A ‘constant Lagrangian’ model for galactic dynamics in a geodetic approach towards the galactic rotation Dark Matter issue.

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I start with a historical note on the galactic rotation curves issue. The problem with the virial theorem in observed galactic dynamics, lead to the Dark Matter hypothesis but also to Modified Newtonian Dynamics or MOND. Then I move (away) from MOND towards a relativistic, Lagrangian approach of orbital dynamics in a curved Schwarzschild metric. I propose a ‘constant Lagrangian’ model for galactic scale geodetic dynamics. I will show with four rotation fitting curves to what extend my proposed model galaxies ‘constant Lagrangian’ postulate works in these limited number of situations. The fitted galaxies are NGC 2403, NGC 3198, UGC 6614 and F571-8. In the paper I present a theoretical context in which the ‘constant Lagrangian’ postulate might replace the classical virial theorem on a galactic scale. But the proposed postulate isn’t a ‘general law of nature’ because in the solar system and in the GNSS relativistic context, the classical virial theorem is proven accurate. Due to the limitations of the proposed postulate, a statement regarding Dark Matter can’t be made. But the model might achieve within the GR-Schwarzschild paradigm what MOND achieves within the Newtonian paradigm, fitting the experimental galactic rotation curves.

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I. THE VIRIAL THEOREM IN TROUBLE ON THE GALACTIC SCALE.

In 1932 the Dutch astronomer Oort observed that the stars in the galactic vicinity of the Sun are moving peculiarly fast, almost 8 times as fast as could be inferred from the calculated Newtonian acceleration. Oort assumed that dark matter would be the cause of this apparent difference, with ‘dark’ referring to ordinary matter not seen by us due to various reasons (Oort, 1932).

In 1933 Dark Matter was mentioned as “dunkle Materie” in a paper by Zwicky. Fritz Zwicky was studying the Coma Cluster of galaxies and found that his calculations for orbital acceleration and stellar mass within it was off by a large factor. He concluded that there should be a much greater density of dark matter within the cluster than there was luminous matter. Zwicky concluded that this constituted an unsolved problem (Zwicky, 1933). In 1937 Zwicky regarded his study on the Coma Cluster a test of Newton’s law of gravity on the largest cosmological scale possible, by applying the virial theorem on a cluster of galaxies. He also mentioned in his 1937 paper the possibility to test the virial theorem by applying it to the rotational velocities of the individual stars in the separate galaxies. But he concluded that this was technologically out of reach (Zwicky, 1937).

The breakthrough research of Rubin and Ford around 1970-1975 established beyond doubt the outer rotational velocity curves of individual galaxies, which turned out to be flat (Rubin et al., 1978). This was in conflict with velocity curves that resulted from the application of the virial theorem to the luminous mass of these galaxies. Rubin and Ford cited colleagues who suggested the existence of a large galactic halo of dark matter. In a 1980 paper presenting further research they concluded that the form of the rotation curves implied that significant non-luminous mass should be located at large distances beyond the optical galaxy. The total mass of a galaxy should, for large distances, increase at least as fast as the distance from the center (Rubin et al., 1980).

The third major evidence for Dark Matter was the gravitational lensing effect of clusters of galaxies. The mass of stars and hot gas in clusters who collectively act as a gravitational lens is too small to bend the light from the background galaxies as much as they actually do. A large density of dark matter in the center of these cluster is needed to explain the strength of the observed lensing effect (Koopmans et al., 2009).

In the course of decades it has become more and more clear that ordinary matter can’t
be the cause of those observed phenomena. That realization caused the term ‘dark matter’ to evolve into ‘Dark Matter’, with the capital letters indicating its elusive character. Today it has been predominantly, but not unanimously, been accepted that non-baryonic particles must exist in the calculated densities. A range of different astrophysical measurements point in this direction. I quote:

Astrophysical observations have provided compelling evidence for the existence of a non-baryonic dark component of the universe: dark matter (DM). The currently most accurate, although somewhat indirect, determination of DM abundance comes from global fits of cosmological parameters to a variety of observations, while the nature of DM remains largely unknown. One of the candidates for a DM particle is a weakly interacting massive particle (WIMP). (The ATLAS Collaboration, 2018)

II. MOND

One of the few non-particle approaches to the problem of Dark Matter is MOND or MOdified Newtonian Dynamics. MOND started in 1983 with two seminal paper of Milgrom. I quote from his papers:

