The Strong Goldbach Conjecture, Klein Bottle And Möbius Strip

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Abstract

This modest article shows the connection between the strong Goldbach conjecture and the topological properties of the Klein bottle and the Möbius strip. This connection is established by functions derived from the number of divisors of the two odd integers whose sum is an even number.

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I. INTRODUCTION

In number theory, the Goldbach conjecture is one of the oldest open problems in mathematics. Sometimes it is described as the most difficult problem in the history of this science. Specifically, G.H. Hardy in 1921 in his famous speech at the Copenhagen Mathematical Society commented that probably the Goldbach conjecture is not only one of the most difficult unresolved problems of number theory, but of all mathematics. Its statement is the following: Any even number greater than 2 can be written as the sum of two prime numbers.

This conjecture had been known by Descartes.The following statement is equivalent to the previous one and is the one originally conjectured in a letter from Goldbach to Euler in 1742:

Every integer greater than five can be written as the sum of three prime numbers.

This conjecture has been investigated by many number theorists and has been checked by computers for all even numbers less than 4 x 10E8. Most mathematicians believe that the conjecture is true, and rely mostly on statistical considerations about the probabilistic distribution of prime numbers in the set of natural numbers: the larger the even integer, the more likely it can be written as the sum of two prime numbers.

In spite of this, the natural numbers are infinite and therefore having demonstrated the conjecture for 4 x 10E8 numbers is not enough since this is only a very small part of the set of numbers.

We know that every even number can be written minimally as a sum of at most six prime numbers. As a result of a work by Vinogradav, any sufficiently large even number can be written as a sum of at most four prime numbers. In addition, Vinográdov showed that almost all even numbers can be written as the sum of two prime numbers (in the sense that the proportion of even numbers that can be written in that way tends to 1). In 1966, Chen Jing-run showed that any even large enough number can be written as a sum of a prime and a number that has at most two prime factors.

In order to generate publicity for the book The Uncle Petros and the Goldbach Conjecture of Apostolos Doxiadis, the British publisher Tony Faber offered in 2000 a prize of one million dollars to that English-speaker who demonstrated the conjecture before April 2002. Nobody he claimed the prize.

Goldbach made two related conjectures about the sum of prime numbers: Goldbach's 'strong' conjecture and Goldbach's 'weak' conjecture. The one discussed here is the strong one, and it is what is often referred to as the "Goldbach conjecture".

Much work has been done on the weak conjecture, culminating in 2013 in a vindication of the Peruvian mathematician Harald Helfgott on his complete demonstration.

II. TOPOLOGICAL PROPERTIES OF KLEIN BOTTLE AND MÖBIUS STRIP

The main topological properties of the klein bottle immersed in three dimensions, and the Möbius strip are the following:

Klein Bottle

Like the Möbius strip, the Klein bottle is a two-dimensional manifold which is not orientable. Unlike the Möbius strip, the Klein bottle is a closed manifold, meaning it is a compact manifold without boundary. While the Möbius strip can be embedded in three-dimensional Euclidean space R3, the Klein bottle cannot. It can be embedded in R4, however.

The Klein bottle is a closed nonorientable surface of Euler characteristic 0 that has no inside or outside, originally described by Felix Klein. It can be constructed by gluing both pairs of opposite edges of a rectangle together giving one pair a half-twist, but can be physically realized only in four dimensions, since it must pass through itself without the presence of a hole.

1) It is a non-orientable closed surface of Euler characteristic equal to 0

$\chi(Kb) = 0 = 2 - g$

Where Kb symbolizes the Klein bottle. And g, is the genre of the surface. Being for Klein's bottle g = 2

2) Dissection.

Dissecting a Klein bottle into halves along its plane of symmetry results in two mirror image Möbius strips, i.e. one with a left-handed half-twist and the other with a right-handed half-twist.

3) Color.

Six colors suffice to color any map on the surface of a Klein bottle; this is the only exception to the Heawood conjecture, a generalization of the four color theorem, which would require seven.

