# Limiting Vectors

*Walter B. Gress V*

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The limit operation applied directly to vectors.

## 1. INTRODUCTION

This paper presents the novel idea of applying limits to vectors themselves.

## 2. THEOREM 1.0

$\lim_{x \to \infty} \boldsymbol{x} = \boldsymbol{q}$. Just as a vector is said to represent direction, the limit of a vector in 2-space is said to be traveling. The limit of **x** is traveling towards **q** where **q** is of magnitude infinity. We call this a limiting vector.

## 3. PROOF 1.0

Where u=v=w ∈ $\mathbb{Z}$ and are scalar values, and **x** = $u\hat{\imath} + v\hat{\jmath} + w\hat{k}$

$$\lim_{h \to q} \boldsymbol{x} = (u+h)\hat{\imath} + (v+h)\hat{\jmath} + \left(w + h\hat{k}\right)$$
$$= \boldsymbol{q}$$

For each increment value of h the vector grows by h until it reaches **q.** Let the vector q be the largest vector, the vector limit, by the following definition:

Let **q =** $\lim_{a,b,c \to \infty} < a\boldsymbol{i}, b\boldsymbol{j}, c\boldsymbol{k} >$

||**q**|| = $\lim_{a,b,c \to \infty} \sqrt{a^2 + b^2 + c^2}$

## 4. THEOREM 2.0

When we add to limiting vectors in 2-space we say the vectors are *sweeping* across a plane and form a quarter plane.

## 5. PROOF THEOREM 2.0

Assume **x** and **y** are limiting vectors. By (1.0) Let **q** and **r** be vector limits.

$$\lim_{x \to q} \boldsymbol{x} + \lim_{x \to r} \boldsymbol{y}$$

Properties of Vectors:
Vectors are added head to tail. (1)
Vectors have an origin (2)

The addition of two limiting vectors forms a quadrant. Take for example, the vectors **x** and **y.** Let **x =** <u, $\hat{\imath}$> and let **y=**<q,$\hat{r}$>. Take the limits

$$\lim_{x \to q} \boldsymbol{x} + \lim_{x \to r} \boldsymbol{y}$$

For every real number u the vector **x** extends in the x direction. For every real number q the vector extends in the y direction. The result fills the upper-right quadrant of $\mathbb{R}^2$.

## 6. THEOREM 3.0

When we perform the cross product over limiting vectors in 3-space, we fill a three-dimensional octant. Essentially quarter planes stacked on top of each other.

## 7. PROOF THEOREM 3.0

The two limited vectors **a** and **b** cross product forms the limited vector **c.** With each increment unit vector **a** and **b** sweep across the quadrant in two-dimensions and vector **c** repeats this quadrant in the perpendicular direction of vectors **a** and **b,** forming the octant..

## 8. THEOREM 4.0

Geometric form of the dot product:

$$\mathbf{a} \cdot \mathbf{b} = \|\mathbf{a}\|\ \|\mathbf{b}\| \cos(\theta),$$

Magnitude of a and b grows while cos of theta remains fixed.

The geometric form of the dot product is as follows:

$$\angle a, b = \arccos \frac{a \cdot b}{||a||||b||}$$

The fraction of the dot product and its magnitudes will change over the course of the limit. The denominator grows gradually, while the numerator grows gradually as well. The ratio gradually converges towards 1.The angle however gradually converges to 0. This is the property we are looking for.

$\lim_{\boldsymbol{a} \to \boldsymbol{q}} \boldsymbol{a} \lim_{\boldsymbol{b} \to \boldsymbol{r}} \boldsymbol{b} \angle a, b = \arccos \frac{a \cdot b}{||a||||b||}$ approaches 0°. It can be inferred that as the limited vectors iterate towards infinity, they grow infinitely close to each other.

## 9. PROOOF THEOREM 4.0

Let **a** and **b** be limiting vectors.
Let $\boldsymbol{a} \cdot \boldsymbol{b} < ||\boldsymbol{a}||||\boldsymbol{b}||$ approaching 1.
By the nature of the arc cos, arc cos is asymptotic at
1 and decreases in degrees as approached.
Therefore, the dot product of the limiting vector converges on 0°.
Better proof?

## 10. THEOREM 5.0

The scalar projection of one vector onto another is

$$a_b = \|\mathbf{a}\| \cos\theta,$$

Where

where θ is the angle between a and b.

If we fix cos ($\vartheta$) and take the limit of a, we result in a sliding effect, where the projection of **a** on to **b** changes in the following manner. The perpendicular drawn from the tip of **a** down to **b** slides along upwards along **b** as **a** grows.

## 11. PROOF 5.0

Let **a** and **b** be limiting vectors. For every iteration of **a**, the perpendicular drawn from **a** to **b** is dropped from the head of **a** perpendicularly down to **b. b** is fixed, so as the vector **a** grows, the perpendicular moves in that direction.