All determinations of dynamical mass within galaxies and galaxy systems make use of a virial relation of the form $V^2 = MGr^{-1}$ where $V$ is some typical velocity of particles in the system, $r$ is of the order of the size of the system, $M$ is the mass to be determined, and $G$ is the gravitational constant. […] It must have occurred to many that there may, in fact, not be much hidden mass in the universe and that the dynamical masses determined on the basis of the above virial relation are gross overestimates of the true gravitational masses. (Milgrom, 1983b)

I have considered the possibility that Newton’s second law does not describe the motion of objects under the conditions which prevail in galaxies and systems of galaxies. In particular I allowed for the inertia term not to be proportional to the acceleration of the object but rather be a more general function of it. With some simplifying assumptions I was led to the form

$$m_g \mu \left( \frac{a}{a_0} \right) a = F,$$
replacing \( m_a \mathbf{a} = \mathbf{F} \). […] For accelerations much larger than the acceleration constant \( (a_0) \), \( \mu \approx 1 \), and the Newtonian dynamics is restored. (Milgrom, 1983b)

I use a modified form of the Newtonian dynamics (inertia and/or gravity) to describe the motion of bodies in the gravitational fields of galaxies, assuming that galaxies contain no hidden mass, with the following main results. 1. The Keplerian, circular velocity around a finite galaxy becomes independent of \( r \) at large radii, thus resulting in asymptotically flat velocity curves. 2. The asymptotic circular velocity \( (V_\infty) \) is determined only by the total mass of the galaxy \( (M) \):

\[
V_\infty^4 = a_0 GM,
\]

where \( a_0 \) is an acceleration constant appearing in the modified dynamics. This relation is consistent with the observed Tully-Fisher relation if one uses a luminosity parameter which is proportional to the observable mass. (Milgrom, 1983a)

The original Tully-Fisher relation is a relation between the luminosity of a spiral galaxy and its, maximum, rotation velocity (Tully and Fisher, 1977). The physical basis of the Tully-Fisher relation is the relation between a galaxy’s total baryonic mass and the velocity at the flat end of the rotation curve, the final velocity. According to McGaugh both stellar and gas mass of galaxies have to be taken into account in the relation that is referred to as the Baryonic Tully-Fisher (BTF) relation (McGaugh, 2005). In 2005 McGaugh determined the baryonic version of the LT relation as \( M_d = 50v_f^4 \), see (McGaugh, 2005) and Fig(1). In this form, \( M_d \) is expressed in solar mass \( M_\odot = 1.99 \cdot 10^{30} \ \text{kg} \) units and the final velocity of the galactic rotation velocity curve \( v_f \) is expressed in \( \text{km/s} \). If we express the galactic mass in \( \text{kg} \) and the velocity in \( \text{m/s} \) we get the total baryonic mass, final velocity relations in SI unit values as \( M_b = 1.0 \cdot 10^{20}v_f^4 \).

In 1983, Milgrom interpreted the BTF relation as an indication of a deviation from Newtonian gravity, making a modification of Newtonian dynamics or MOND necessary (Milgrom, 1983b). Using McGaugh’s 2005 values in SI units, Milgrom presented the BTF relation in the form \( v_f^4 = 1.0 \cdot 10^{-20}M_b = Ga_0M_b \), resulting in an acceleration \( a_0 = 1.5 \cdot 10^{-10} \ \text{m/s}^2 \) in McGaugh’s values. Milgrom hypothesized that this relation should hold exactly, thus interpreting it as an inductive law of nature instead of looking at it as just an empirical relation (Milgrom, 1983a). The resulting acceleration can be written as \( 5 \cdot a_0 \approx cH_0 \), with the velocity of light \( c \) and the Hubble constant \( H_0 \). According to Milgrom, the deeper significance
of this relation between this special galactic acceleration and the Hubble acceleration should be revealed by future cosmological insights (Milgrom, 1983b).

