Refutation of Liouville's theorem as not invertible

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We assume Meth8/VŁ4 where the designated proof value is tautology. The truth table is repeating fragments of 16-values, row major and horizontal. LET pqtw ABRTW; \( \rightarrow \), transition

We rely on: inside.mines.edu/~tohno/teaching/PH505_2011/liouville_dvorak.pdf

Allow \( W(A) \) to denote the phase volume of macrostate \( A \), i.e. \( W(A) \) is the number of microstates that realize macrostate \( A \); we can immediately conclude that \( W(\overset{\text{T}}{A}) = W(A) \). (1.1)

\[
((w\&p)\Rightarrow p) > ((w\&(r\&p)) = (w\&p)) ; \quad TTTT\ TTTT\ TTTT\ TTTT\ TTTT\ TTTT\ TTTT\ TTTT
\]

Consider two distinct macrostates \( A \) and \( B \) in the same phase space. Let \( \Gamma \) denote the microscopic path through phase space that realizes the macroscopic transition \( A \rightarrow B \). Denote the transformed macrostate \( A \) as \( \overset{\text{T}}{A} \) for time evolved \( A \). Liouville’s theorem preserves phase space volumes. Therefore, \( W(\overset{\text{T}}{A}) = W(A) \). We now consider only cases where the transition \( A \rightarrow B \) is experimentally reproducible. For [Figure 3: Evolution of macrostates in a dynamical system.] this to be true, \( \overset{\text{T}}{A} \) must lie entirely in \( B \). We cannot control which microstate the system evolves into, but we require that all evolved microstates \( \overset{\text{T}}{A} \) are a subset of \( B \). This condition implies that \( W(\overset{\text{T}}{A}) < W(B) \). The number of microstates for macrostate \( B \) is greater than that of macrostate \( A \). But Liouville’s theorem tells us \( W(\overset{\text{T}}{A}) = W(A) \), so experimental reproducibility of \( A \rightarrow B \) means that \( W(A) < W(B) \). This condition depends only on the initial configuration of the system because phase space volume is conserved. This is the requirement for experimental reproducibility and one explanation for entropy, \( S/\ln W \). (2.1)

\[
(((w\&(t\&p))=(w\&q)) & ((w\&(t\&p)) < (w\&p))) > ((p>q)=(w\&p) < (w\&q)) ;
\quad TTTT\ TTTT\ TTTT\ TTTT\ TTTT\ TTTT\ TTTT\ TTTT
\]

Consider the reverse transition: why does macrostate \( B \) not evolve into \( A \). This is equivalent to the transition \( \overset{\text{RB}}{B} \rightarrow \overset{\text{RA}}{A} \). This transition requires additional information about the initial microstate of \( \overset{\text{RB}}{B} \) to transform it into the proper sub-region of \( \overset{\text{RA}}{A} \) - information we don’t typically have. Because \( W(\overset{\text{RA}}{A}) < W(\overset{\text{RB}}{B}) \), this transformation is not experimentally reproducible. Liouville’s theorem connects the time evolved state to the initial state - their phase space volume are the same. Therefore, Liouville’s theorem places the requirement for experimental reproducibility (second law) on initial and final states \( S(A) < S(B) \). Interestingly, nowhere does any notion of time enter this argument. In this derivation, increasing entropy is a requirement only for experimental reproducibility, not a forward direction in time.

\[
(w\&(r\&p)) < (w\&(r\&q)) > \sim ((q>p) = ((r\&q) >(r\&p))) ;
\quad TTTT\ TTTT\ TTTT\ TTTT\ TTTT\ TTTT\ TTTT\ TTTT\ TTTT\ TTTT\ TTTT\ TTTT\ TTTT\ TTTT\ TTTT\ TTTT
\]

Eq. 1.2 is not tautologous: it is not a theorem. Eq. 2.2 is tautologous: it is a constructive theorem.

However, Eq. 3.2 is not tautologous: as the reverse of Eq.2.2, it is not a theorem. This means Liouville's theorem is not invertive and hence not a reversible theorem.