) approach at the end of this paper.

The structure of the paper is as follows: In Section 2, some basic definitions and Neutrosophic goal geometric programming problem (NGGPP) method is introduced; In section 3, a bridge network reliability model is introduced and provide NGGPP method for solving the proposed model. In Section 4, numerical examples are solved and compared with the existing method .Finally the conclusions are drawn in section 4.

2. **Neutrosophic goal geometric programming problem (NGGPP):**

Definition 2.1. Let X be a space of points and $x \in$ X. A neutrosophic set (NS) $\tilde{A}^N$ in X having the form

$\tilde{A}^N = \{< x \quad \mu_A(x), \nu_A(x) \,,\, \sigma_A(x) > | \; x \; \in X\}$,
where $\mu_A(x), \nu_A(x)$ and $\sigma_A(x)$ denote the truth membership degree , falsity membership degree and indeterminacy membership degree of x respectively and they are real standard or non-standard subsets of $]\,0^{-},1^{+}[$ i.e.

$$\mu_A(x){:}\, X \;\rightarrow\;]\,0^{-},1^{+}[$$

$$\nu_A(x){:}\, X \;\rightarrow\;]\,0^{-},1^{+}[$$

and $\sigma_A(x){:}\, X \;\rightarrow]\,0^{-},1^{+}[$

There is no restriction on the sum of $\mu_A(x), \nu_A(x) \; and \; \sigma_A(x)$. So,

$$0^{-} \le \sup\, \mu_A(x) + \sup\, \nu_A(x) + \sup \sigma_A(x) \le 3^{+}.$$

Ye [14] ,[15] reduced NSs of non-standards intervals into a kind of simplified neutrosophic sets of standard intervals that will preserve the operations of NSs.

Definition 2.2. [17] Let X be a space of points with a generic element x in X. A single-valued neutrosophic set (SVNS) $\tilde{A}^N$ in X is characterized by $\mu_A(x), \nu_A(x)$ and $\sigma_A(x)$, and of the form

$$\tilde{A}^N = \{< x : \mu_A(x), \nu_A(x), \sigma_A(x) > | \ x \in X\}$$

Where $\mu_A(x): X \to [0,1]$

$\nu_A(x): X \to [0,1]$

and $\sigma_A(x): X \to [0,1]$ with $0 \le \mu_A(x) + \nu_A(x) + \sigma_A(x) \le 3$ for all $x \in X$.

Here we consider neutrosophic goal geometric problem as an extension of intuitionistic fuzzy goal geometric programming problem. In NGGPP , degree of indeterminacy is also taken into consideration for neutrosophic goal programming objectives together with the degree of acceptance and degree of rejection.

A multi-objective non-linear neutrosophic goal geometric programming problem with k objective functions can be taken as follows-

Find $X = (x_1, x_2, \dots x_m)$ so as to

$\widetilde{Minimize}$ $(Z_{01}(x) = \sum_{p=1}^{N_{01}} C_{0ip} \prod_{j=1}^{m} x_j^{\alpha_{0ipj}}$ satisfying target goal achievement value C_{01} with acceptance tolerance ${t_{01}}^{acc}$, rejection tolerance ${t_{01}}^{rej}$ and indeterminacy tolerance ${t_{01}}^{ind}$.

$\widetilde{Minimize}$ $(Z_{02}(x) = \sum_{p=1}^{N_{02}} C_{0ip} \prod_{j=1}^{m} x_j^{\alpha_{0ipj}}$ satisfying target goal achievement value C_{02} with acceptance tolerance ${t_{02}}^{acc}$, rejection tolerance ${t_{02}}^{rej}$ and indeterminacy tolerance ${t_{02}}^{ind}$.

: :

$\widetilde{Minimize}$ $(Z_{0k}(x) = \sum_{p=1}^{N_{0k}} C_{0ip} \prod_{j=1}^{m} x_j^{\alpha_{0ipj}}$ satisfying target goal achievement value C_{0k} with acceptance tolerance ${t_{0k}}^{acc}$, rejection tolerance ${t_{0k}}^{rej}$ and indeterminacy tolerance ${t_{0k}}^{ind}$.

Subject to,

$$Z_r(x) = \sum_{p=1+T_{(r-1)k}}^{N_{rk}} C_{rp} \prod_{j=1}^{m} x_j^{\alpha_{rpj}},$$

$$r = 1,2,\dots,l\ , \quad x = (x_1, x_2, \dots\dots, x_m) > 0.$$

Where we have …(2.1)

$C_{0ip} > 0$, (for p = 1,2,3 , .. , N_{0i} ; $i = 1,2,\dots,k$),

$C_{rp} > 0$, (for k = $1 + N_{0k}, \dots, N_{1k}, N_{1k} + 1, \dots, T_{lk}$; $r = 1,2,\dots,l$) ,

α_{0ipj} (p = 1,2, .. , N_{0i}; $i = 1,2,\dots,p$; j = 1,2, …,m)

and α_{rpj} (k = $1 + N_{0k}, \dots, N_{1k}, N_{1k} + 1, \dots, N_{lk}$; j = 1,2, …,m) are real numbers.