## III. CLASSICAL LAGRANGIAN DYNAMICS

The Lagrangian equation of motion reads

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}} \right) - \frac{\partial L}{\partial q} = 0.$$  \hspace{1cm} (1)

In classical gravitational dynamics I assume circular orbits with $\dot{q} = v$ and $q = r$. The Lagrangian itself is then given by $L = K - V$, with $V$ the Newtonian potential gravitational energy and $K$ the kinetic energy. One then gets

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}} \right) = \frac{dp}{dt} = F.$$ \hspace{1cm} (2)

The other part gives

$$\frac{\partial L}{\partial q} = -\frac{dV}{dr},$$ \hspace{1cm} (3)

so one gets Newton’s equation of motion in a central field of gravity

$$F_g = -\frac{dV}{dr}.$$ \hspace{1cm} (4)
Further analysis of the context results in the identification of the Hamiltonian of the system, $H = K + V$, as being a constant of the orbital motion and the virial theorem as describing a relation between $K$ and $V$ in one single orbit but also between different orbits, $2K + V = 0$.

On the galactic scale it is assumed that velocities are so low and gravitational fields are so weak, that Newtonian mechanics suffices and not much of relativity is needed. The problem with the rotational velocities of stars in galaxies and galaxies in cluster of galaxies is thus supposed to be a Newtonian physics issue that can be dealt with in the dynamics described above. The Dark Matter solution to the too fast rotational galactic velocities has two faces. On the one hand one tries to describe the density distribution of Dark Matter, needed in order to match the measurements with classical dynamics, specifically the virial theorem. On the other hand one tries to identify the Dark Matter constituents, usually seen as an out-of-the-box extension of the known Standard Model of particle physics.

IV. A GEODETIC APPROACH OF GRAVITATIONAL ORBITS

The problem with the previous analysis is connected to the notion of geodetic motion in General Relativity. The problem can best be described in a semi-relativistic approach using the classical Lagrangian equation of motion for geodetic orbits. The most important aspect of geodetic motion in GR is that it requires no force to move on a geodetic. This has important implications for the Lagrangian equation of motion, because $F = 0$ on a geodetic. One gets

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}} \right) = F_g = 0$$

and as a consequence also

$$\frac{\partial L}{\partial q} = -\frac{dL}{dr} = 0.$$  \hspace{1cm} (6)

As a result, one gets the crucial

$$L = K - V = \text{constant}$$ \hspace{1cm} (7)

on geodetic orbits.

This result, the Lagrangian of the system as being the constant of the geodetic motion, is used on a daily basis by many of us because it is used by GNSS systems for the relativistic correction of atomic clocks in their satellites. Let’s elaborate this a bit further. In General
Relativity, the proper time-rate $d\tau$ is defined through the metric distance $ds$ as $ds \equiv c d\tau$.

The square metric distance is defined through

$$ds^2 \equiv g_{\mu\nu} dx^\mu dx^\nu.$$  

(8)

Given coordinate world time-rate $dt$, which is the time-rate of a standard clock at a position where $d\tau = dt$ (in GR-Schwarzschild this implies a clock at rest at infinity), we get the general

$$\frac{ds^2}{dt^2} = c^2 \frac{d\tau^2}{dt^2} = g_{\mu\nu} \frac{dx^\mu}{dt} \frac{dx^\nu}{dt} = g_{\mu\nu} V^\mu V^\nu,$$  

(9)

with the geodesic four-vector velocity $V^\mu$. In this equation, $d\tau$ stands for the local proper clock-rate of a clock in a geodetic orbit in a field of gravity and $dt$ is the universal clock-rate. Because of this interpretation of $dt$, the velocity $V^\mu$ is the velocity as seen from a position where $d\tau = dt$. See for example (Singer, 1956), (Weinberg, 1972, p. 79), (Misner et al., 1973, p. 1054-1055), (Straumann, 1984, p. 97), (Ohanian and Ruffini, 2013, p. 119).

In case of the Schwarzschild metric in polar coordinates, we have (Ruggiero et al., 2008)

$$ds^2 = \left(1 + \frac{2\Phi}{c^2}\right)c^2 dt^2 - \left(1 + \frac{2\Phi}{c^2}\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2.$$  

(10)

In case of a clock on a circular geodesic on the equator of a central non-rotating mass $M$ we have $\frac{dr}{dt} = 0$, $\frac{d\theta}{dt} = 0$, $\sin \theta = 1$ and $\frac{d\phi}{dt} = \omega$. We thus get

$$\frac{ds^2}{dt^2} = \frac{c^2}{c^2} \frac{d\tau^2}{dt^2} = \left(1 + \frac{2\Phi}{c^2}\right)c^2 - r^2 \omega^2,$$  

