Newton's First Law Revisited

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Abstract

Newton's first law is expressed in textual form. It states that, *unless acted upon by a net unbalanced force, an object will remain at rest, or move uniformly forward in a straight line*. Accordingly, "inertial motion" means uniform rectilinear motion, while uniform circular motion is considered to be a noninertial, accelerated motion. This differentiation between the two types of motion has resulted in the aftermath in different analytical treatments of the two types, both in classical and relativistic kinematics.

In this short note, we show, based on Newton's kinematics, that, contrary to the conventional differentiation between rectilinear and circular systems of motion, the two are *dynamically equivalent*, such that the set of laws describing the dynamics of one system correspond to an identical set of laws describing the dynamics of the second. An immediate corollary for the special case of uniform motion is that Newtonian kinematics are inconsistent with Newton's first law and are, instead, consistent with Galileo's definition of inertial motion. To rectify the apparent inconsistency, we propose a natural modification of the first law, which incorporates the case of uniform circular motion. We formulate the modified law textually and mathematically and comment briefly on the implication of our modification to the theory and education of classical and relativistic physics.

Keywords: Newton's first law; inertial systems; system's equivalence; rectilinear motion; circular motion.

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1. Introduction

Newton's first law is unique in more than one aspect, i.e., it exists solely in textual form, thus making it difficult to eliminate the ambiguity regarding what it exactly tells us.[1] Nonetheless, it articulates a basic principle of nature, which, together with the third law, is the only law of Newton's mechanics that has passed untouched to relativity theories and all modern physics. In a commonly used English translation of Newton's seminal *Principia,* [2] the first law states that *"every body continues in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed upon it"* (p. 13). Another commonly used translation of the law states that, *unless acted upon by a net unbalanced force, an object will remain at rest, or move uniformly forward in a straight line.* [3] Yet there are many other translations that diverge from each other due to the inherent vagueness in the original text and the different nuances opted by different translators.[1] However, despite such differences, all statements of the law agree that an inertial motion is *motion in uniform velocity in a straight line*. Accordingly, uniform circular motion is considered to be a noninertial, accelerated motion. This differentiation between uniform rectilinear and uniform circular motions is not semantic. In fact, it is the source of the different analytical treatments of the two types of motion, both in classical and relativistic kinematics. In special relativity theory (SRT),[4] the formalization of the "relativity axiom" and derivation of the theory transformations assume *constant rectilinear motion* and, thus, are not supposed to apply to other types of motion, such as circular motion, which, according to Newton's first law and Einstein SRT, is a noninertial, accelerated system.

2. On the equivalence between rectilinear and circular motion

Here, we challenge this convention by demonstrating that there is a one-to-one correspondence between the kinematics of rectilinear and circular types of motion. In the language of systems analysis, the two types of motion are completely *equivalent systems*.[5,6] The proof for our claim is trivial. Consider a dynamical system of any type (physical, biological, social, etc.), which could be completely defined by a set of dynamical parameters p_i (i = 1, 2, .. , n), and a set of equations R defined in (1):

$$R = \{p_2 = \dot{p_1}, p_3 = \ddot{p_1}, p_5 = p_3\, p_4, p_6 = \int p_5 dp_1, p_7 = \frac{1}{2} p_4\, {p_2}^2\}. \tag{1}$$

If we think of p_1, p_2, p_3 as representing rectilinear position x, velocity v, and acceleration a, respectively, and of p_4, p_5, p_6, p_7 as mass m, rectilinear force F, work W, and kinetic energy E, respectively; then, the dynamical system defined by R gives a full description of a classical *rectilinear motion* (see Table 1,

Appendix A). Alternatively, if we think of p_1, p_2, p_3 as representing angular position θ, velocity w, and acceleration α, respectively, and of p_4, p_5, p_6, p_7 as radial inertia I, torque τ, work W, and kinetic energy E, respectively (see Table 1, Appendix A), then the dynamical system defined by R gives a full description of a classical *circular motion* (Q.E.D.).