Now using the concept of neutrosophic sets, construct the truth membership function $\mu_i(Z_{oi}(x))$, indeterminacy membership function $\sigma_i(Z_{oi}(x))$ and falsity membership function $\vartheta_i(Z_{oi}(x))$ of NGP objectives are given by –

$$\mu_i(Z_{oi}(x)) = \begin{cases} 1 & , \ Z_{oi} \le C_{0i}; \\ 1 - \frac{Z_{oi}(x) - C_{0i}}{{t_{0i}}^{acc}} & , C_{0i} \le Z_{oi} \le C_{0i} + {t_{0i}}^{acc} \\ 0 & , Z_{oi} \ge C_{0i} + {t_{0i}}^{acc}; \end{cases} \quad \dots(2.2)$$

$$\vartheta_i(Z_{oi}(x)) = \begin{cases} 0 & , \ Z_{oi} \le C_{0i}; \\ \frac{Z_{oi}(x) - C_{0i}}{{t_{0i}}^{rej}} & , \ C_{0i} \le Z_{oi} \le C_{0i} + {t_{0i}}^{rej} \\ 1 & , \ Z_{oi} \ge C_{0i} + {t_{0i}}^{rej}; \end{cases} \quad \dots(2.3)$$

and

$$\sigma_i(Z_{oi}(x)) = \begin{cases} 1 & , \ Z_{oi} \le C_{0i}; \\ 1 - \frac{Z_{oi}(x) - C_{0i}}{{t_{0i}}^{ind}} & , C_{0i} \le Z_{oi} \le C_{0i} + {t_{0i}}^{ind} \\ 0 & , \ Z_{oi} \ge C_{0i} + {t_{0i}}^{ind}; \end{cases} \quad \dots(2.4)$$

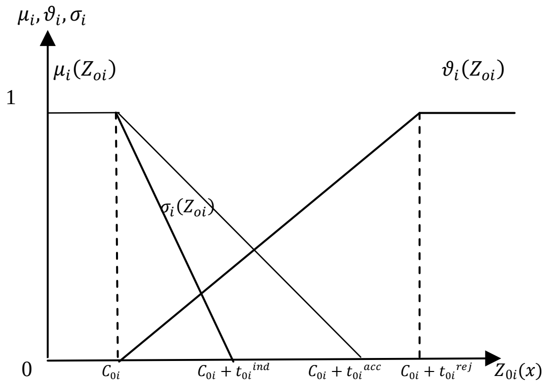

Fig (1) : truth membership function, indeterminacy membership function and falsity membership function for the objective functions $Z_{0i}(x)$.

Now the above NGP model (3.1) can be reduced to a crisp model by maximizing the degree of acceptance, degree of indeterminacy as well as minimizing the degree of falsity of NGP objective functions. Hence we have

Maximize $\mu_i\left(Z_{oi}(x)\right)$ for i = 1,2,…,k

Minimize $\vartheta_i\left(Z_{oi}(x)\right)$ for i = 1,2,…,k

Maximize $\sigma_i\left(Z_{oi}(x)\right)$ for i = 1,2,…,k

Subject to, $Z_r(x) \leq b_r \;; r = 1,2,\ldots,l$

$$0 \leq \mu_i(Z_{oi}) + \vartheta_i(Z_{oi}) + \sigma_i(Z_{oi}) \leq 3,$$

$$\vartheta_i(Z_{oi}) \geq 0,$$

$$\mu_i(Z_{oi}) \geq \vartheta_i(Z_{oi}),$$

$$\mu_i(Z_{oi}) \geq \sigma_i(Z_{oi}) \text{ , for i = 1,2,…,p}$$

and $X = (x_1, x_2, \ldots\ldots, x_m) > 0$. … (2.5)

Now (2.5) is equivalent to-

Maximize α Minimize β Maximize γ

Subject to, $\mu_i\left(Z_{oi}(x)\right) \geq \alpha$

$$\vartheta_i\left(Z_{oi}(x)\right) \leq \beta$$

$$\sigma_i\left(Z_{oi}(x)\right) \geq \gamma \quad \text{, for i = 1,2,…,}$$

$$Z_r(x) \leq b_r \;; \quad r = 1,2,\ldots,l$$

$$0 \leq \alpha + \beta + \gamma \leq 3 \;,\; \alpha \geq \beta \;,\; \alpha \geq \gamma,$$
$$\alpha, \beta, \gamma \in [0,3] \;,$$

and $X = (x_1, x_2, \ldots\ldots, x_m) > 0$. ….(2.6)