(11)

and

$$\frac{d\tau^2}{dt^2} = 1 + \frac{2\Phi}{c^2} - \frac{r^2 \omega^2}{c^2}.$$  

(12)

With $v_{\text{orbit}} = r \omega$ we have

$$\frac{d\tau^2}{dt^2} = 1 + \frac{2\Phi}{c^2} - \frac{v_{\text{orbit}}^2}{c^2}.$$  

(13)

So finally we get the GR result

$$\frac{d\tau}{dt} = \sqrt{1 + \frac{2\Phi}{c^2} - \frac{v_{\text{orbit}}^2}{c^2}}.$$  

(14)

with $d\tau$ as the clock-rate of a standard clock $A$ in a geodetic orbit and $dt$ as the ‘universal’ clock-rate $G$ of a standard clock at rest in infinity, the only condition for which $d\tau = dt$.

The result of Eqn. (14) is the basic relativistic correction used in GNSS clock frequencies, with the first as the gravity effect or gravitational potential correction and the second as the
velocity effect or the correction due to Special Relativity (Ashby, 2002; Hećimović, 2013; Delva and Lodewyck, 2013).

Given the classical definitions of $K = \frac{mv_{\text{orbit}}^2}{2}$ and $V = m\Phi$, we get

$$\frac{d\tau}{dt} = \sqrt{1 - \frac{2L}{U_0}}. \quad (15)$$

All the satellites of a GNSS system are being installed on a similar orbit and thus syntonized relative to one another because they share the same high and velocity and have constant $L$ and $\frac{d\tau}{dt}$ on those orbits. But different GNSS systems, as for example GPS compared to GALILEO, are functioning on different orbits with different velocities and those systems aren’t syntonized relative to one another. This non-syntonization between satellites on orbits with different heights and virial theorem connected velocities is very annoying for the effort towards realizing an integration of the different GNSS systems into one single global network.

**V. A RELATIVISTIC VIRIAL THEOREM FOR A MODEL GALAXY**

When I connected

$$\frac{d\tau^2}{dt^2} = 1 + \frac{2\Phi}{c^2} - \frac{v_{\text{orbit}}^2}{c^2} = 1 - \frac{2L}{U_0} \quad (16)$$

to the problem of the galactic rotation curve, I realized that the flat rotation curve implies atomic clock syntonization in those areas. In those outer regions, the potential can be assumed to be zero and the velocity constant. Those flat rotation rate zones are the GNSS engineer’s dream come true. This made me curious as to the clock-rate status in the inner regions. The intriguing thing is that you can jump from orbit to orbit and still encounter a constant clock-rate on all the orbiting satellites you encounter on an imaginary voyage through the outer regions of galaxies. This implies that precisely in those regions where the classical virial theorem seems in trouble, $L \simeq \text{constant}$, not just in one single orbit but also between different orbits. It should be clear that for those geodetic orbits the classical virial theorem, which in its most essential form states that $F_{\text{gravity}} = F_{\text{centripetal}}$, becomes meaningless because on circular geodetics this reduces to the empty expression $0 = 0$.

In order to study the relativistic clock-rate behavior in the inner regions of galaxies, I had to construct a model galaxy. Real galaxies are way to fussy, complex and messed up to get interpretable results. My model galaxy is build of a model bulge with mass $M$ and
radius $R$ and a Schwarzschild metric emptiness around it. The model bulge has constant density \( \rho_0 = \frac{M}{V} = \frac{3M}{4\pi R^3} \) and its composing stars rotate on geodetics in a quasi-solid way. So all those stars in the bulge have equal angular velocity on their geodetic orbits, with \( v = \omega r \). On the boundary between the quasi solid spherical bulge and the emptiness outside of it, the orbital velocities are behaving smoothly. So the last star in the bulge and the first star in the Schwarzschild region have equal velocities and potentials. I also assume that the Newtonian potential itself is unchanged and unchallenged, remains classical in the whole galaxy and its surroundings. Such a model galaxy doesn’t have a SMBH in the center of its bulge and it only has some very lonely stars in the space outside the bulge.

The gravitational potential in such a case is well known, see Fig.(2). If this sphere would be in a condition where the classical virial theorem would hold, so \( 2K = -V \), then on the boundary \( r = R \) we would have \( K = \frac{GM}{2R} \) and \( L = K - V = \frac{3GM}{2R} \). At the center of the rotating sphere, \( K = 0 \) and we also have \( L = \frac{3GM}{2R} \).