3. Implications to the case of uniform motion

The proven equivalence between rectilinear and circular dynamical systems holds in general and is not restricted to the special case of uniform types of motion. However, the uniform motion is particularly interesting due to its intimate relation to Newton's first law and to special relativity theory. An immediate corollary of the above proven equivalence is that Newton's first law, as articulated by Newton and adopted by Einstein and others, is inconsistent with Newtonian kinematics, which shows without any doubt that the dynamics of circular uniform motion are identical to the dynamics of rectilinear uniform motion; thus, if the latter is inertial, then so is the former. In fact, we can "translate" Newton's first law from its linear coordinates to radial coordinates simply by replacing, in the original statement of the law, the words "straight line" by the word "circle," thus yielding the following law:

Every body continues in its state of uniform rotation in a circle, unless it is compelled to change that state of motion

Quite interestingly, we found that our view of what defines an inertial system is in complete agreement with Galileo's interpretation of inertia. In Galileo's words: "All external impediments removed, a heavy body on a spherical surface concentric with the earth will maintain itself in that state in which it has been; if placed in movement toward the west (for example), it will maintain itself in that movement."[7] This notion, which is termed "circular inertia" or "horizontal circular inertia" by historians of science, is a precursor to Newton's notion of rectilinear inertia.[8,9]

Note that a pure rectilinear inertial motion cannot exist in reality because there are always forces acting on a body with mass. The closest approximation of a rectilinear inertial motion is the motion of a body on a perfectly horizontal and frictionless surface, e.g., a billiard ball on a frictionless pool table. What maintains the inertial motion in this situation is the fact that the force of gravity applied on the body by Earth is always balanced by an equal force in opposite direction applied on the body by the table. The case of inertial circular motion differs. Here, the centripetal force, which supports the circular motion, is always orthogonal to the tangential velocity vector.

4. Restatement and formalization of Newton's first law

Encouraged by the agreement between our view of inertial motion and the Galileo's view, we dare to put forward the following formal definition of an inertial motion, which encompasses both the rectilinear and the circular types of motion. According to the proposed definition:

A rigid body is said to be in a state of inertial motion if and only if the scalar product between the sum of all the forces acting on the body and its velocity vector is always equal to zero.

Or in mathematical notation:

$$(\sum \vec{\boldsymbol{F}_i}(t)).\, \vec{\boldsymbol{v}}(t) = 0, \text{ for all } t. \quad (2)$$

Note that the condition in Eq. (2) is satisfied (under ideal conditions) only by a state of rest, as well as by uniform rectilinear and circular types of motion. According to equality in Eq. (2), an inertial state requires either that $\|\vec{\boldsymbol{v}}(t)\| = 0$, which describes a state of rest, or when $\|\vec{\boldsymbol{v}}(t)\| \neq 0$, but $\left\|\sum \vec{\boldsymbol{F}_i}(t)\right\| = 0$, which describes the case of uniform rectilinear motion with zero net force, and also when $\|\vec{\boldsymbol{v}}(t)\| \neq 0$, and $\left\|\sum \vec{\boldsymbol{F}_i}(t)\right\| \neq 0$, but the net force acting on the body is always orthogonal to the velocity vector, which is the case of uniform circular motion.

4. Some implications to classical and relativistic physics

Accepting the above restatement of Newton's first law has significant implications to classical and relativistic physics. For example, in classical analysis of many body systems in circular motion, as in fiber-optic gyroscopes (FOGs)[10-12] or in approximation of planetary dynamics,[13-15] one can simplify the analysis by first solving a rectilinear identical system and then "translating" the solution from one systems of coordinates to another, simply by using a "dictionary" like the one depicted in Table A1, i.e., simply by replacing in the derived solution the variables x with θ, v with w, and so forth.

Another implication to classical physics concerns the teaching of rotational motion, where a fictitious centrifugal force, which has no actor, is usually added to balance the real centripetal force acting on the body.[16-17] Our proposed definition in Eq. (2) makes such artificial and nonphysical addition superfluous. With regard to relativistic physics, the system's equivalence between rectilinear and circular motion means that SRT should also apply to uniform circular motion. For Einstein, this would have not been news. In fact, in his seminal 1905 paper, he emphasized that SRT's solution of the twin paradox is independent of whether the travel path is comprised of straight lines or of a closed curve of any shape. In Einstein's words: "If there are two synchronous clocks at A, and one of them is moved along a closed curve with constant velocity [v] until it has returned to A, which takes, say t seconds, then this clock will lag on its arrival at A by $\frac{1}{2}\,t\,(\frac{v}{c})^2$ seconds behind the clock that has not been moved."[4]

Notwithstanding, we argue that the comprehensive definition of inertial motion advanced here poses a serious problem to SRT, as it sharpens the debated contradiction between the theory and the Sagnac effect,[18,19] by making it possible to pit the two against each other – not only in rectilinear motion[20,21] but also in circular motion.