Now by geometric mean method , the above model (2.6) can be written as –

Minimize $\beta(1-\alpha)(1-\gamma)$

Subject to,

$$Z_{oi}(x) \leq C_{0i} + a_{0i}{}^{acc} \times a_{0i}{}^{rej} \times a_{0i}{}^{ind}\left(\frac{\beta(1-\alpha)(1-\gamma)}{3}\right) ,$$

(for i = 1,2,….,k.)

$$\frac{1}{b_r} Z_r(x) \leq 1, \qquad r = 1,2,\ldots,l.$$

$$0 \leq \alpha + \beta + \gamma \leq 3 \;,\; \alpha \geq \beta \;,\; \alpha \geq \gamma,$$
$$\alpha, \beta, \gamma \in [0,3] \;,$$

and $X = (x_1, x_2, \ldots\ldots, x_m) > 0$. …(2.7)

Let , $\beta(1-\alpha)(1-\gamma) = w > 0$, then the above model becomes-

Minimize w

Subject to , $\frac{Z_{oi}(x)}{C_{0i}+a_{0i}{}^{acc} \times a_{0i}{}^{rej} \times a_{0i}{}^{ind} \times w} \leq 1$,

(for i = 1,2,….,k);

$$\frac{1}{b_r} Z_r(x) \leq 1, \quad r = 1,2,\ldots,l.$$

$$X = (x_1, x_2, \ldots\ldots, x_m) > 0. \ldots(2.8)$$

From (3.8) we construct the dual programming model as –

Maximize

$$\left(\frac{w}{\delta_{00}}\right)^{\delta_{00}} \prod_{i=1}^{k} \prod_{p=1}^{N_{0i}} \left[\frac{C_{0ip}}{(C_{0i} + a_{0i}{}^{acc} \times a_{0i}{}^{rej} \times a_{0i}{}^{ind} \times w)\,\delta_{0ip}}\right]^{\delta_{0ip}}$$

$$\times \prod_{r=1}^{l} \prod_{q=1+T_{(r-1)k}}^{N_{rk}} \left[\frac{C_{rq}}{C_r\,\delta_{orq}}\right]^{\delta_{orq}} \left(\sum_{k=1}^{N_{0i}} \delta_{0ip}\right)^{(\sum_{k=1}^{N_{0i}} \delta_{0ip})}$$

$$\times \left(\sum_{q=1+T_{(r-1)k}}^{N_{rk}} \delta_{0rq}\right)^{\sum_{q=1+T_{(r-1)k}}^{T_{rk}} \delta_{orq}}$$

$$\left.\begin{array}{l} \text{Subject to, } \delta_{00} = 1 \\ \sum_{p=1}^{N_{0i}} \delta_{0ip} = 1 \\ \sum_{q=1+T_{(r-1)k}}^{N_{rk}} \delta_{0rq} = 1, \end{array}\right\} \text{Normality Condition (for i =1,2,…,k; } r = 1,2,\dots,l)$$

$$\sum_{i=1}^{k}\sum_{p=1}^{N_{0i}} \alpha_{0ipj}\delta_{0ip} + \sum_{r=1}^{l}\sum_{q=1+(r-1)k}^{N_{rk}} \alpha_{rpj}\,\delta_{0rq} = 0.$$

(for $j = 1,2,\dots,m.$) Orthogonality condition

where
$\delta_{0ip} > 0$ (for $p = 1,2,\dots,N_{0i}$; $i = 1,2,\dots,k$)
$\delta_{0rq} > 0$ (for $q = 1 + N_{(r-1)k},\dots,T_{rk}$; $r = 1,2,\dots,l$)
(Positivity conditions) … (2.9)

Let there are total T number of terms in the above primal problem. Then the degree of difficulty (DD) of the single objective geometric programming problem is T – (m+1).

Case I : for T > (m+1) , a solution vector exists for the dual variables.

Case II : T < (m+1) , generally no solution vectors exist for the dual variables, but we can get the approximate solution for this system using different methods.

Now to find out the solution of the geometric programming model (2.8) , firstly we have to find out the optimal solution of the dual problem (2.9) .Hence from the primal-dual relationship, the corresponding values of the primal variable vector x can be easily obtained. The LINGO-16.0 software is used here to find optimal dual variables from the equations of (2.9).