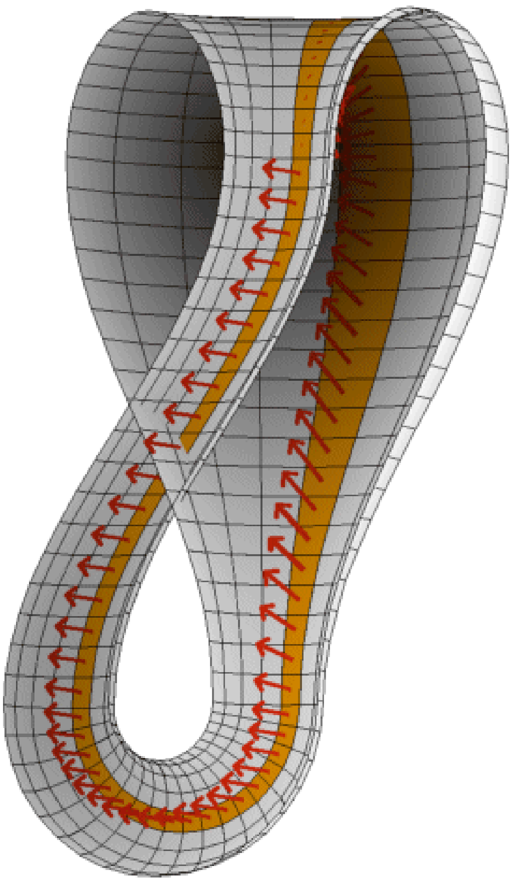


Figure 1:

Möbius Strip

1) The Möbius strip, also called the twisted cylinder, is a one-sided nonorientable surface obtained by cutting a closed band into a single strip, giving one of the two ends thus produced a half twist, and then reattaching the two ends. But can also be cut into a single Möbius strip (Gardner 1984, pp. 14 and 17)

2) The Möbius strip has Euler characteristic $\chi(Ms) = 0$ Where Ms symbolizes Möbius strip

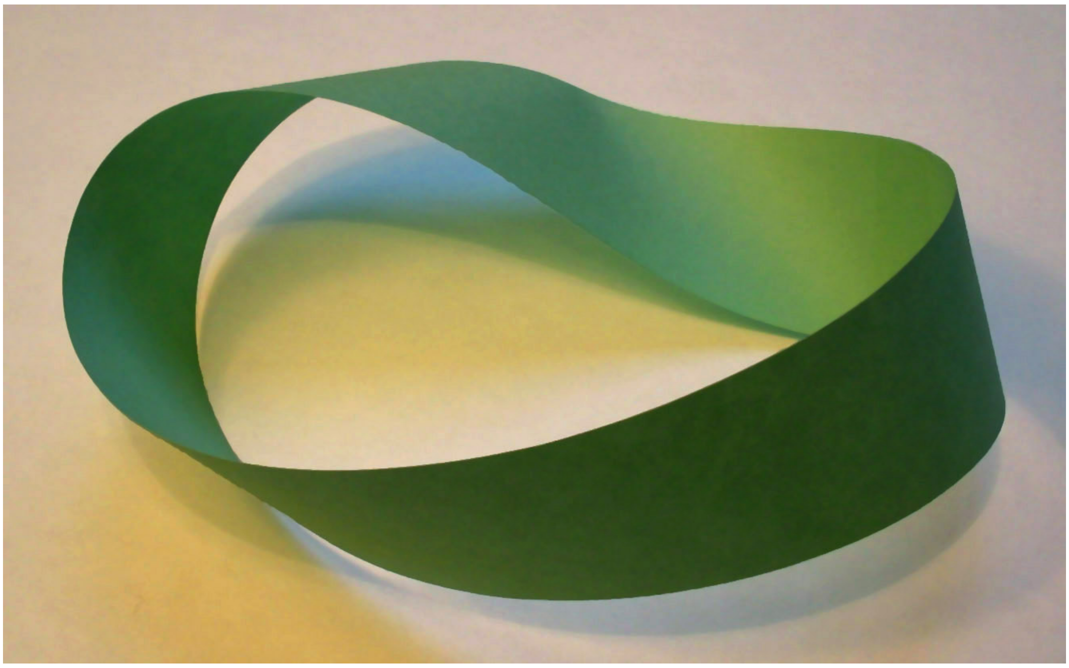
3) Color

Any set of regions on the Möbius strip can be colored using only six colors.

From the above characteristics, the following **common properties of the Klein bottle and the Möbius strip** are derived:

1) Euler characteristic.

$\chi(Kb) = \chi(Ms) = 0$

Figure 2: 

2) A single surface (surface and side for the Möbius strip): 1

3) Minimum number of colors needed to color the Klein bottle and the Möbius strip (without two regions with a common border having the same color): 6 colors.

III. CONNECTING THE STRONG GOLDBACH CONJECTURE AND THE COMMON FUNDAMENTAL PROPERTIES OF THE KLEIN BOTTLE AND THE MÖBIUS STRIP

A. Choice of the appropriate infinite set of Klein bottles immersed in three dimensions

Be the infinite set Klein's bottles, whose dissection by its axis of symmetry generates two Möbius strips whose surface (when converted into normal strips), is equal to a strip of smaller side, unit, and of greater side equal to a even integer. In the following figure, the two Möbius strips converted into two rectangles are shown; with a surface equal to side unit x side equal to an even number.

