Change in kinetic energy of the body in the environment with constant energy, Mpemba effect

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***Abstract**: The algorithm of change in kinetic energy of the body consisting of elastic spheres in the environment with constant energy is received. The software application is written. Physical experiments on the cooling of bodies in the constant temperature air are made. Mpemba effect is shown. Qualitative coincidence of calculated and experimental data is shown.*

Keywords: kinetic energy; algorithm; experiment.

introduction

At the end of the 17th century Isaac Newton studied cooling of bodies. Experiments showed that the cooling rate is approximately proportional to the difference between the temperature of the body and the temperature of the environment. This fact can be written down in the form of a differential equation:

$$\frac{dQ}{dt} = \alpha A(T_S - T), \qquad (1)$$

where Q - the thermal energy, A - the area of the surface transferring heat, T - the temperature of the body, T_S − the temperature of the environment, α − the heat transfer coefficient depending on body geometry, surface state, heat transfer behavior and others.

The heat transfer coefficient contains a numerous undefined factors. Some factors, such as body geometry, surface state, heat transfer behavior are specified or can be measured, but others, such as molecular vibration frequency in solids and liquids or velocity of the molecules in gases, are not specified since molecular and kinetic properties of substance were not known in the 17th century.

William Thompson stated and solved a problem of cooling or heating of a semi-infinite half-space. [1].

1. A model for the exchange of kinetic energy of a body consisting of absolutely elastic particles in the environment with constant energy.

Let's consider this problem in terms of the kinetic theory of gasses taking into consideration the statement of N.N. Pirogov about the interaction of molecules of the gas and the environment and the influence of the edges [2]. It is well known that the heat can be transferred in three ways: kinetically, convection or radiation. To eliminate the convection let's consider the body to be solid or a liquid at rest. The influence of the radiation is small and can be omitted. Also let's consider coincides of molecules are elastic.

Taking all this limitation into consideration the problem may be simplified as following.

The plane is filled with rigid spheres (b). All spheres are oscillating with the same frequency v_b. Let's place on the plane a rectangle filled rigid spheres (r) oscillating with the frequency $v_{r0} > v_b$. Let only one edge of the rectangle is interacting with the environment. All spheres have the same weight. Pic. 1 The oscillation in any of four direction may happen with the equal probability. The spheres inside the rectangle get oscillating with the frequency v_{r0} or v_b. We need to find the time period after which the frequencies of the oscillation of the spheres inside the rectangle and the environment will become equal.

0	**1**	**2**	**3**	**.**	***n***	
		j		.		
b	*r*	*r*	*r*	.	*r*	
b	*r*	*r*	*r*	.	*r*	
b	*r*	*r*	*r*	.	*r*	
b	*r*	*r*	*r*	.	*r*	***i***
b	*r*	*r*	*r*	.	*r*	
b	*r*	*r*	*r*	.	*r*	
b	*r*	*r*	*r*	.	*r*	
b	*r*	*r*	*r*	.	*r*	
b	*r*	*r*	*r*	.	*r*	
				.		

Figure 1

Solution.

Molecules exchange moment when impact on the top or bottom edge. Impacting on the left or right edge lead to the elastic bounce. The sum of the moment remains constant that's why we can consider the rectangle as one row of the spheres Figure 2

0	**1**	**2**	**3**	**.**	***n***	
		j		.		
b	*r*	*r*	*r*	.	*r*	

Figure 2

$p_i(r_j)$ is the probability that the sphere has the speed v_r at step *j*.

Problem 1. $p_0(r_0) = 0, p_0(r_1) = 1,\ p_0(r_2) = 1$

0	**1**	**2**
b	*r*	

Figure 3

$p_1(r_1) = \frac{3}{4}$

$p_2(r_1) = \frac{1}{2}p_1(r_1) + \frac{1}{4}p_1(r_0) + \frac{1}{4}p_1(r_2) = \frac{1}{2}\frac{3}{4} + \frac{1}{4}0 + \frac{1}{4}1 = \frac{5}{8}$