Lemma 3.1: The ranges of truth , indeterminacy and falsity membership function of neutrosophic goal geometric programming problem will satisfy if ${t_{0i}}^{rej} > 2{t_{0i}}^{ind}$ and ${t_{0i}}^{acc} > {t_{0i}}^{ind}$, where ${t_{0i}}^{acc}, {t_{0i}}^{rej}$ and ${t_{0i}}^{ind}$ are acceptance tolerance, rejection tolerance and indeterminacy tolerance respectively of the NGP objective functions.

Proof: From the equations (3.5) we have –

$$\mu_i(Z_{oi}) \geq \sigma_i(Z_{oi})$$

implies $1 - \frac{Z_{oi}(x)-C_{0i}}{{t_{0i}}^{acc}} \geq 1 - \frac{Z_{oi}(x)-C_{0i}}{{t_{0i}}^{ind}}$

or, $(Z_{oi}(x) - C_{0i})\left(\frac{1}{{t_{0i}}^{ind}} - \frac{1}{{t_{0i}}^{acc}}\right) \geq 0$ (i)

In the above mentioned neutrosophic goal programming problem , we consider each objective functions $Z_{oi}(x)$ satisfying target achievement value C_{0i} and also from the relation – $\vartheta_i(Z_{oi}) \geq 0$

or, $\frac{Z_{oi}(x)-C_{0i}}{{t_{0i}}^{rej}} \geq 0$

or, $(Z_{oi}(x) - C_{0i}) \geq 0$ (ii)

Thus the relation (i) is true if

$$\left(\frac{1}{{t_{0i}}^{ind}} - \frac{1}{{t_{0i}}^{acc}}\right) \geq 0$$

i.e. ${t_{0i}}^{acc} > {t_{0i}}^{ind}$ (iii)

Hence from relation (iii), we have in neutrosophic goal geometric programming problem, acceptance tolerance ${t_{0i}}^{acc}$ should be greater than indeterminacy tolerance ${t_{0i}}^{ind}$.

Again from the relation $\mu_i(Z_{oi}) \geq \vartheta_i(Z_{oi})$ and $\mu_i(Z_{oi}) \geq \sigma_i(Z_{oi})$

we have, $1 - \frac{Z_{oi}(x)-C_{0i}}{{t_{0i}}^{acc}} \geq \frac{Z_{oi}(x)-C_{0i}}{{t_{0i}}^{rej}}$ (iv)

and $1 - \frac{Z_{oi}(x)-C_{0i}}{{t_{0i}}^{acc}} \geq 1 - \frac{Z_{oi}(x)-C_{0i}}{{t_{0i}}^{ind}}$ (v)

Adding the above inequalities (iv) and (v) , we get-

$$1 - \frac{Z_{oi}(x)-C_{0i}}{{t_{0i}}^{acc}} \geq \frac{1}{2} + \frac{(Z_{oi}(x)-C_{0i})}{2}\left(\frac{1}{{t_{0i}}^{rej}} - \frac{1}{{t_{0i}}^{ind}}\right)$$

(vi)

Now from (3.5) using the relation

$\mu_i(Z_{oi}) \geq \vartheta_i(Z_{oi}) \geq 0$ and

$\mu_i(Z_{oi}) + \vartheta_i(Z_{oi}) + \sigma_i(Z_{oi}) \leq 3$

we get , $\sigma_i(Z_{oi}) \leq 3$

or, $1 - \frac{Z_{oi}(x) - C_{0i}}{{t_{0i}}^{ind}} \leq 3$

or, $Z_{oi}(x) - C_{0i} \geq -2{t_{0i}}^{ind}$

or $\frac{1}{Z_{oi}(x) - C_{0i}} \leq -\frac{1}{2{t_{0i}}^{ind}}$ (vii)

Hence from $\mu_i(Z_{oi}) + \vartheta_i(Z_{oi}) + \sigma_i(Z_{oi}) \leq 3$ using (vi) and (vii) –

$\frac{1}{2} + \frac{(Z_{oi}(x) - C_{0i})}{2}\left(\frac{1}{{t_{0i}}^{rej}} - \frac{1}{{t_{0i}}^{ind}}\right) + \frac{Z_{oi}(x) - C_{0i}}{{t_{0i}}^{rej}} + 1 - \frac{Z_{oi}(x) - C_{0i}}{{t_{0i}}^{ind}} \leq 3$ gives ${t_{0i}}^{rej} > 2{t_{0i}}^{ind}$.

Thus from the above relation it is clear that in neutrosophic goal geometric programming problem half of the rejection tolerance ${t_{0i}}^{rej}$ should be greater than the indeterminacy tolerance ${t_{0i}}^{ind}$.

