# Logical Errors of Special Relativity

by

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**Abstract**

In this paper, a fundamental logical error of special relativity is exposed. It is shown that the lengths measured by clocks do not differ between two systems moving relative to each other, in strict derivation from the concept of special relativity itself. Furthermore, it is shown that special relativity de facto relates two independent events and derives from this the loss of simultaneity.

## 1. Introduction

Most laymen, but also more and more experts, confronted with the ideas of special relativity have a quiet feeling of lack of logical reason about the basic structure of the theory. In particular, the assertion of the invariance of the speed of light and the equivalence of the relative uniformity of motion are under suspicion. Although mathematics and the limited capacity for abstraction make a complete understanding of special relativity difficult, we will point out the basic fallacy at the outset. We will strictly follow the original work of Albert Einstein and try to visualize the mathematical assumptions for clarification.

## 2. Clock Synchronisation

We first cite from “On the Electrodynamics of Moving Bodies”, Chapter A,1.:

> If there is a clock at point $A$ in space, then an observer located at $A$ can evaluate the time of events in the immediate vicinity of $A$ by finding the positions of the hands of the clock that are simultaneous with these events. If there is another clock at point $B$ that in all respects resembles the one at $A$, then the time of events in the immediate vicinity of $B$ can be evaluated by an observer at $B$. But it is not possible to compare the time of an event at $A$ with one at $B$ without a further stipulation. So far we have defined only an “$A$-time” and a “$B$-time,” but not a common “time” for $A$ and $B$. The latter can now be determined by establishing *by definition* that the “time” required for light to travel from $A$ to $B$ is equal to the “time” it requires to travel from $B$ to $A$. For, suppose a ray of light leaves from $A$ for $B$ at “$A$-time” $t_A$, is reflected from $B$ toward $A$ at “$B$-time” $t_B$, and arrives back at $A$ at “$A$-time” $t'_A$. The two clocks are synchronous by definition if
>
> $$t_B - t_A = t'_A - t_B.$$
>
> We assume that it is possible for this definition of synchronism to be free of contradictions, and to be so for arbitrarily

Excerpt 1: Synchronization of two clocks

We now illustrate the principle of synchronization of two clocks in **A** and **B** in a reference system at rest which strictly follows the description of special relativity. To be consistent with the text, we have chosen to denote the system at rest as **S'**. The lightray is orange, the rod is black, and the clocks involved, which read the time at each instant, are highlighted in red:

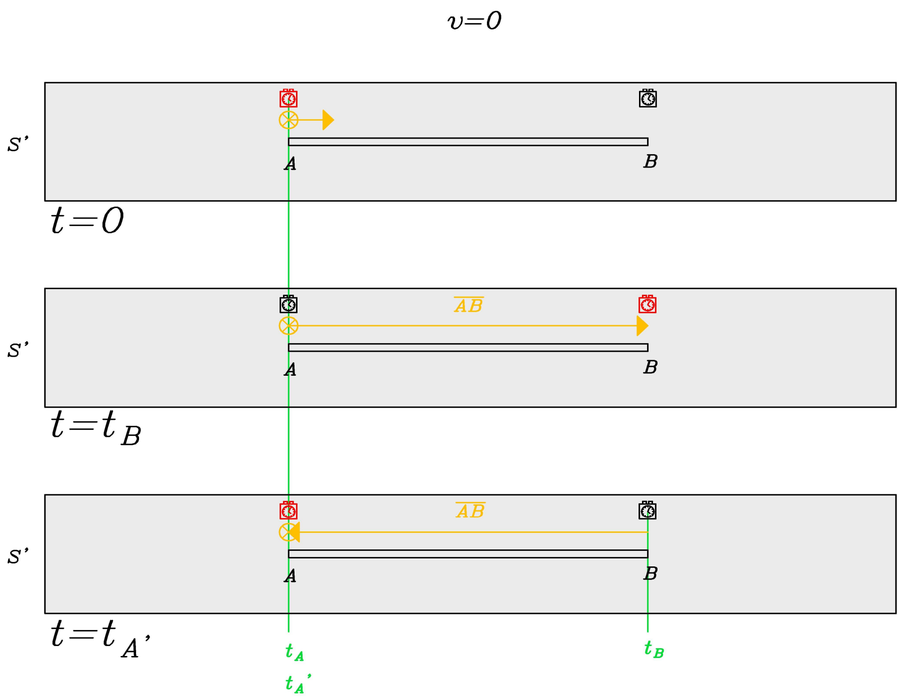


Fig. 1: Synchronization of two clocks, so far within a stationary reference system. Light is going forth and back, and synchronicity of the two clocks is defined by equal runtimes of both lightrays