$p_3(r_1) = \frac{1}{2}p_2(r_1) + \frac{1}{4}p_2(r_0) + \frac{1}{4}p_2(r_2) = \frac{1}{2}\frac{5}{8} + \frac{1}{4}0 + \frac{1}{4}1 = \frac{9}{16}$

Problem 2. $p_0(b_0) = 1 \leftrightarrow p_0(r_0) = 0, p_0(r_1) = 1, p_0(r_2) = 1, p_0(r_3) = 1$

0	**1**	**2**	**3**
b	*r*	*r*	

Figure 4

$p_1(r_1) = \frac{3}{4},\ p_1(r_2) = 1$

$p_2(r_1) = \frac{1}{2}p_1(r_1) + \frac{1}{4}p_1(r_0) + \frac{1}{4}p_1(r_2) = \frac{1}{2}\frac{3}{4} + \frac{1}{4}0 + \frac{1}{4}1 = \frac{5}{8}$

$p_2(r_2) = \frac{1}{2}p_1(r_2) + \frac{1}{4}p_1(r_1) + \frac{1}{4}p_1(r_3) = \frac{1}{2}1 + \frac{1}{4}\frac{3}{4} + \frac{1}{4}1 = \frac{1}{2} + \frac{3}{16} + \frac{1}{4} = \frac{8+3+4}{16} = \frac{15}{16}$

$p_i(r_2) = \frac{1}{2}p_{i-1}(r_2) + \frac{1}{4}p_{i-1}(r_1) + \frac{1}{4}p_{i-1}(r_3)$

Problem 3. $p_0(b_0) = 1 \leftrightarrow p_0(r_0) = 0, p_0(r_j) = 1, j = 1, 2, 3, \ldots$ k - number of columns

$p_i(r_j) = \frac{1}{2}p_{i-1}(r_j) + \frac{1}{4}p_{i-1}(r_{j-1}) + \frac{1}{4}p_{i-1}(r_{j+1})$, $i = 1, 2, 3, \ldots$ n - number of steps (2)

Problem 4. In the 1-3 problems we calculated the probability of colliding of neighboring spheres depending on the direction of the collision. The speed (frequency) was not taken into consideration. It follows from all above that the probability in the formula (2) is reduced by a defined amount.

Let the frequency (speed) be put into the formula (2)

$ps_n(r_k) = p_n(r_k) - s * (1 - p_n(r_k))$

$1 > s > 0$ - coefficient showing the influence of the speed on the probability

$ps_n(r_k)$- the new probability

$$\begin{cases} p_i(r_j) = \frac{1}{2}p_{i-1}(r_j) + \frac{1}{4}p_{i-1}(r_{j-1}) + \frac{1}{4}p_{i-1}(r_{j+1}) \\ ps_i(r_j) = \frac{1}{2}p_{i-1}(r_j) + \frac{1}{4}\left(p_{i-1}(r_{j-1}) - s * \left(1 - p_{i-1}(r_{j-1})\right)\right) + \frac{1}{4}\left(p_{i-1}(r_{j+1}) - s * \left(1 - p_{i-1}(r_{j+1})\right)\right) \end{cases} \quad (3)$$

$i, j = 1, 2, 3, \ldots.$

In Figures 5, 6 below we see the results of applying the formula (3) to the system on two equal elastic s having different start speed.