Theorem 3.1: x^* is a pareto optimal solution to NGGPP (3.1) iff x^* is a pareto optimal solution to fuzzy goal geometric programming problem (FGGPP) which is of the form

Minimize $(Z_{01}(x), Z_{02}(x), \ldots, Z_{0k}(x))$

Subject to, $Z_r(x) \leq b_r$, $r = 1,2, \ldots, l$

$$\mathrm{X} = (x_1, x_2, \ldots \ldots, x_m) > 0. \quad \text{….(2.10)}$$

Proof:

Definition: x^* is said to be a pareto optimal solution to the neutrosophic goal geometric programming problem (2.1) iff there does not exist another x such that $\mu_i(Z_{oi}(x)) \geq \mu_i(Z_{oi}(x^*))$, $\vartheta_i(Z_{oi}(x)) \leq \vartheta_i(Z_{oi}(x^*))$ and $\sigma_i(Z_{oi}(x)) \geq \sigma_i(Z_{oi}(x^*))$ for all $\mathrm{i} = 1,2, \ldots, \mathrm{k}$ with strict inequality holds for at least one i.

If x^* be a pareto optimal solution of the FGGPP (2.10) then there does not exist any x such that $Z_{oi}(x) \leq Z_{oi}(x^*)$ for all i=1,2,…,k. and $Z_{oi}(x^*) \neq Z_{oi}(x)$ for at least one i.

Then we have for all $X = (x_1, x_2, \ldots \ldots, x_m)$

$$Z_{oi}(x) \leq Z_{oi}(x^*) \quad \text{…. (A)}$$

with strict inequality hold for at least one i.

i.e. $Z_{oi}(x) - C_{0i} \leq Z_{oi}(x^*) - C_{0i}$

or, $\frac{Z_{oi}(x) - C_{0i}}{{t_{0i}}^{acc}} \leq \frac{Z_{oi}(x^*) - C_{0i}}{{t_{0i}}^{acc}}$

or, $1 - \frac{Z_{oi}(x) - C_{0i}}{{t_{0i}}^{acc}} \geq 1 - \frac{Z_{oi}(x^*) - C_{0i}}{{t_{0i}}^{acc}}$ implies $\mu_i(Z_{oi}(x)) \geq \mu_i(Z_{oi}(x^*))$.

Similarly from (A) we have

$\frac{Z_{oi}(x) - C_{0i}}{{t_{0i}}^{rej}} \leq \frac{Z_{oi}(x^*) - C_{0i}}{{t_{0i}}^{rej}}$ which implies $\vartheta_i(Z_{oi}(x)) \leq \vartheta_i(Z_{oi}(x^*))$

and also $\frac{Z_{oi}(x) - C_{0i}}{{t_{0i}}^{ind}} \leq \frac{Z_{oi}(x^*) - C_{0i}}{{t_{0i}}^{ind}}$

or, $1 - \frac{Z_{oi}(x) - C_{0i}}{{t_{0i}}^{ind}} \geq 1 - \frac{Z_{oi}(x^*) - C_{0i}}{{t_{0i}}^{ind}}$

or, $\sigma_i(Z_{oi}(x)) \geq \sigma_i(Z_{oi}(x^*))$. Hence from the definition of pereto optimal solution to the NGGPP , we have x^* is the pareto optimal solution of (2.1).

Conversely, let x^* is a pareto optimal solution to NGGPP (2.1), then from the expression of membership function given in (2.2) we get

$$1 - \frac{Z_{oi}(x) - C_{0i}}{{t_{0i}}^{acc}} \geq 1 - \frac{Z_{oi}(x^*) - C_{0i}}{{t_{0i}}^{acc}}$$

i.e. $Z_{oi}(x) \leq Z_{oi}(x^*)$.

Again using (3.3) we have

$\frac{Z_{oi}(x)-C_{0i}}{t_{0i}{}^{ind}} \leq \frac{Z_{oi}(x^*)-C_{0i}}{t_{0i}{}^{ind}}$ which implies $Z_{oi}(x) \leq Z_{oi}(x^*)$.

Similarly, using (3.4) ,

$1-\frac{Z_{oi}(x)-C_{0i}}{t_{0i}{}^{ind}} \geq 1-\frac{Z_{oi}(x^*)-C_{0i}}{t_{0i}{}^{ind}}$ gives

$Z_{oi}(x) \leq Z_{oi}(x^*)$.

Thus we have $Z_{oi}(x) \leq Z_{oi}(x^*)$ with strict inequality hold for at least one i , $i \in \{1,2,\dots,k\}$ and which shows that x^* is a pareto optimal solution of (2.10).