This infinite set of Klein bottles will be reduced to the infinite subset of Klein bottles whose dissection along its axis of symmetry generates two Möbius strips; which, when we convert them into two rectangular strips, we divide them into two rectangles (two for each strip); so that one of them always has the surface equal to an odd prime number, with a smaller unit side. This is seen in Figure 3 .

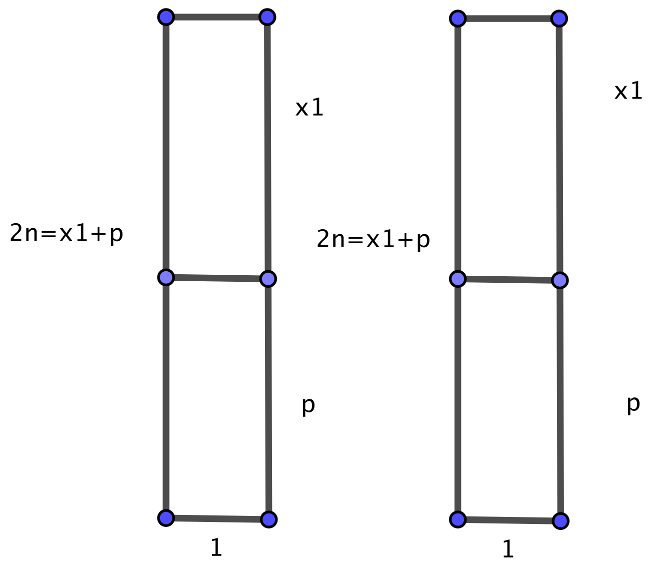


Figure 3:

Where, x1 is an odd number, and, p is a prime number.

B. Algorithm of verification of prime numbers and constructibility of the two Möbius strips of the Klein bottle dissected by its axis of symmetry.

An algorithm will be used to verify that, x1, is a prime number or it is not; and that also allows the construction of the surface of the two Möbius's strips (equal surfaces).

An algorithm that allows the verification of primality or not, of an odd number; and at the same time build surfaces (Infinite subset of Klein bottles specified in section A) , it is as follows: Be an odd number. Let be the integer part of the square root of this number. We divide this number by each prime number $p \leq \left\lfloor \sqrt{2n-p=x_1} \right\rfloor$, $p \geq 3$

The algorithm ends when the divisibility is verified. If the number is prime, the rectangular surface (strip) is constructed as: $x_1 = p \times 1$. If the number is not prime, the surface is constructed as: $x_1 = x_2 \times p$

At this point; you can duplicate the surfaces or continue with an algorithm that defines the generation of the second surface that must have a unitary side, one side an even number, and be divided into two rectangles with the same prime number, p, and equal x_1

If you choose to finish the algorithm with the simple duplication of the surfaces; then it follows automatically that only when the even number is the sum of two odd prime numbers; the Klein bottle generated belongs to the set of Klein bottles specified above.

C. Functions of the number of divisors of the two prime numbers whose sum is an even number and its equivalence with the main topological properties of the Klein bottle.

Be an even number greater than 4. Let the number of divisors of a number, with $d > 1$: $\sigma_0(n) = \sum\limits_{d>1|n} d^0$

Lemma 1. *Only when an even number (greater than 4) can be expressed as the sum of two prime numbers, is it met:* $\left(\sum\limits_{d>1|p_1} d^0\right) - \left(\sum\limits_{d>1|p_2} d^0\right) = \chi(Ms) = \chi(kb) = 0 \,; 2n = p_1 + p_2$

Proof. If only one of the numbers is prime, you have to at least: $2n = \overline{p_1} + p_2$; $\left(\sum\limits_{d>1|\overline{p_1}} d^0\right) - \left(\sum\limits_{d>1|p_2} d^0\right) = 3 - 1 = 2$ Example: 32=15+17; $\left(\sum\limits_{d>1|15} d^0\right) = 3$; $\left(\sum\limits_{d>1|17} d^0\right) = 1$ And only when the two numbers are prime numbers is it met: $\left(\sum\limits_{d>1|p_1} d^0\right) - \left(\sum\limits_{d>1|p_2} d^0\right) = 1 - 1 = \chi(Ms) = \chi(kb) = 0$ □

Lemma 2. *Unsing the lemma 1, it follows automatically that only when an even number is expressed as the sum of two prime numbers, is it met: a single surface,* $1 = \left(\sum\limits_{d>1|p_1} d^0\right) \cdot \left(\sum\limits_{d>1|p_2} d^0\right)$