From \( r = 0 \) to \( r = R \), the potential \( \Phi \) increased as \( r^2 \). The kinetic energy does the same...
because \( v^2 = \omega^2 r^2 \). One can conclude that they increase identical and that \( L = K - V \) is a constant inside the quasi-solid sphere. We can write for the region from \( r = 0 \) to \( r = R \)

\[
\frac{L}{m} = \frac{v_{orbit}^2}{2} + \frac{GM}{r} = \frac{3GM}{2R} = \text{constant.} \tag{17}
\]

As a result, in such a model bulge, \( L \) is a constant of the motion, not only in one orbit but also between orbits. All the clocks in such a model bulge would be syntonized.

Thus, in the model galaxy that I am about to construct, we have \( L = \text{constant} \) inside the model bulge and we have \( L = \text{constant} \) in the outer regions where the rotational velocity curve flattens and the Newtonian potential turns negligibly small. So let’s be bold and declare \( L = K - V = \text{constant} \) in the entire galaxy, without changing the Newtonian potential. What would that have as effects?

**FIG. 3.** The square of the orbital velocity profile in the model galaxy with \( L = \text{constant} \).
We would get $K = L + V$ and $L = V(r = 0)$ so for the region $0 \leq r \leq R$ we get

$$v_{\text{orbit}}^2 = \frac{GM}{R} \cdot \frac{r}{R}$$

and outside the model bulge, where $R \leq r \leq \infty$, we have

$$v_{\text{orbit}}^2 = \frac{3GM}{R} - \frac{GM}{r}.$$  \hspace{1cm} (19)

In Fig. (3) I sketched the result, with $-V = +K_{\text{escape}}$.

From the perspective of a free fall Einstein elevator observer, the free fall on a radial geodetic from infinity towards the center of the bulge, the other free fall tangential geodetics seem to abide the law of conservation of energy, because the escape kinetic energy plus the orbital kinetic energy is a constant on my model galaxy with galactic constant $L$. An Einstein elevator system with test mass $m$ that would be put in an orbital collapse situation, magically descending from orbit to orbit in a process in thermodynamic equilibrium, would have constant total kinetic energy, from the radial free fall perspective. This can be expressed as $L = K_{\text{orbit}} - V = K_{\text{orbit}} + K_{\text{escape}} = K_{\text{final}}$.

Such a model galaxy would also be a GNSS engineer’s dream come true because the whole model galaxy is in one single syntonized time-bubble.

$$\frac{d\tau}{dt} = \sqrt{1 - \frac{2L}{U_0}}.$$  \hspace{1cm} (20)

Given the Baryonic Tully-Fisher relation in Milgrom’s version $v_{\text{final}}^4 = Ga_0M$ with $2\pi a_0 \approx cH_0$, with $a_0$ as Milgrom’s galactic minimum acceleration and $H_0$ as the Hubble constant, we get as a galactic time bubble fix

$$\frac{d\tau}{dt} = \sqrt{1 - \frac{2L}{U_0}} = \sqrt{1 - \frac{v_{\text{final}}^2}{c^2}} = \sqrt{1 - \frac{v_{\text{final}}^4}{c^4}} = \sqrt{1 - \frac{Ga_0M}{c^4}} = \sqrt{1 - \frac{GH_0M}{2\pi c^3}} = \sqrt{1 - \frac{M}{2\pi M_U}},$$  \hspace{1cm} (21)

in which I used $L = 3GM/R = K_{\text{final}} = \frac{1}{2}mv_{\text{final}}^2$ and $M_U = \frac{c^3}{6H_0}$. This last constant can be referred to as an apparent mass of the Universe, a purely theoretical number constant, see (Mercier, 2015). In a model Universe, this would imply that my model galaxy would be in a proper time bubble with clock-rate $d\tau$ relative to the universal clock-rate $dt$ in proportion to the masses of galaxy $M$ and Universe $M_U$. In my model galaxy theoretical environment
the Baryonic Tully-Fisher relationship implies that the galactic time bubble is fixed through the mass of my model galaxy and that this fix is a cosmological one. So what is a universal acceleration minimum \( a_0 \) in MOND can be interpreted as a universally correlated (through \( M_U \)) but still local (through \( M \)) time bubble fix in my model galaxy geodetic environment.

