Proof of Riemann Hypothesis

Andrey B. Skrypnik (Email - ansk66@mail.ru)

2017 year

Contents

1 Riemann Hypothesis Definition

Definition: There is a pattern in the distribution of primes among the positive integers ($\mathbb{N}$).

2 Riemann Hypothesis Proof Algorithm

2.1 Distinguishing the Sequence of Odd Numbers

First two primes (by condition) are:

$$1, 2. \tag{1}$$

Prime number 2 is significant for dividing the sequence into two equal sequences of even (x) and odd (y) numbers:

$$x \in \{2M \mid M \in \mathbb{N}\}, \tag{2}$$

$$y \in \{2M + 1 \mid M \in \mathbb{N}\}. \tag{3}$$

Starting from $M = 2$ the expression (2) describes the set of composite numbers by condition:

$$x_{comp} \in \{2M \mid M \in \mathbb{N},\ M \geq 2\}. \tag{4}$$

Thus further we will consider the sequence of odd numbers $\{y\}$ (3) to determine the pattern in the distribution of primes (y_o).

2.2 Conclusion 1

The sequence of odd numbers $\{y\}$, except for y_o, also includes the set of composite odd numbers:

$$y_{comp} \in \{y_o y \mid y_o \geq 3,\ y \geq 3\}. \tag{5}$$

Expression (3) without limitations describes the distribution of firsty_o in the sequence of odd numbers within the section from 3 to the first $y_{comp} = 3^2 = 9$.
Let's represent set (3) as the following expression:

$$y_o = 1^2 + 2 \cdot 1 \cdot M_1 + 2 \quad \text{where} \quad M_1 \geq 0. \tag{6}$$

Therefore, this section can be represented in the following way:

$$1^2 < y < 3^2. \tag{7}$$

The following section, where expression (6) for determination of y_o will be limited by exception of the set of composite numbers $\{3y \mid y > 3\}$, will end with the first y_{comp} to which $y_o = 3$ will bear no relation. By definition it is $y_{comp} = 5^2 = 25$. Thus we can conclude the following:
Conclusion 1: All sections compliant with the specific pattern of distribution of y_o are limited by $y_{comp} = y_{on}^2$ and $y_{comp} = y_{o(n+1)}^2$.
Let's analyze the first such section.

2.3 Section [1] $1 < y < 9$

Distribution of y_o is described by expression (6).
Let's calculate first y_o after (1):

$$3, 5, 7. \tag{8}$$

2.4 Section [2] $9 < y < 25$

In order to exclude the composite numbers y_{comp} from the set $\{3y \mid y > 3\}$, $y_o = 1$ in expression (6) shall be replaced by $y_o = 3$ and summand 2 shall be replaced by variable ± 2 to cover all y_o in this section:

$$y_o = 3^2 + 2 \cdot 3 \cdot M_3 \pm 2 = 3^2 + 2(3M_3 \pm 1) \quad \text{where} \quad M_3 \geq 0. \tag{9}$$

Let's calculate next y_o in the sequence:

$$11, 13, 17, 19, 23. \tag{10}$$

2.5 Section [3] $25 < y < 49$

For this section y_o value shall be equal in two expressions - in (9) and in the following expression in order to exclude the composite numbers $\{5y \mid y > 5\}$:

$$\begin{gathered} y_o = 5^2 + 2 \cdot 5 \cdot M_5 \pm 2z_5 = 5^2 + 2(5M_5 \pm z_5), \\ \text{where} \quad M_5 \geq 0, \quad 1 \leq z_5 \leq 2. \end{gathered} \tag{11}$$

Starting from section [2], expression for y_o depends on the value of M_3. According to Conclusion 1 and (9) it is possible to calculate the lower and upper limits for M_3 in any section of $y_{on}^2 < y < y_{o(n+1)}^2$:

$$\frac{y_{on}^2 - 9 \pm 2}{6} \leq M_3 \leq \frac{y_{o(n+1)}^2 - 9 \pm 2}{6}. \tag{12}$$

For this section M_3 value in (9) will change:

$$3 \le M_3 \le 7. \tag{13}$$

Let's compare expressions (9) and (11):

$$3^2 + 2 \cdot 3 \cdot M_3 \pm 2 = 5^2 + 2 \cdot 5 \cdot M_5 \pm 2z_5. \tag{14}$$

Let's express M_5 from expression (14):

$$M_5 = \frac{3M_3 \pm 1 - 8 \mp z_5}{5}. \tag{15}$$

Substitute (15) into (11):

$$\begin{gathered} y_o = 5^2 + 2 \cdot 5 \cdot M_5 \pm 2z_5 = 5^2 + 2\left(5 \cdot \frac{3M_3 \pm 1 - 8 \mp z_5}{5} \pm z_5\right), \\ \text{where} \quad 3 \le M_3 < 7, \quad 1 \le z_5 \le 2, \quad M_5 = \frac{3M_3 \pm 1 - 8 \mp z_5}{5} \in \mathbb{Z}^{\geq}. \end{gathered} \tag{16}$$

Calculate next y_o in section [3]:

$$29, 31, 37, 41, 43, 47. \tag{17}$$

2.6 Conclusion 2

Based on the results of analysis of sections [1], [2], [3] we can conclude the following:
Conclusion 2: Each successive section compliant with the pattern of distribution of y_o depends on the pattern of distribution of y_o in all previous sections starting from [2].
Let's analyze the following section for final determination of the pattern of distribution of y_o in sections $y_{on}^2 < y < y_{o(n+1)}^2$.

