A Note on Diophantine Relations at or near the beginning of Prime sequences

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If you look at the set of primes, at the beginning of the sequence there are several Diophantine equations, which probably are well known. Here are those examples:

2 - 3 + 5 + 7 - 11 = 0

3 + 5 + 7 - 11 + 13 - 17 = 0

5 + 7 + 11 + 13 - 17 - 19 = 0   5^2 - 7^2 + 11^2 - 13^2 -17^2 + 19^2 = 0

5 - 7 + 11 - 13 + 17 - 19 - 23 + 29 = 0

11 + 13 - 17 - 19 - 23 + 29 - 31 + 37 = 0

There are other ways of course of sorting the primes to produce other particular prime sequences. Examples are the set of primes ending in a prime number.

Prime numbers ending in the prime number 3   OEIS A030431

3, 13, 23, 43, 53, 73 …

At or near the beginning of this sequence is a Diophantine equation

13 - 23 - 43 + 53 = 0

Prime numbers ending in the prime 7   OEIS A030432

7, 17, 37, 47, 67, 97…

At or near the beginning of this sequence is a Diophantine equation

17 - 37 - 47 + 67 = 0

Prime numbers ending in the prime 11

11, 211, 311, 811, 911, 1511…

211 - 311- 811 + 911 = 0

Prime numbers ending in prime 31   OEIS A167388

31, 131, 331, 431, 631, 1031…

131 - 331 - 431 + 631 = 0

Many more examples like this where you obtain 4 integer Diophantine equations

E.g. Prime numbers ending in 9, 19, 41, 47, 101 etc.   Also, primes ending in the number 1

Other prime numbers show an increase in the number of integer values for the Diophantine solutions. It usually appears that the increase in the number of integer solutions are even valued such as 4 to 6 to eight to ten and so on. These Diophantine equations are always at or near the beginning of the sequence. Here are other examples.

Prime numbers ending in prime 71   OEIS A167441

71, 271, 571, 971, 1171, 1471, 1571, 1871…

271 - 571 - 971 + 1171 -1471 + 1571 = 0

Prime numbers ending in prime 59

59, 359, 659, 859, 1259, 1459, 1559…

359 - 659 - 859 + 1259 + 1459 - 1559 = 0

We can make it more interesting by merging two different prime number sets together

Alternating numbers that end in primes 47 and 59

47, 59, 347, 359, 547…

47 - 59 - 347 + 359 = 0

Alternating numbers that end in primes 7 and 9

19, 37, 29, 47, 67…

19 - 37 - 29 + 47 = 0

How about a more spread out alternating set

Alternating numbers that end in primes 31 and 101

31, 101, 131, 5101, 331, 8101, 431, 12101, 631, 15101, 1031…

We then have two 8 valued Diophantine equations

131 + 5101 - 331 - 8101 - 431 - 12101 + 631 + 15101 = 0

131 - 5101 - 331 + 8101 - 431 + 12101 + 631 - 15101 = 0

Here are examples involving the complete prime set and a set ending with a specific prime number or the number 9 in this case

2, 19, 3, 59, 5, 79, 7, 89, 11, 109, 13…

19 -79 + 3 + 59 + 5 - 7 = 0

Ending in the prime number 3

2, 3, 3, 13, 5, 23, 7, 43, 11, 53, 13…

2 + 3 + 3 + 13 - 5 -23 +7 = 0   Notice 7 integer solution here.

There seems to be no end to this process of finding Diophantine equations at the front of prime sequences. Let us end this by including a much larger prime set and a moonshine set.

Prime numbers ending in the prime 196561

196561, 7196561, 16196561, 18196561, 19196561, 21196561, 43196561, 45196561, 46196561, 58196561, 63196561, 66196561, 70196561, 78196561, 84196561, 106196561…

Yields 2 two 10 valued Diophantine equations

21196561 - 43196561 - 45196561 + 46196561 + 58196561 - 63196561 - 66196561 -70196561 + 78196561 + 84196561 = 0

21196561 - 43196561 - 45196561 + 46196561 - 58196561 - 63196561 + 66196561 + 70196561 - 78196561 + 84196561 = 0

How about the prime sequence, the supersingular primes of moonshine theory

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 41, 47, 59, 71

3 + 5 + 7 - 11 + 13 - 17 + 19 + 23 -29 -31 - 41 + 47 - 59 + 71 = 0

Being that there is a (pseudo?) randomness to primes is there an explanation for these beginning Diophantine sequences, or is this expected?