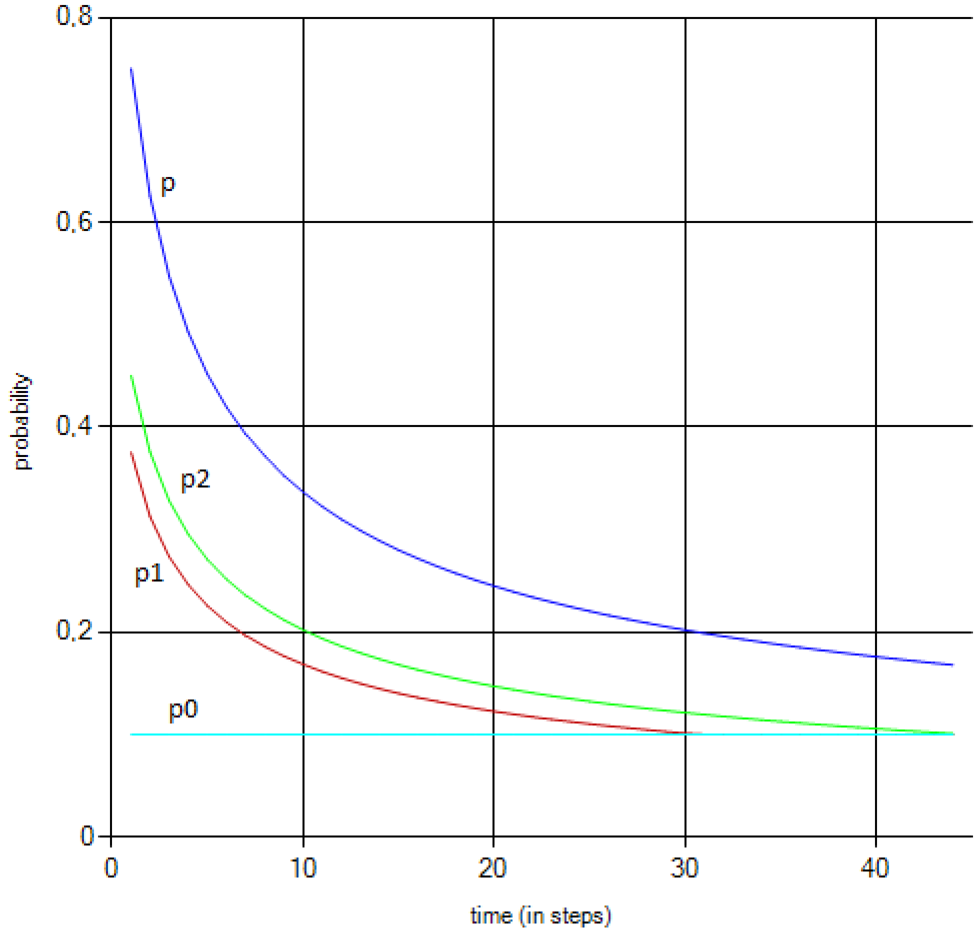
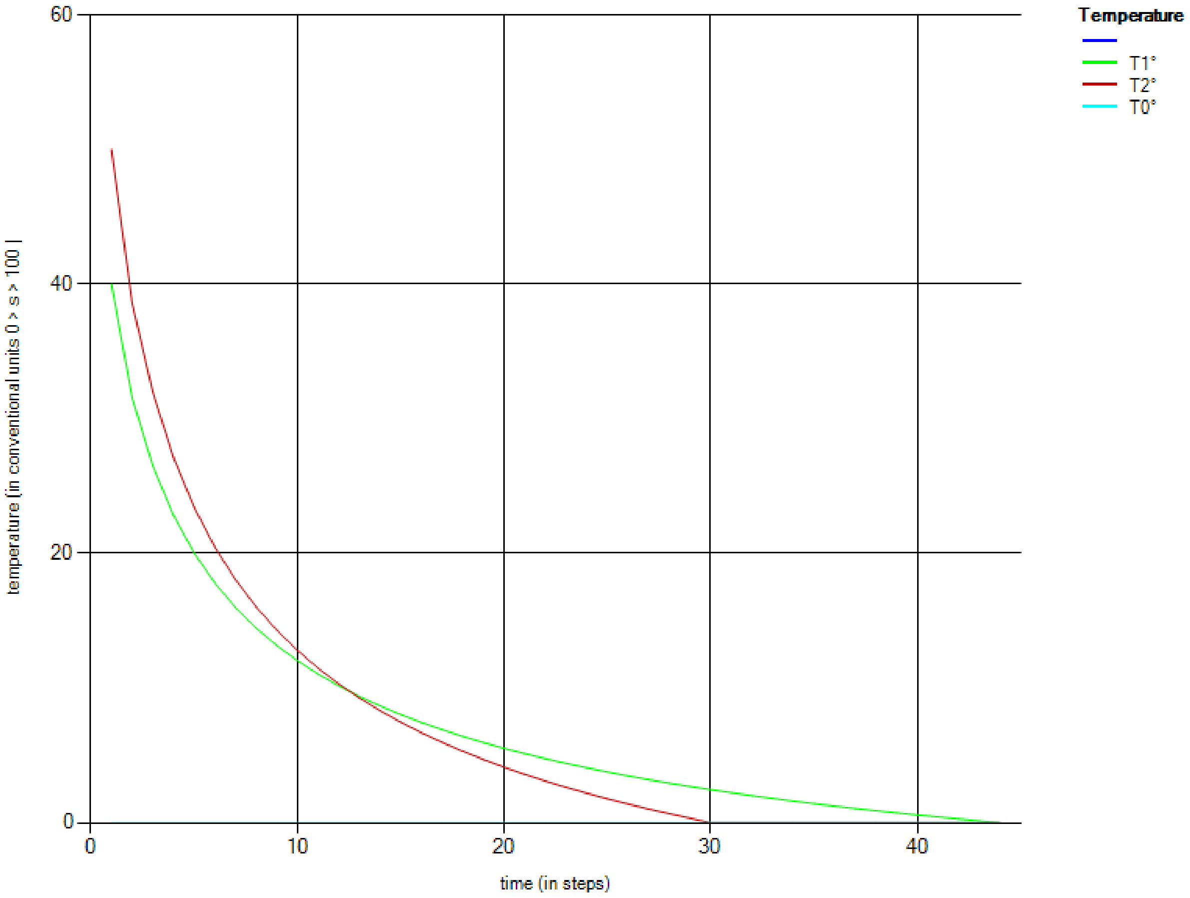