Again we cite Special Relativity regarding synchronization of additional clocks:

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many points, and therefore that the following relations are generally valid:

1. If the clock at $B$ runs synchronously with the clock at $A$, the clock at $A$ runs synchronously with the clock at $B$.
2. If the clock at $A$ runs synchronously with the clock at $B$ as well as with the clock at $C$, then the clocks at $B$ and $C$ also run synchronously relative to each other.

Excerpt 2: Synchronization of two more clocks

We now imagine to have two more clocks at **C** and **D** and we again synchronize such clocks with **B** and therefore according to Special Relativity also with **A**. Positions of **C** and **D** appear random and we will show its purpose later:

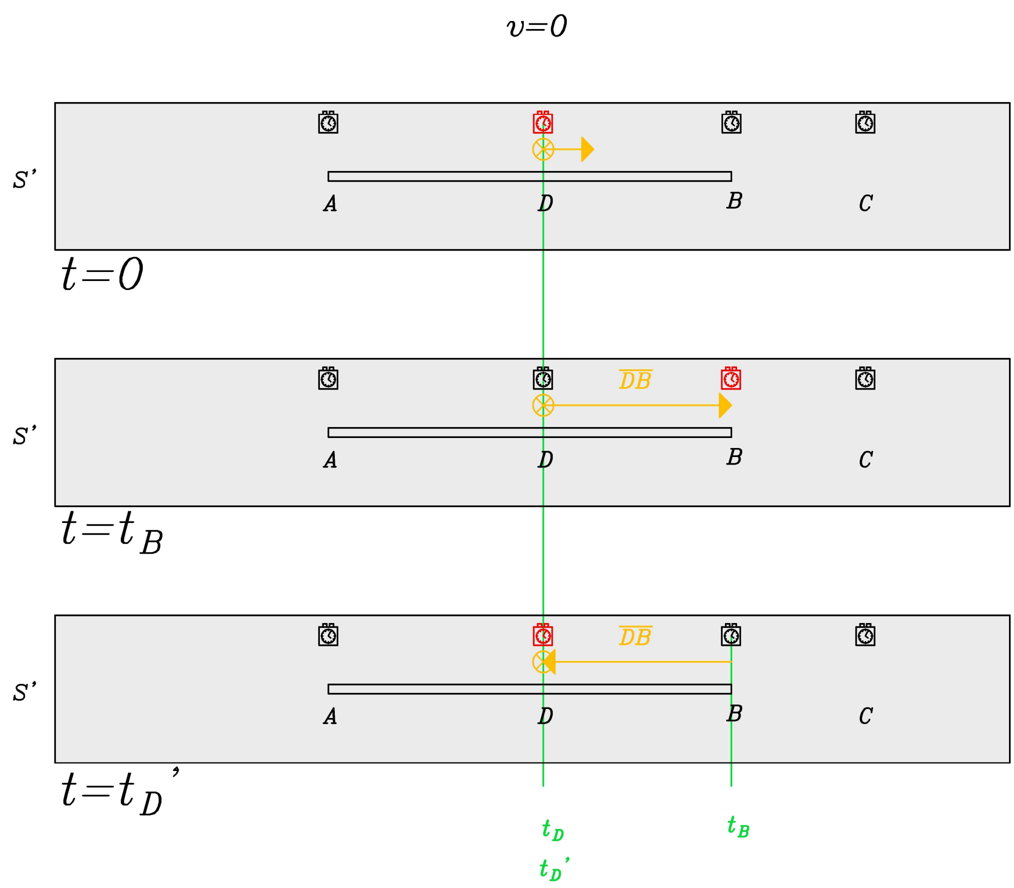


Fig. 2: Synchronization of two more clocks, still within a stationary system. Clock D is synchronized utilizing the same procedure as per synchronization of clocks A and B before. Clocks A, B, C and D are synchronized eventually

Strictly according to Special Relativity we now have four synchronized clocks in **A**, **B**, **C** and **D**.