3. Numerical Example:

3.1. Bridge network Model [16] :

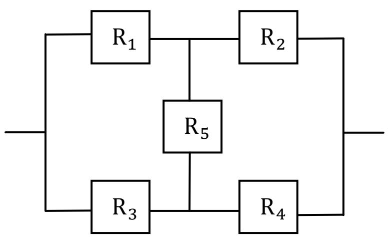

Fig (2) : A five-component complex bridge network system

Here a bridge network system as shown in the figure(3) has been considered, each having a component reliability R_j , j = 1,2,...,5 .

Based on the simple probability theorem

$\Pr(X \cup Y) = \Pr(X) + \Pr(Y) - \Pr(X \cap Y)$ (3.1)

the system reliability $R_S(R)$ of the bridge network system is given by as follows:

Now to use equation (3.1) , it is required to found all possible paths from the input node to output node. The system will operate if the components in any one the following sets

$\{R_1, R_2\}$, $\{R_3, R_4\}$, $\{R_1, R_5, R_4\}$ and $\{R_3, R_5, R_2\}$ operate.

Thus the system reliability is given by

$R_S(R) = \Pr(\{R_1, R_2\} \cup \{R_3, R_4\} \cup \{R_1, R_5, R_4\} \cup \{R_3, R_5, R_2\})$

Since all the components operate independently , thus-

$\Pr(\{R_1, R_2\}) = R_1R_2$,

$\Pr(\{R_3, R_4\}) = R_3R_4$,

$\Pr(\{R_1, R_5, R_4\}) = R_1R_5R_4$,

$\Pr(\{R_3, R_5, R_2\}) = R_3R_5R_2$.

Now using equation (3.1) ,

$\Pr(\{R_1, R_2\} \cup \{R_3, R_4\})$

$= \Pr(\{R_1, R_2\}) + \Pr(\{R_3, R_4\}) - \Pr(\{1,2\} \cap \{3,4\})$

$= R_1R_2 + R_3R_4 - R_1R_2R_3R_4$

Similarly

$\Pr(\{R_1, R_2\} \cup \{R_3, R_4\} \cup \{R_1, R_5, R_4\}) =$

$R_1R_2 + R_3R_4 - R_1R_2R_3R_4 + R_1R_5R_4 - R_1R_2R_5R_4 - R_1R_3R_5R_4 + R_1R_2R_3R_5R_4$.

$\Pr(\{R_1, R_2\} \cup \{R_3, R_4\} \cup \{R_1, R_5, R_4\} \cup \{R_3, R_5, R_2\})$
$= R_1R_2 + R_3R_4 + R_1R_5R_4 + R_3R_5R_2 - R_1R_2R_3R_4 - R_1R_2R_5R_4 - R_1R_3R_5R_4 - R_1R_3R_5R_2 - R_2R_3R_5R_4 + 2R_1R_2R_3R_5R_4$

Thus the multi- objective reliability optimization model becomes

Maximize $R_S(R) = R_1R_2 + R_3R_4 + R_1R_5R_4 + R_3R_5R_2 - R_1R_2R_3R_4 - R_1R_2R_5R_4 - R_1R_3R_5R_4 - R_1R_3R_5R_2 - R_2R_3R_5R_4 + 2R_1R_2R_3R_5R_4$

Minimize $C_S(R) = \sum_{i=1}^{n} C_i R_i{}^{a_i}$

$0 < R_j \leq 1$, $0 \leq R_S \leq 1$, j =1,2, ... ,5. (3.2)

Where $C_S(R)$ denote the cost of the system and C_{lim} is the available cost of the system.

3.1. Application of Neutrosophic Goal Geometric Programming on Bridge Network Reliability Model:

To solve the above multi-objective problem using geometric programming approach , the problem should be in minimization form. Thus , the suitable form of optimization model is taken as

Minimize $R_S{}'(R) = -\mathrm{R}_1\mathrm{R}_2 - \mathrm{R}_3\mathrm{R}_4 - \mathrm{R}_1\mathrm{R}_5\mathrm{R}_4 - \mathrm{R}_3\mathrm{R}_5\mathrm{R}_2 + \mathrm{R}_1\mathrm{R}_2\mathrm{R}_3\mathrm{R}_4 + \mathrm{R}_2\mathrm{R}_5\mathrm{R}_4 + \mathrm{R}_1\mathrm{R}_3\mathrm{R}_5\mathrm{R}_4 + \mathrm{R}_1\mathrm{R}_3\mathrm{R}_5\mathrm{R}_2 + \mathrm{R}_2\mathrm{R}_3\mathrm{R}_5\mathrm{R}_4 - 2\mathrm{R}_1\mathrm{R}_2\mathrm{R}_3\mathrm{R}_5\mathrm{R}_4$

satisfying target achievement value R_0 with acceptance tolerance $t_R{}^{acc}$, rejection tolerance $t_R{}^{rej}$ and indeterminacy tolerance $t_R{}^{ind}$.