Figure 5



Change of probability for the left sphere depending on coefficient s. s1 > s2

Figure 6

The executed files are to the address [3].

2. Physical experiments on the conditions of the algorithm.

We can see that the bigger is the initial speed the quicker the speed of the spheres inside the rectangle become equal to the speed of the spheres of the environment.

Let's consider an experiment.

The experiment was provided using a domestic refrigerator. The temperature was measured by Digital LCD Thermometer Temperature Sensor Fridge Freezer Thermometer Beliebt

Experiment 1. Take two plastic ping-pong ball with diameter of 40 mm. Both balls are filled with water 29 ml. Thermal detectors are placed inside the balls. The temperature of the environment is -15,7°C. The interval of measurements is 2 minutes. Tab. 1, Figure 7.

1	39,6	32
2	34,2	25,7
3	30,8	22,8
4	28,7	21,2
5	26,3	19,2
6	25	18,1
7	22,5	16,2
8	20,3	14,5
9	18,8	13,4
10	17,4	12,2
11	15,8	11
12	13,7	9,3
13	12,4	8,3
14	10,6	6,9
15	9	5,6
16	8,2	5
17	6,7	3,9
8	5,9	3,4
19	4,8	2,3
20	3,3	1,3
21	1,8	0,1

Tab. 1

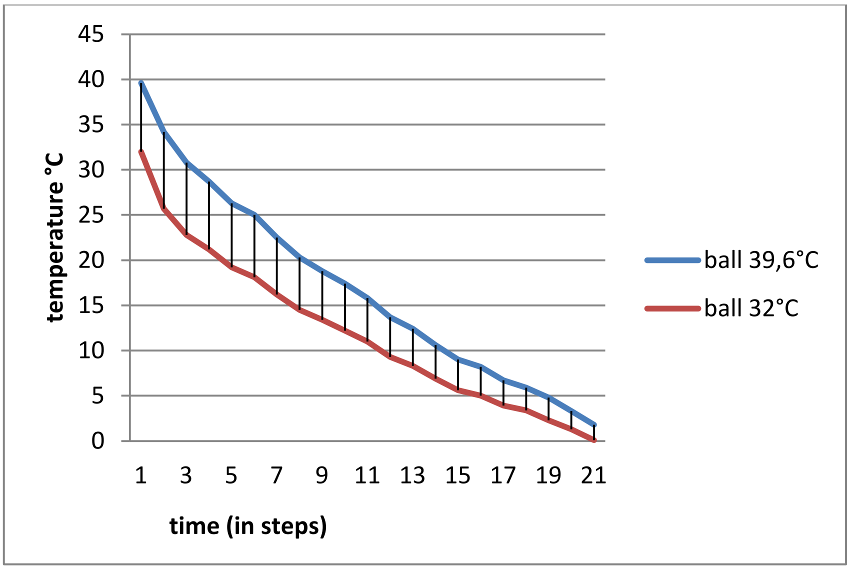


Figure 7

Experiment 2. Take two aluminium cylinders with 20mm diameter. Thermal detectors are put into 24mm hollows in each cylinder. The surrounding temperature is -16,4°C. The measurement step is 2 minutes. Tab. 2, Figure 8.