## 3. Length Measurement Methods

Special Relativity is proposing two different ways of length measurement. The first method is to simply have a scale and measure the length **l** of a rod from within any moving reference system **S**:

Take a rigid rod at rest; let its length, measured by a measuring rod that is also at rest, be $l$. Now imagine the axis of the rod placed along the $X$-axis of the rest coordinate system, and the rod then set into uniform parallel translational motion (with velocity $v$) along the $X$-axis in the direction of increasing $x$. We now inquire about the length of the *moving* rod, which we imagine to be ascertained by the following two operations:

a. The observer moves together with the aforementioned measuring rod and the rigid rod to be measured, and measures the length of the rod by laying out the measuring rod in the same way as if the rod to be measured, the observer, and the measuring rod were all at rest.

Excerpt 3: Measurement using scale(operation a.)

Again the visualization for this, measurement of the rod length **l** from inside the moving system **S** (bluish). The rod is blue, the measuring scale green:

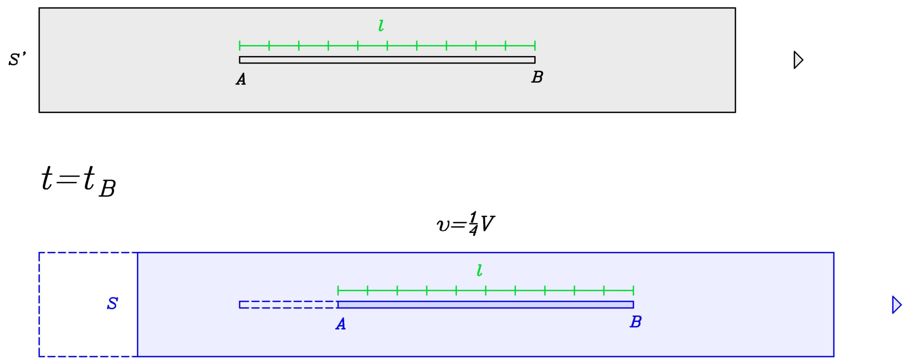


Fig. 3: Measurement using a scale (operation a.). System S' is stationary, whereby system S is moving to the right, dragging the scale along and showing no different measurement (seen from the co-moving observer) compared to S'

The second method is measuring the length of the rod $r_{AB}$ by means of the position of the two ends of the rod at a given time t, to be measured with synchronized clocks from a stationary system $S'$:

b. Using clocks at rest and synchronous in the rest system as outlined in section 1, the observer determines at which points of the rest system the beginning and end of the rod to be measured are located at some given time $t$. The distance between these two points, measured with the rod used before—but now at rest—is also a length that we can call the "length of the rod."

Excerpt 4: Measurement using clocks (operation b.)

It should be emphasized that so far we are concerned only with the measurement of a rigid rod which is in relative motion, once from the moving system **S** (bluish), once from the stationary system **S'** (gray). We are not concerned with measuring the length of any light beam. It must also be emphasized that the measurement of the moving rod from the stationary system **S'** is made within an instant, as if the two clocks were stopwatches (measurement of the rod with a scale according to special relativity "at some given time **t**", i.e. instant). The clocks involved in the measurement are highlighted in red:

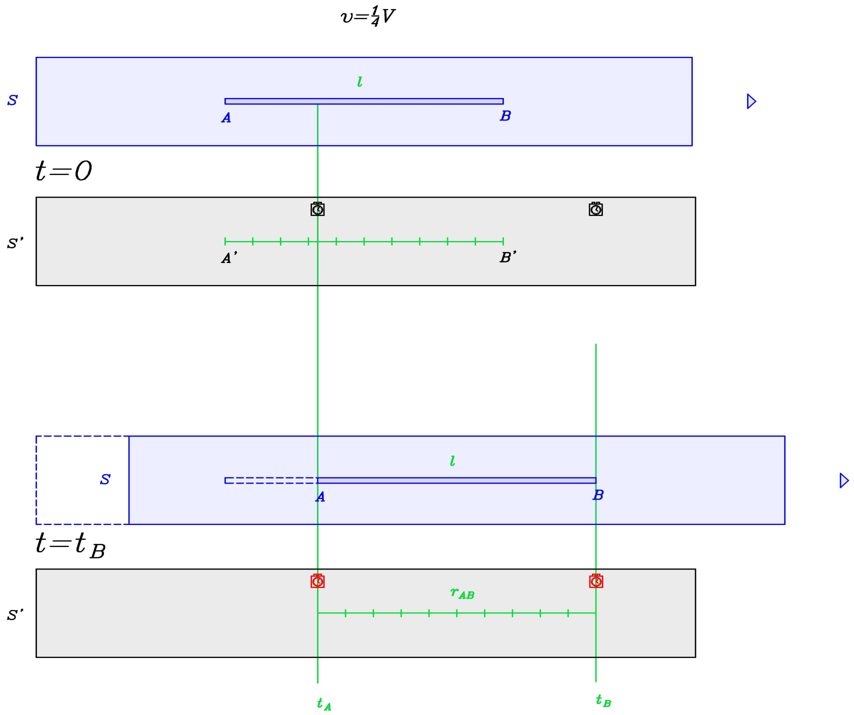


Fig. 4: Measurement using clocks in stationary system (operation b.). Top: S starts moving. Bottom: The moving rod in S is being measured by the stationary observer in S' by means of two stationary clocks, being already synchronized according to the aforementioned procedure. Readings of both clocks being highlighted in red take place at one and the same instant.

Now special relativity claims that it will show that the two measurement methods give different lengths for the rod. It also claims that classical physics assumes that the two measurement methods must be the same. We will prove the opposite, that they are not the same according to classical physics, but according to special relativity.

The length determined using operation (b), which we shall call "the length of the (moving) rod in the rest system," will be determined on the basis of our two principles, and we shall find that it differs from $l$.

Current kinematics tacitly assumes that the lengths determined by the above two operations are exactly equal to each other, or, in other words, that at the time $t$ a moving rigid body is totally replaceable, in geometric respects, by the *same* body when it is *at rest* in a particular position.

Excerpt 5: Differing lengths in moving system

## 4. Non- Simultaneity within two reference systems moving relative to each other

Now Special Relativity introduces a lightray that is traveling along the moving rod towards a mirror from which it is being reflected back.

> Further, we imagine the two ends ($A$ and $B$) of the rod equipped with clocks that are synchronous with the clocks of the rest system, i.e., whose readings always correspond to the "time of the system at rest" at the locations the clocks happen to occupy; hence, these clocks are "synchronous in the rest system."
>
> We further imagine that each clock has an observer co-moving with it, and that these observers apply to the two clocks the criterion for the synchronous rate of two clocks formulated in section 1. Let a ray of light start out from $A$ at time[2] $t_A$; it is reflected from $B$ at time $t_B$, and arrives back at $A$ at time $t'_A$. Taking into account the principle of the constancy of the velocity of light, we find that
>
> $$t_B - t_A = \frac{r_{AB}}{V - v}$$

Excerpt 6: Measurement using clocks in moving system

The difference **$\Delta t_1$ ($t_B$-$t_A$)** and **$\Delta t_2$ ($t_A$'-$t_B$)** is computed strictly according to classic speed addition (rather than on the assumption of "the principle of the constancy of the velocity of light"), as shown by the following visualization.

The source of the lightray (orange) travels together with the moving system **S** (bluish), the traveling length of the ray is being measured by classic speed addition, i.e. light travels from **A** to the right with speed **V** while the rod (blue) is traveling to the right with speed **v** together with the moving system. According to classic physics (and to Special Relativity) movement of the source is irrelevant.

The ray then hits the mirror on the moving rod in **B**. Special Relativity has not mentioned at this how the measurement is done at the stationary frame at this same instant, and we show the stationary system with its measuring scale for clarification.

