Duality transform between black and white psychological profiles.

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Abstract
It is shown that the Fourier transformation is the appropriate defining characteristic of black and white polarization states in psychological archetypes.

1 Introduction.
In previous work, I have described the necessity for at least a black-white theory of psychological profiles. In principle, psychological archetypes are described by means of a real vectorspace $V = \mathbb{R}^n$ given that no obvious extremal states need a priori exist. Such assumption would lead to a description by means of convex spaces. Black is an effective charge described by a delta peak distribution on $V$ whereas a pure white state is described by its Fourier transform $F$. They are extremal weak distributional states in $L^2(V, \mu)$ the Hilbertspace of square integrable functions on $V$ with respect to the measure $\mu$. Circularly polarized states are then defined as “black content equals white content” which is the space of eigenstates of the Fourier transform.

2 Elaboration.
In what follows, we take the pure theory and assume $n = 1$. $V$ then is spanned by means of the black states given by $\delta(x - a)$, white states are of the form $e^{ikx}$. As such, no information loss occurs and black-white are just different configurations of the same substance. Hence, given $g(x) = \int dy g(y) \delta(y - x)$ we have that $(Fg)(k) = \int dy g(y) e^{iky}$.

We now look for states for which $|g(x)|^2 \sim |(Fg)(x)|^2$ or, more precisely, $(Fg)(x) \sim e^{ikx} g(x)$

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for some $d$. Taking into account that

$$(F(g))(k) = \int dx e^{ikx} g(x)$$

the aforementioned class is given by the Gaussian functions

$$e^{-a(x-b)^2}$$

since

$$\int dx e^{ikx} e^{-a(x-b)^2} = \int dx e^{ikb} e^{-a((x-b)-ik)^2} = \frac{\pi}{\sqrt{a}} e^{-ab^2} e^{-\frac{1}{4a}(k-2ia)^2}$$

and for this function to satisfy our criterion it is necessary and sufficient that $a = \frac{1}{2}$ and $b$ is freely chosen. Hence, the diagonal in the white-black plane is one dimensional and parametrized by $b$ just as the black line interval is. Taking $n > 1$ would entail a definition of blackness given by $\delta(|x|-a)$ thereby suppressing $n-1$ dimensions. Taking $a$ to zero and scaling the Gaussian functions appropriately leads to the pure black states whereas taking $a$ to infinity provides for the pure white states. The appropriate duality is therefore $a \to \frac{1}{4a}$. Hence one has to consider the operators

$$P_{a,b} = e^{2ab(1-i)} \circ F \circ S_{\frac{1}{\alpha}}$$

and notice that

$$(P_{a,b})^2 = e^{x^{2ab(1-i)}} \circ F \circ S_{\frac{1}{\alpha}} \circ e^{x^{2ab(1-i)}} \circ F \circ S_{\frac{1}{\alpha}}$$

$$= 2ae^{x^{2ab(1-i)}} \circ F \circ e^{x^{2b(1-i)}} \circ F = 2ae^{x^{2ab(1-i)}} \circ T_{-b(1+i)} \circ F$$

and consider eigenvectors $v_{d,\alpha}$ with the appropriate eigenvalue $2\pi\sqrt{a}$ to be calculated above (hint: $\alpha = \frac{1}{2a}$ for the diagonal states). All these functions are eigenvectors of the latter operator with eigenvalue $4\pi^2a$. Here, $e^{iax}$ is the multiplication operator and $S_{\alpha}$ the scaling operator defined by $$(S_{\alpha}g)(x) = g(ax).$$

Generally, $d = (i+1)2ab$ and there are some interesting commutation relations

$$e^{-iax} F = FT_d$$

where

$$(T_d g)(x) = g(x+d)$$

and

$$(FS_{\alpha} g)(x) = \frac{1}{\alpha}(S_{\frac{1}{\alpha}} F g)(x).$$

Hence,

$$P_{d,\alpha}^\dagger P_{d,\alpha} \sim 1$$

given that $e^{idx}$ is unitary and

$$S_{\alpha}^\dagger S_{\alpha} = \frac{1}{\alpha} 1.$$

Notice that

$$x F x + \partial_x F \partial_x = i F$$

and therefore, the Heisenberg algebra is equivalent to

$$X^\dagger X - P^\dagger P = i1$$
on inproduct spaces with a complex valued bilinear form gauged by $X^\dagger = P$. The Fourier transform is then recuperated by finding a unitary operator $\mathcal{F}$ on a Hilbert space representation of $X, P$ such that $X^\dagger = \mathcal{F}X^H\mathcal{F}$ with $X^H = X$. In general, the angle $\alpha$ between the mixed states and the “axis” of black extremal states should be given by a self dual function, that is

$$\frac{1}{f(x)} = f\left(\frac{1}{4x}\right)$$

giving rise to

$$f(x) = \frac{1}{\sqrt{2x}}.$$  

Therefore,

$$\tan\left(\frac{\pi}{2} - \alpha\right) = f(x)$$

and the magnitude is determined by the shift parameter $b$. Henceforth, a compactification of the infinite dimensional function space to a real two dimensional Hilbert space with black-white extremal states is given by

$$|a, b\rangle = b (\cos(\alpha)|b\rangle + \sin(\alpha)|w\rangle).$$

There is no information loss but the correspondence is not a linear one for sure given that any linear combination on the right hand side determines a Gaussian but not so for linear combinations of the left hand side. It remains a task to find other suitable, higher, “pictorial” charges such as shapes, including tigers, cats, dogs and sharks, to further specify the psychological mapping to relative moral values.