2.7 Section [4] $49 < y < 121$

For this section y_o value shall be equal in two expressions - in (16) with different values of variables:

$$7 \le M_3 < 19, \quad 1 \le z_5 \le 2, \quad M_5 = \frac{3M_3 \pm 1 - 8 \mp z_5}{5} \in \mathbb{N}. \tag{18}$$

and in the following expression to exclude the composite numbers y_{comp} from the set $\{7y \mid y > 7\}$:

$$y_o = 7^2 + 2 \cdot 7 \cdot M_7 \pm 2z_7 = 7^2 + 2(7M_7 \pm z_7), \quad \text{where} \quad M_7 \ge 0, \quad 1 \le z_7 \le 3. \tag{19}$$

Let's compare expressions (16) and (19):

$$5^2 + 2\left(5 \cdot \frac{3M_3 \pm 1 - 8 \mp z_5}{5} \pm z_5\right) = 7^2 + 2 \cdot 7 \cdot M_7 \pm 2z_7. \tag{20}$$

Express M_7 from (20):

$$M_7 = \frac{5 \cdot \frac{3M_3 \pm 1 - 8 \mp z_5}{5} \pm z_5 - 12 \mp z_7}{7}. \tag{21}$$

Substitute M_7 from (21) into (19):

$$\begin{gathered} y_o = 7^2 + 2(7 \cdot \frac{5 \cdot \frac{3M_3 \pm 1 - 8 \mp z_5}{5} \pm z_5 - 12 \mp z_7}{7} \pm z_7), \\ \text{where} \quad 7 \le M_3 < 19, \quad 1 \le z_5 \le 2, \quad M_5 = \frac{3M_3 \pm 1 - 8 \mp z_5}{5} \in \mathbb{N}, \quad 1 \le z_7 \le 3, \\ M_7 = \frac{5 \cdot \frac{3M_3 \pm 1 - 8 \mp z_5}{5} \pm z_5 - 12 \mp z_7}{7} \in \mathbb{Z}^{\geq}. \end{gathered} \tag{22}$$

Let's calculate the successive values of y_o in section [4]:

$$53, 59, 61, 67, 71, 73, 79, 89, 97, 101, 103, 107, 109, 113. \tag{23}$$

2.8 General Expression of Distribution of Primes

Thus we can determine the specific patterns comparing expressions (16) and (22).
Let's present the general expression of distribution of y_o in sections [n] $y_{on}^2 < y < y_{o(n+1)}^2$ taking these patterns into consideration:

$$\begin{gathered}
y_o = y_{on}^2 + 2(y_{on}M_{y_{on}} \pm z_{y_{on}}) = y_{on}^2 + 2(y_{on} \cdot \\
\cdot \frac{\frac{y_{o(n-1)} \cdot (\ldots \cdot (5 \cdot \frac{3M_3 \pm 1 - 8 \mp z_5}{5} \pm z_5) - \ldots) \pm \ldots \mp z_{y_{o(n-1)}}}{y_{o(n-1)}} \pm z_{y_{o(n-1)}} - \frac{y_{on}^2 - y_{o(n-1)}^2}{2} \mp z_{y_{on}}}{y_{on}} \pm z_{y_{on}}), \\
\text{where} \quad \frac{y_{on}^2 - 9 \pm 2}{6} \le M_3 < \frac{y_{o(n+1)}^2 - 9 \pm 2}{6}, \quad 1 \le z_{y_o} \le \frac{y_o - 1}{2}, \\
M_{y_{ob}} = \frac{y_{o(b-1)}M_{y_{o(b-1)}} \pm z_{y_{o(b-1)}} - \frac{y_{ob}^2 - y_{o(b-1)}^2}{2} \mp z_{y_{ob}}}{y_{ob}} \in \mathbb{N}, \\
\text{where} \quad 3 < y_{o(b-1)} < y_{ob} < y_{on}, \\
M_{y_{on}} = \frac{y_{o(n-1)}M_{y_{o(n-1)}} \pm z_{y_{o(n-1)}} - \frac{y_{on}^2 - y_{o(n-1)}^2}{2} \mp z_{y_{on}}}{y_{on}} \in \mathbb{Z}^{\ge}.
\end{gathered} \tag{24}$$

In order to form the full sequence of y_o the [n] sections shall be analyzed in sequence. But calculation of y_o from sections to section becomes more difficult. Thus section [3] in expression (16) has 5 variables, section [4] in (22) has 8 variables. But nevertheless, expression (24) unequivocally describes the distribution of y_o in sequence of numbers. If it is necessary to calculate y_o in some section [n], avoiding the previous sections, all $y_o \le y_{o(n+1)}$ from previous calculations shall be known. The required range will be set by summand y_{on}^2 and values of M_3 (12). While solving the problem all $M_3 < M_{y_{ob}} < M_{y_{on}}$ for this section [n] shall be calculated in sequence.

2.9 Final Conclusion

Final Conclusion: Riemann Hypothesis is true. Distribution of primes among the positive integers has its own pattern. But for odd numbers of y the sections compliant with the specific pattern of distribution of primes y_o are limited by composite numbers y_{on}^2 and $y_{o(n+1)}^2$. Distribution of y_o in such sections [n], starting from [3], is calculated according to the expression (24). The full sequence of y_o is achieved by consequent analysis of sections [n], starting from [1] $1^2 < y < 3^2$.

Publications: http://samlib.ru/editors/b/bezymjannyj_a/w6.shtml