Also , we Minimize $C_S(R) = \sum_{i=1}^{n} C_i R_i{}^{a_i}$ satisfying target achievement value C_0 with acceptance tolerance $t_C{}^{acc}$, rejection tolerance $t_C{}^{rej}$ and indeterminacy tolerance $t_C{}^{ind}$. Now , construct the truth membership function , falsity membership function and indeterminacy membership function as follows –

$$\mu_{R_S{}'}(R) = \begin{cases} 1 & , \quad R_S{}'(R) \le \mathrm{R}_0; \\ 1 - \frac{R_S{}' - \mathrm{R}_0}{t_R{}^{acc};} & , \mathrm{R}_0 \le R_S{}'(R) \le \mathrm{R}_0 + t_R{}^{acc}; \\ 0 & , \quad R_S{}'(R) \ge \mathrm{R}_0 + t_R{}^{acc}; \end{cases}$$

$$\vartheta_{R_S{}'}(R) = \begin{cases} 0 & , \quad R_S{}'(R) \le \mathrm{R}_0; \\ \frac{R_S{}' - \mathrm{R}_0}{t_R{}^{rej}} & , \quad \mathrm{R}_0 \le R_S{}'(R) \le \mathrm{R}_0 + t_R{}^{rej} \\ 1 & , \quad R_S{}'(R) \ge \mathrm{R}_0 + t_R{}^{rej}; \end{cases}$$

$$\sigma_{R_S{}'}(R) = \begin{cases} 1 & , \quad R_S{}'(R) \le \mathrm{R}_0; \\ 1 - \frac{R_S{}' - \mathrm{R}_0}{t_R{}^{ind}} & , \mathrm{R}_0 \le R_S{}'(R) \le \mathrm{R}_0 + t_R{}^{ind} \\ 0 & , \quad R_S{}'(R) \ge \mathrm{R}_0 + t_R{}^{ind}; \end{cases}$$

$$\mu_{C_S}(R) = \begin{cases} 1 & , \quad C_S(R) \le \mathrm{C}_0; \\ 1 - \frac{C_S - \mathrm{C}_0}{t_C{}^{acc}} & , \mathrm{C}_0 \le C_S(R) \le \mathrm{C}_0 + t_C{}^{acc} \\ 0 & , \quad C_S(R) \ge \mathrm{C}_0 + t_C{}^{acc} \end{cases}$$

$$\vartheta_{C_S}(R) = \begin{cases} 0 & , \quad C_S(R) \le \mathrm{C}_0; \\ \frac{C_S - \mathrm{C}_0}{t_C{}^{rej}} & , \mathrm{C}_0 \le C_S(R) \le \mathrm{C}_0 + t_C{}^{rej} \\ 1 & , \quad C_S(R) \ge \mathrm{C}_0 + t_C{}^{rej}; \end{cases}$$

$$\sigma_{C_S}(R) = \begin{cases} 1 & , \quad C_S(R) \le \mathrm{C}_0; \\ 1 - \frac{C_S - \mathrm{C}_0}{t_C{}^{ind}} & , \quad \mathrm{C}_0 \le C_S(R) \le \mathrm{C}_0 + t_C{}^{ind} \\ 0 & , \quad C_S(R) \ge \mathrm{C}_0 + t_C{}^{ind}; \end{cases}$$

Now using (2.5), the above model (3.2) reduces to the following form –

Maximize $\mu_{R_S{}'}$ Maximize μ_{C_S}

Maximize $\sigma_{R_S{}'}$ Maximize σ_{C_S}

Minimize $\vartheta_{R_S{}'}$ Minimize ϑ_{C_S}

Subject to , $0 \le \mu_{R_S{}'} + \vartheta_{R_S{}'} + \sigma_{R_S{}'} \le 3$,

$0 \le \mu_{C_S} + \sigma_{C_S} + \vartheta_{C_S} \le 3$,

$\vartheta_{R_S{}'} \ge 0$, $\vartheta_{C_S} \ge 0$

$\mu_{R_S{}'} \ge \vartheta_{R_S{}'}$, $\mu_{C_S} \ge \vartheta_{C_S}$,

$\mu_{R_S{}'} \ge \sigma_{R_S{}'}$, $\mu_{C_S} \ge \sigma_{C_S}$,

$0 < R_i \le 1$; i=1,2,…,n; …..(3.1.1)