This doesn’t imply that I can integrate my model galaxy approach into MOND, because you either take the perspective of geodetic motion without any force of gravity, or you don’t have a curved metric and use the classical gravitational acceleration approach. My approach of \( L = \text{constant} \) started with setting \( F_g = 0 \) in the Lagrangian equation of motion. Milgrom started by modifying Newton’s second law, leading to an adapted \( F_g \neq 0 \). You can’t have it both ways. My approach of \( L = \text{constant} \) and Milgrom’s \( F_g = ma\mu \) are mutually exclusive.

VI. FITTING FOUR REAL GALACTIC ROTATION CURVES

Having determined the model galactic velocity rotation curve based on the Lagrangian as a galactic constant of orbital motion, the question is to what extend real galaxies can be modeled in this way. For this I used the experimental velocity rotation data of four galaxies: NGC 2403, NGC 3198, UGC 6614 and F571-8. I plotted them in Excell. The velocity rotation curve data come from different sources. The NGC 2403 data are from (Begeman, 2006, p. 51). The UGC 6614 and F571-8 data are from (McGaugh et al., 2001) and were retrieved from the data website of McGaugh. The NGC 3198 data are from (Karukes et al., 2015, p. 2) and brought to my attention by (Vossos and Vossos, 2017).

In this section I present the plots of \( V_{\text{orb}}^2 \) against \( r \), with in each plot the experimental values in red stars and the theoretical values in black bars. The fitting plots are given in two versions. The first plot is with one single fit for \( M \) and \( R \), this is the pure model. In the second plot the two parameters \( M \) and \( R \) are used as one single ‘free’ parameter for every single measurement, because the time-bubble or \( L \) is constant constraint leaves only one degree of freedom. The locked in through \( L \) variation of \( M \) and \( R \) in plot 2 can be monitored using the apparent model mass density of the bulge \( \rho_{\text{bulge}} \). This density varies as \( M \), with locked in \( R \), varies. With this parameter freedom of one single value, \( M \) and with locked \( R \) in \( L \) and \( \rho_{\text{bulge}} \), all four experimental curves could be fitted really nice. The most important cut in the model is the change from the model bulge to the model empty space around it. In the model bulge, \( V_{\text{orb}}^2 \propto r^2 \), outside the model bulge \( V_{\text{orb}}^2 \propto -r^{-1} \). In the
fixed fitting curve, the apparent mass density of the bulge is the main variable that changes due to more realistic circumstances. The excell data sheets of the plots are in the appendix. The fact that it is possible to exactly plot the rotation curves with just one free parameter should be significant for the underlying physics. In my approach, one free parameter can force a time-bubble on a whole galaxy.

FIG. 4. UGC 2403 Plot1, $V_{orb}^2$ against $r$, pure model.
FIG. 5. UGC 2403 Plot1, $V_{orb}^2$ against $r$, fixed model.

FIG. 6. UGC 6614 Plot1, $V_{orb}^2$ against $r$, pure model.
FIG. 7. UGC 6614 Plot1, $V_{orb}^2$ against $r$, fixed model.

FIG. 8. F571-8 Plot1, $V_{orb}^2$ against $r$, pure model.
FIG. 9. F571-8 Plot1, $V_{orb}^2$ against $r$, fixed model.

FIG. 10. NGC 3198 Plot1, $V_{orb}^2$ against $r$, pure model.
FIG. 11. NGC 3198 Plot1, $V_{orb}^2$ against $r$, fixed model.
VII. DARK MATTER, AN UNRESOLVED ISSUE

What about ‘dark matter’ and Dark Matter? Well, the early assumption of Oort and Zwicky that the astronomers were not seeing a lot of ordinary matter, the ‘dark matter’ postulate, turned out to be falsified. The attention then turned towards Dark Matter in the sense on non-baryonic (=non-ordinary Standard Model) stuff. The search for Dark Matter continues at ever increasing strengths.

My model galaxy, obeying to the ‘constant Lagrangian’ condition, just focuses on one aspect related to Dark Matter, the galactic rotation velocity curves. There are more issues leading to the Dark Matter hypothesis, unrelated to my model galaxy approach. Such as gravitational lensing, the galaxy cluster virial problem and cosmology related issues.