NN	T1	T2	NN	T1	T2	NN	T1	T2	NN	T1	T2
1	55,7	42,6	16	-8,5	-8,5	31	-14,2	-13,8	46	-16,4	-16
2	43,5	34	17	-9,6	-9,6	32	-14,2	-13,9	47	-16,5	-16
3	34,7	26,7	18	-10,5	-10,4	33	-14,2	-13,9	48	-16,5	-16
4	26,4	20,3	19	-11,2	-11	34	-14,2	-13,9	49	-16,5	-16
5	20,2	15,5	20	-11,9	-11,6	35	-14,3	-14	50	-16,5	-16
6	15,1	11,4	21	-12,3	-12,1	36	-14,5	-14,2			
7	10,8	7,9	22	-12,8	-12,5	37	-14,8	-14,5			
8	7	4,6	23	-13,2	-12,8	28	-15,2	-14,8			
9	3,5	1,7	24	-13,4	-13,1	39	-15,4	-14,9			
10	2,1	1	25	-13,6	-13,3	40	-15,8	-15,3			
11	0,8	-0,6	26	-13,8	-13,5	41	-15,9	-15,5			
12	-1,6	-2,7	27	-14	-13,6	42	-16,1	-15,6			
13	-3,8	-4,6	28	-14,1	-13,7	43	-16,2	-15,8			
14	-5,8	-6,2	29	-14,1	-13,8	44	-16,3	-15,9			
15	-7,2	-7,4	30	-14,2	-13,8	45	-16,4	-15,9			

Tab. 2

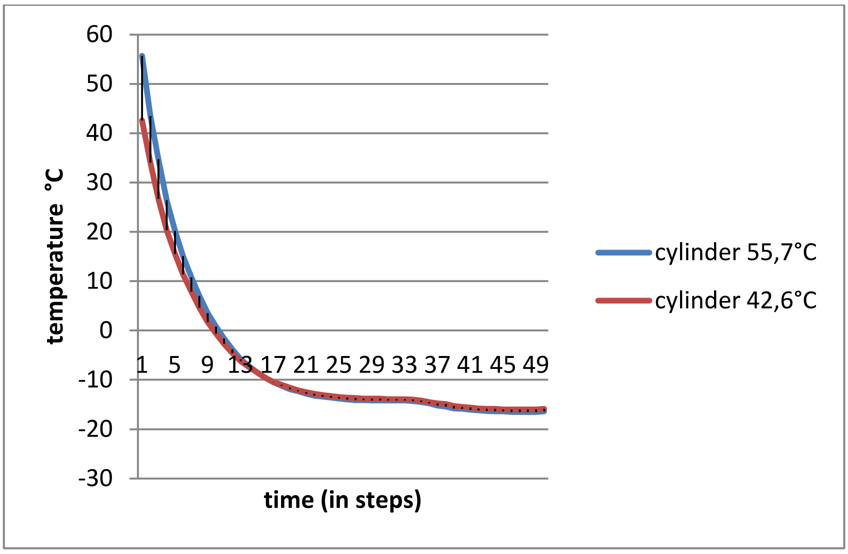


Figure 8

Conclusion

On the plots above we can see the qualitative coincidence between the temperature change in the experiment and the momentum change in the numerical modeling. In the second experiment we can observe the Aristotle-Mpemba effect. This effect should take place under certain conditions no matter what stuff is used.

To obtain a practically useful formula the calculations have to be extended on 3D, also some corrections have to be made. To achieve these goals a set of additional experiments need to be carried out.

Literature

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