The above model (3.1.1) is equivalent to

Maximize α, Minimize β, Maximize γ

Subject to, $\mu_{R_S{}'} \geq \alpha$, $\mu_{C_S} \geq \alpha$,

$\vartheta_{R_S{}'} \leq \beta$, $\vartheta_{C_S} \leq \beta$,

$\sigma_{R_S{}'} \geq \gamma$, $\sigma_{C_S} \geq \gamma$,

$0 \leq \alpha+\beta+\gamma \leq 3$, $\alpha \geq \beta$, $\alpha \geq \gamma$,

$0 \leq \alpha, \beta, \gamma \leq 1$ …..(3.1.2)

Using geometric mean method (4.1.8) becomes-

Minimize w

Subject to ,

$$\frac{-R_1R_2 - R_3R_4 - R_1R_5R_4 - R_3R_5R_2 + R_1R_2R_3R_4 + R_2R_5R_4 + R_1R_3R_5R_4 + R_1R_3R_5R_2 + R_2R_3R_5R_4 - 2R_1R_2R_3R_5R_4}{R_0 + a_{0i}{}^{acc} \times a_{0i}{}^{rej} \times a_{0i}{}^{ind} \times w} \leq 1 \quad ;$$

$$\frac{\sum_{i=1}^{5} C_i R_i{}^{a_i}}{C_0 + a_{0i}{}^{acc} \times a_{0i}{}^{rej} \times a_{0i}{}^{ind} \times w} \leq 1;$$

$0 < R_i \leq 1$; i=1,2,…,5; ..(3.1.3)

where we take $w = \beta(1-\alpha)(1-\gamma) > 0$ as a parameter. The degree of difficulty (D.D) of (4.1.9) is $(5+2)-(5+1) = 1\ (> 0)$.

Now using (2.9) , the above model (3.1.3) can be solved by geometric programming technique after finding its dual.

4 Numerical Example

Here we consider the bridge network reliability optimization model for the numerical exposure. Thus the model (4.1) becomes-

Maximize $R_S(R) = R_1R_2 + R_3R_4 + R_1R_5R_4 + R_3R_5R_2 - R_1R_2R_3R_4 - R_1R_2R_5R_4 - R_1R_3R_5R_4 - R_1R_3R_5R_2 - R_2R_3R_5R_4 + 2R_1R_2R_3R_5R_4$

Minimize $C_S(R) = \sum_{i=1}^{n} C_i R_i{}^{a_i}$

$0 < R_j \leq 1$, $0 \leq R_S \leq 1$, j =1,2, … ,5. …(4.1)

Table (1) : The input data for the neutrosophic goal geometric programming problem (5.1) is given as follows –

C_1	C_2	C_3	C_4	C_5	C_0	$t_C{}^{acc}$	$t_C{}^{rej}$	$t_C{}^{ind}$	$t_R{}^{acc}$	$t_R{}^{rej}$	$t_R{}^{ind}$	$a_i, \forall i$	R_0
12	10	15	18	16	100	8	14	6	0.3	0.52	0.25	0.15	0.2

Table (2): Comparison of optimal solutions of (4.1) by NGGPP method with fuzzy goal geometric programming problem (FGGPP) approach and intuitionistic fuzzy goal geometric programming (IFGGPP) approach:

Method	R_1	R_2	R_3	R_4	R_5	$R_S(R)$	$C_S(R)$
FGGPP	0.905917	0.905923	0.896927	0.796213	0.948311	0.970147	69.702
IFGGPP	0.812514	0.992162	0.992359	0.992842	0.892531	0.998364	70.313
NGGPP	0.967124	0.992981	0.993162	0.965927	0.985742	0.999519	70.786

The above table describes the comparison of results of objective functions for primal problem of the proposed neutrosophic goal geometric programming approach with the FGGPP and IFGGPP approach. It is clear from the above table (2) that NGGPP approach gives better result than the IFGGPP approach in perspective of system reliability. But in view of system cost the proposed approach gives a little bit higher value than the IFGGPP and FGGPP method.

5. **Conclusion and future work:**

A new concept to non-linear multi-objective optimization problem in neutrosophic environment is discussed in this paper. In this work we have introduced NGGPP technique to find the best optimal solution of the multi-objective bridge network reliability model in which system reliability and system cost are chosen as two objective function. Finally an illustrative numerical example is provided by comparing the result obtained in NGGPP technique with IFGGPP and FGGPP approach to demonstrate the efficiency of the proposed method. Thus the proposed method is an efficient and modified optimization technique and can construct a highly reliable system than the other existing method. The method presented here is quite general and can be applied to the typical problems in other areas of Operation Research and Engineering Sciences, like Transportation problems, Inventory problems, Structural optimization, etc.

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