Lemma 3. *Be the function; of the product of the prime number p, and of the number* $2n - p = x_1$ *;defined by:* $\sum\limits_{d>1|x_1 \cdot p_1} d^0$ *. Only when 2n can be expressed as the sum of two prime numbers (* $2n = x_1 + p_1$; $x_1 = p_2$ *); you can color any Möbius strip and Klein bottle with a minimum of six colors; applying the following algorithm: Be the three divisors; greater* *than unity; of the product of the two prime numbers whose sum is an even number. We define a color as the amount of imaginary paint obtained by the convention of three primary colors (red, blue and green); in such a way that a color-tone is obtained as the mixture of the permutations of quantity of paint of the three primary colors. The following figure shows the complete algorithm to obtain the 6 minimum colors needed to color a Möbius strip and a Klein bottle, and function of the divisors of the number* $p_1 \cdot p_2$ *; as follows: Number of permutations of* $\sum_{d>1|p_1 \cdot p_2} d^0$; $P\left(\sum_{d>1|p_1 \cdot p_2} d^0\right) = 6$

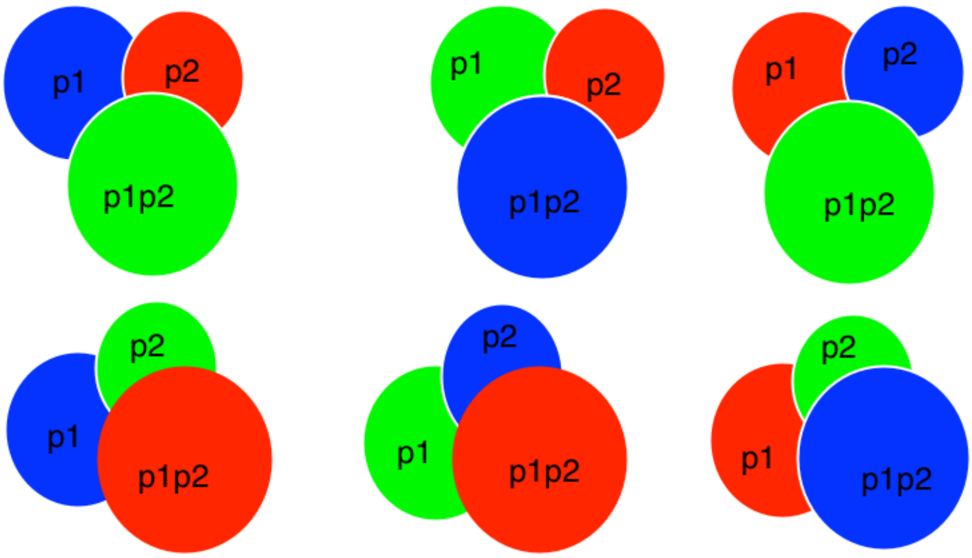


Figure 4:

D. Duplication algorithm of the surfaces to be included in the algorithm of section B

As we mentioned in section B, we can include a selective algorithm to duplicate the equal surfaces (surface and unit sides) that is a function of the prime factors of x1 and at the same time depends on the number of divisors of numbers p and x1, whose sum is an even number 2n

This algorithm will be the following: Let the number of divisors function, minus one unit (the factor 1), of the numbers p and x1 (whose sum is the number 2n). Be the sum $\left(\sum_{d>1|p_1} d^0\right) + \left(\sum_{d>1|x_1} d^0\right)$

If the number x1 is prime, according to the algorithm of section B, then a number of equivalent surfaces are generated, to the number of permutations of the previous function. This is: permutations $\left[\left(\sum_{d>1|x_1=p_2} d^0\right)+\left(\sum_{d>1|p_1} d^0\right)\right]=2\rightarrow 2$ equal surfaces of unitary minor side, and greater side the even number; whose sum are two prime numbers.

If on the other hand, the number x1 is not prime; then two surfaces are constructed as: A side surface $x_1 \times 1$. A second surface of sides $x_2 \times p$

With the previous selective algorithm, it is evident that it is not possible to build a Klein bottle whose dissection along its axis of symmetry are two unitary side Möbius strips; since in the case that x_1 is not a prime number, there are two different Möbius strips. And these two different Möbius strips $(p_1 \times 1, x_2 \times p)$ can not generate the required Klein bottle.

If x1 is not a prime, then: $permutations \left[\left(\sum_{d>1|x_1=x_2\times p} d^0\right)+\left(\sum_{d>1|p_1} d^0\right)\right]>2$

Theorem 1. *The application of lemmas 1,2 and 3, together with the selective algorithm of section D; implies that only when an even number (greater than 4) can be expressed as the sum of two prime numbers, then it is possible to generate a bottle of Klein from the set specified in section A.*

Acknowledgments

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