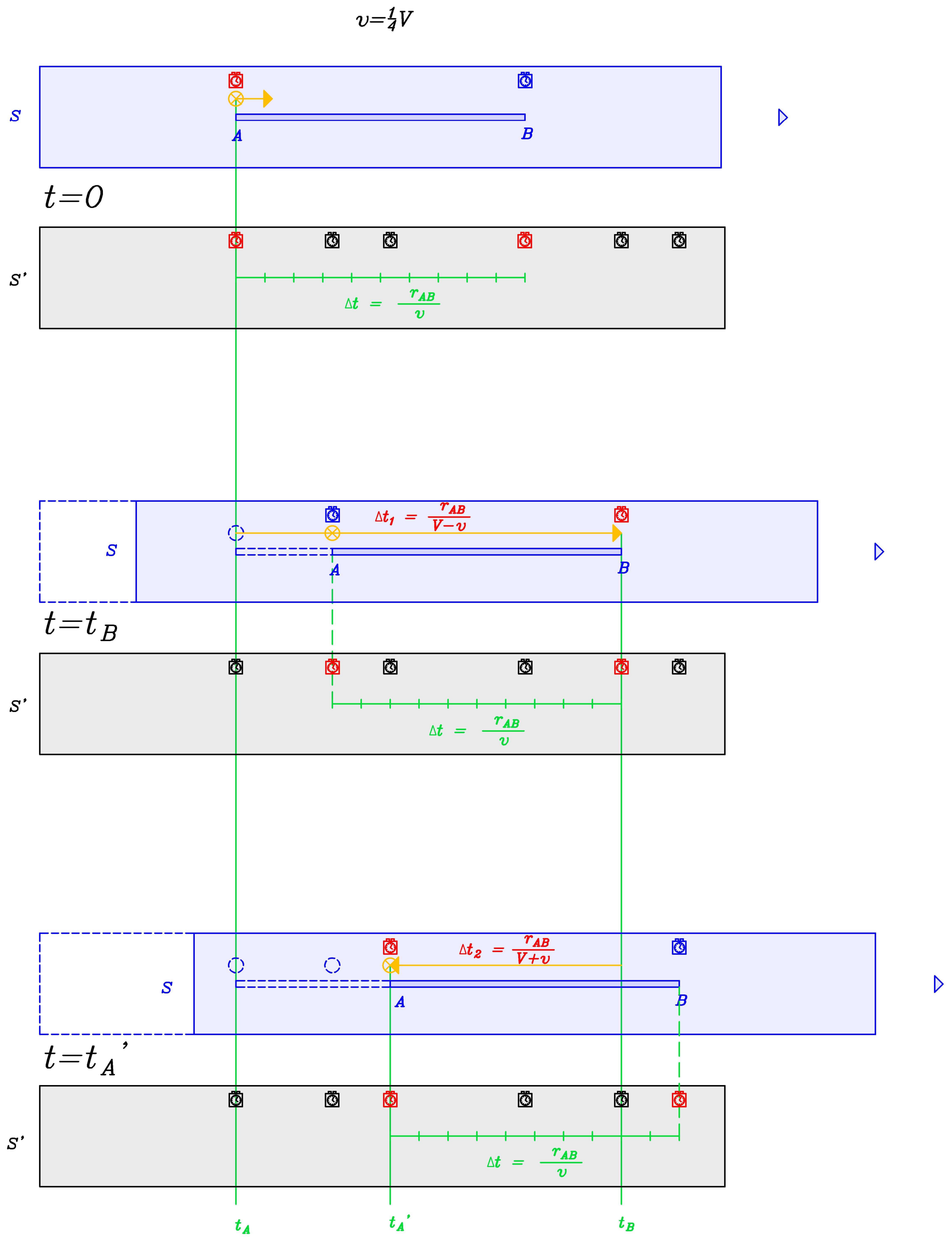


Fig. 5: Measurement using clocks in moving system. Clocks that do measurements at one instant in each event are highlighted in red. The principle is strictly following the original text. Reading of co-moving, synchronized clocks A and B in system S results in differing runtimes of the lightray, fully in accordance with classic speed addition (bluish). At the other hand, the length of the rod (not the lightray!), passing by against system S', is being measured by the stationary observer in S', using "stopwatches" at the point where the rod will pass at a given instant. In this case readings do not differ whether back or forth.

Now Special Relativity claims that **Δt₁ ($t_B$-$t_A$)** in one direction is different from **Δt₂ ($t_A$'-$t_B$)** in the opposite direction, hence the principle outlined in excerpt 1 and fig. 1 is violated and thus synchronicity of clocks is broken.

and

$$t'_A - t_B = \frac{r_{AB}}{V+v},$$

where $r_{AB}$ denotes the length of the moving rod, measured in the rest system. Observers co-moving with the rod would thus find that the two clocks do not run synchronously, while observers in the system at rest would declare them to be running synchronously.