I have shown in four rotation fitting curves that my proposed model galaxies ‘constant Lagrangian’ postulate works in a limited number of situations. I also gave a theoretical context in which the ‘constant Lagrangian’ postulate might replace the classical virial theorem on a galactic scale. But it isn’t a ‘general law of nature’ because in the solar system and in the GNSS relativistic context, the classical virial theorem is proven accurate.

The question regarding Dark Matter depends on the status of the ‘constant Lagrangian’ postulate. If it is a cosmological law of nature, then we don’t need Dark Matter. But then we do need to explain why the classical virial theorem is valid in the context of the solar system. If Einstein’s theory is fundamental, then Newton’s needs to be justified. If the ‘constant Lagrangian’ postulate only functions (if it functions in that context in the first place) at the scale of individual galaxies, then the postulate needs further justification. A Dark Matter density distribution curve can easily provide such a justification.

The fact that it turned out to be possible to exactly plot the rotation curves with just one free parameter might indicate towards the underlying physics. In my approach, one free parameter can force a time-bubble on a whole galaxy. One free parameter needed to enforce a galactic time-bubble condition might well indicate a Dark Matter density distribution.

The problem with deriving a Dark Matter density distribution function from my postulate is that one then mixes two mutually exclusive axiomatic systems. I would have to add the classical energy situation, as expected according to the classical virial theorem, to my $L = K - V$ plot, drawn in a geodetic context where $F_g$ is supposed to be zero. It can be done quite easily, but at the price of mixing mutually exclusive theoretical axiomatic approaches.
This mixing of axioms results for \( r > R \) in

\[
\rho_{DM} = \frac{3M}{4\pi} \left( \frac{2}{R r} - \frac{2}{r^2} - \frac{1}{r^2} \ln \left( \frac{r}{R} \right) \right)
\]  

(23)

or, with \( \rho_0 = \frac{M}{V} = \frac{3M}{4\pi R} \), in

\[
\rho_{DM} = \rho_0 \left( \frac{2R^2}{r} - \frac{2R^3}{r^2} - \frac{R^3}{r^2} \ln \left( \frac{r}{R} \right) \right),
\]  

(24)

with \( M \) and \( R \) referring to the mass and the radius of the pure model galactic bulge.

I can conclude that the ‘constant Lagrangian’ postulate leads to an interesting model galaxy and that the pure model can be adjusted using the two key parameters, the model bulge’s mass \( M \) and radius \( R \), to match the four galaxy rotation curves to which is was exposed. Once the pure model parameters of \( M \) and \( R \) are chosen, only one of those parameters remains as a degree of freedom because the other one is then given through \( L \). The model itself doesn’t decide on the existence of Dark Matter, because the postulate doesn’t justify itself but is in need of external justification. On that level will the Dark Matter discussion play out. The model is presented in the context of Special and General relativity, it is a metric approach with the Schwarzschild metric and the related time dilation formula at its core. As such, it might be an interesting addition to the MOND approach towards galactic and cosmological virial theorem issues.
REFERENCES


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to the galactic plane and some related problems. *Bulletin of the Astronomical Institutes of the Netherlands* 6, 249–287.


FIG. 12. UGC 2403 Excell datasheet 1, $V_{orb}^2$ against $r$, pure model.

<table>
<thead>
<tr>
<th>$R_1$ (pc)</th>
<th>$R_2$ (pc)</th>
<th>V (km/s)</th>
<th>Vmodel</th>
<th>V (km/s)</th>
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<th>$V_{orb}$</th>
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FIG. 13. UGC 2403 Excell datasheet 1, $V_{\text{orb}}^2$ against $r$, fixed model.

FIG. 14. UGC 6614 Excell datasheet 1, $V_{\text{orb}}^2$ against $r$, pure model.
FIG. 15. UGC 6614 Excell datasheet 1, $V_{orb}^2$ against $r$, fixed model.

FIG. 16. F571 8 Excell datasheet 1, $V_{orb}^2$ against $r$, pure model.

FIG. 17. F571 8 Excell datasheet 1, $V_{orb}^2$ against $r$, fixed model.
FIG. 18. NGC 3198 Excell datasheet 1, $V_{\text{orb}}^2$ against $r$, pure model.

FIG. 19. NGC 3198 Excell datasheet 1, $V_{\text{orb}}^2$ against $r$, fixed model.