References

1. Galili, I., & Tseitlin, M. *Science & Education* 2003, 12 (1), 45–73 (2003) https://doi.org/10.1023/A:1022632600805.

2 Newton, I. *Newton's Principia: The Mathematical Principles of Natural Philosophy*, 1845 (Translated into English by A. Motte, New York: Daniel Adee).

3. Newton, I. *Mathematical Principles of Natural Philosophy* (University of California Press, Berkeley, CA, revised edition of Motte's translation by F. Cajori, 1934).

4. Einstein, A. On the electrodynamics of moving bodies (1905). In J. Stachel (Ed.). *The collected papers of Albert Einstein,* 2, 304 (Princeton, NJ: Princeton University Press, 1989).

5. Kalman, R. *J.S.I.A.M. Control* 1 (2), 152-192 (1963).

6. Ogata, K. *System Dynamics* (Prentice Hall, 4th. Edition, 2003).

7. Drake, S. *Discoveries and Opinions of Galileo* **(**Doubleday Anchor, New York, 113–114. 1957).

8. Chalmers, A. Galilean relativity and Galileo's relativity. In: *Correspondence, Invariance and Heuristics: Essays in Hounour of Heinz Post*. S. French, & H. Kamminga (Eds.), pp. 199–200 (Kluwer Academic Publishers, Dordrecht, 1991).

9. Dijksterhuis, E. J. *The mechanisation of the world picture* (Oxford University Press, Oxford, 1961).

10. W. Leeb, G. Schiffner, and E. Scheiterer, E. *Appl. Opt.* 18, 1293 (1979).

11. H. Lefevre, *The Fiber-Optic Gyroscope* (Artech House: Boston, 1993).

12. W. K. Burns, *Fiber Rotation Sensing* (Academic Press, Boston, 1994).

13. Carl D. Murray, Stanley F. Dermot. *Solar System Dynamics* (Cambridge University Press, 1999).

14. Laughlin, G., Chambers, J., & Fischer, D. *The Astrophysical Journal*, 579, 455–467 (2002).

15. Roberto Sanchis-Ojeda, Daniel C. Fabrycky, Joshua N. Winn, Thomas Barclay, Bruce D. Clarke, Eric B. Ford, Jonathan J. Fortney, John C. Geary, Matthew J. Holman, Andrew W. Howard, Jon M. Jenkins, David Koch, Jack J. Lissauer, Geoffrey W. Marcy, Fergal Mullally, Darin Ragozzine, Shawn E. Seader, Martin Still, & Susan E. Thompson. *Nature*, 487, 449–453 (2012).

16. Rothman, M. A. *Discovering the Natural Laws: The Experimental Basis of Physics* (Courier Dover Publications, 1989).

17. Borowitz, S., & Bornstein, L. A. *A Contemporary View of Elementary Physics*. (McGraw-Hill, 1968).

18. Sagnac, M. G. C. *R. Acad. Sci***.** 157, 708–710 (1913).

19. Ashby, N. *Relativity in Rotating Frames* (Springer, 2004).

20. Wang, R., Zheng, Y., Yao, A., & Langley, D. *Phys. Lett***.** A, 312 7–10 (2003).

21. Wang, R., Zheng, Y., & Yao, A. *Phys. Rev. Lett.* 93 (14), 143901 (3 pages) (2004).

Appendix A

Table 1

Dynamical equations of rectilinear and circular systems

Variable	Rectilinear	Circular	General
Position	x	θ	p_1
Velocity	$v = \frac{dx}{dt}$	$\omega = \frac{d\theta}{dt}$	$p_2 = \frac{dp_1}{dt}$
Acceleration	$a = \frac{dv}{dt}$	$\alpha = \frac{d\omega}{dt}$	$p_3 = \frac{dp_2}{dt}$
Mass/Inertia	M	I	p_4
Newton's second law	$F = ma$	$\tau = I\,\alpha$	$p_5 = p_4\,p_3$
Work	$W = \int F dx$	$W = \int \tau\, d\theta$	$p_6 = \int p_5 dp_1$
Kinetic energy	$E = \frac{1}{2} mv^2$	$E = \frac{1}{2} I\,\omega^2$	$p_7 = \frac{1}{2} p_4\,{p_2}^2$
	…….	…….	…….