Thus we see that we cannot ascribe *absolute* meaning to the concept of simultaneity; instead, two events that are simultaneous when observed from some particular coordinate system can no longer be considered simultaneous when observed from a system that is moving relative to that system.

Excerpt 7: Violation of simultaneity

## 5. Critic

At first we have to have a closer look to the mathematic scheme, in order to see what is being herewith presented to us. As for excerpt 1 we have:

$$t_B - t_A = t_A' - t_B \qquad (1)$$

And as per excerpt 6 and 7 we have:

$$t_B - t_A = \frac{r_{AB}}{(V-v)}$$
$$\textit{and} \qquad (2)$$
$$t_A' - t_B = \frac{r_{AB}}{(V+v)}$$

Now it is suggesting itself to do a simple operation, we insert the two formulae from (2) into (1):

$$t_B - t_A = t_A' - t_B$$
$$\Rightarrow \frac{r_{AB}}{(V-v)} = \frac{r_{AB}}{(V+v)}$$

$$\Rightarrow (V-v) = (V+v)$$

The above could be true only if light speed were infinite. In any other case we obtain:

$$\boxed{v = -v}$$

This already gives us a faint glimpse of the problem's root.

We clarify now, how such could happen. At first, we have added the two additional clocks to the stationary system (thus **A'**, **B'**, **C'**, **D'**) that we have already synchronized according to fig. 2, and we have added them equally on both systems.

Secondly we get rid of the basic mistake that is to relate <u>the traveling length of one lightray</u> within the moving system against <u>the measured length of a rigid rod</u> within the stationary system, i.e. we get rid of the fact that Special Relativity does mix an instantaneous event (measuring a length **l** at one given time **t**) with a period event (measuring the length of a lightray in one period $\mathbf{t_B}$-$\mathbf{t_A}$ ), i.e. by no means the same events.

Instead, let us examine the question of how one and the same lightray within the moving system **S** is actually measured in both systems (**S** and **S'**) that are in relative motion to each other.

The first event is the emergence of the ray from the source in **A**. In both systems clocks in **A** and **A'** give equal and simultaneous readings. The second event is when the lightray hits the mirror on the rod in **B** while the rod has moved with system **S**, and again we get equal and simultaneous readings, but this time from clocks **B** and **C'** (keep in mind that all clocks are synchronized). At the third event the lightray hits back to the source in **A** and we obtain equal and simultaneous readings this time from clocks in **A** and **D'**. Obviously the distances that have been covered by the lightray are equal in both systems, represented by the orange ray in **S** and the green scale in **S'**. We have highlighted in red such clocks (being all synchronized according to Special Relativity's rule) that are involved with measurement in each reference system at the respective simultaneous events:

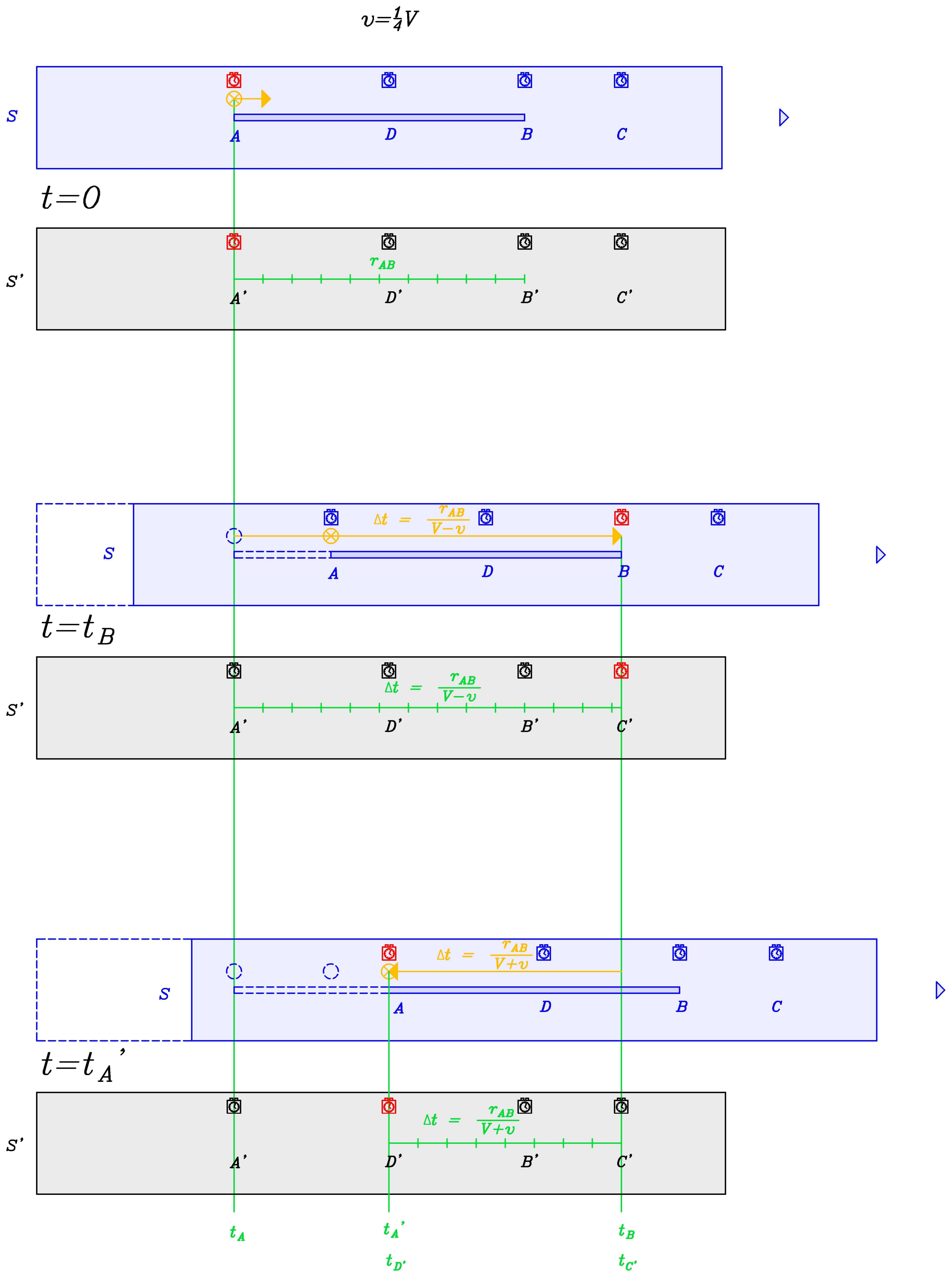


Fig. 6: Scheme with clock readings as it should be done properly in two systems in relative motion. Instead of mixing a duration measurement (in S) against a length measurement at one instant (in S') as was conducted in special relativity, we correctly compare measurements of solely durations in both systems. Clocks that do measurements at one instant in each event are highlighted in red. All clocks in S are synchronized and also synchronized with all clocks in S'. Corresponding clocks in S and S' at a given instant (e.g. lightray hits mirror in B) show equal readings. Equal runtimes for any event and therefore equal distances in both systems. No time dilation, no length contraction.

## 5. Conclusion

As we can easily see, we have now the following results:

1) All clocks are synchronized.
2) Clocks in **A** and **A'** are showing the same reading for the same event and instant and are on the very same locality. Likewise for clocks in **B** and **C'**and clocks in **A** and **D'**for their respective events, instances and localities.
3) Clock readings in both reference systems result in the same traveling length of the lightray.
4) Equal events and instants take place simultaneously and at the same locality.
5) Light speed is equal and constant in both reference systems.
6) Both reference systems are equivalent.
7) Simultaneous events are observed as simultaneous events in both reference systems.

We have shown that Special Relativity de facto puts events into relation that are not linked together in any way. Measurement of a duration representing a distance on one side, measuring a distance using stopwatches on the other side, which leads to the claim that simultaneity does not exist within two systems in uniform relative motion. All further conclusions of Special Relativity must be put on the touchstone.

References and Acknowledgments:

[1] Einstein, Albert, Zur Elektrodynamik bewegter Körper, Annalen der Physik und Chemie. 17, 1905, S. 891–921, 1905.