Circular and rectilinear Sagnac effects are dynamically equivalent and contradictory to special relativity theory

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Abstract: The Sagnac effect, named after its discoverer, is the phase shift occurring between two beams of light, traveling in opposite directions along a closed path around a moving object. A special case is the circular Sagnac effect, known for its crucial role in GPS and fiber-optic gyroscopes. It is often claimed that the circular Sagnac effect does not contradict special relativity theory (SRT) because it is considered an accelerated motion, while SRT applies only to uniform, non-accelerated motion. It is further claimed that the Sagnac effect, manifested in circular motion, should be treated in the framework of general relativity theory (GRT). We counter these arguments by underscoring the fact that the dynamics of rectilinear and circular types of motion are completely equivalent, and that this equivalence holds true for both non-accelerated and accelerated motion. With respect to the Sagnac effect, this equivalence means that a uniform circular motion (with constant w) is completely equivalent to a uniform rectilinear motion (with constant v). We support this conclusion by convincing experimental findings, indicating that an *identical* Sagnac effect to the one found in circular motion, exists in rectilinear uniform motion. We conclude that the circular Sagnac effect is fully explainable in the framework of inertial systems, and that the circular Sagnac effect contradicts special relativity theory and calls for its refutation.

Keywords: Sagnac Effect; Special Relativity Theory; Lorentz Invariance; Systems Equivalence; GPS.

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I. Introduction

The Sagnac effect is a phase shift observed between two beams of light traveling in opposite directions along the same closed path around a moving object. Called after its discoverer in 1913[1], the Sagnac effect has been replicated in many experiments[2-5].

The circular Sagnac effect is a special case of the general Sagnac effect, which has crucial applications in fiber-optic gyroscopes (FOGs)[6-10] and in navigation systems such as GPS[2,2,11]. The amount of the circular Sagnac effect is calculated using a Galilean summation of the velocity of light and the velocity of the rotating frame ($c \pm \omega r$). The difference in time intervals of two light beams sent clockwise and counterclockwise around a closed path on a rotating circular disk is $\Delta t = \frac{2\,v\,l}{c^2}$, where R is the radius of the circular path, $v = \omega R$ is the speed of the circular motion, and $l=2\pi R$ is the circumference of the circle. In fact, the Galilean summation of c and $\pm\, wr$ contradict SRT's second axiom and the Lorentz transformations. Nonetheless, it is consensual that the Sagnac effect does not falsify special relativity theory (SRT)[12], because it is manifested in circular motion, which is considered an accelerated motion[13-15], while SRT applies only to inertial (non-accelerated) systems. Based on this consensus, in the GPS, concurrent corrections for the Sagnac effect and SRT's time dilation are made. Moreover, some theoreticians claimed that the Sagnac effect manifest in circular motion, should be treated in the framework of general relativity theory (GRT) and not SRT[16,17].

The view that the Sagnac effect is a property of rotational systems is strongly disproved by Wang and his colleagues[18-20] who conducted experiments demonstrating that an *identical* Sagnac effect, to the one found in circular motion, exists in rectilinear uniform motion[21]. Using an optical fiber conveyor, the authors measured the travel-time difference between two counter propagating light beams in a uniformly moving fiber. Their finding revealed that the travel-time difference in a fiber segment of length Δl moving at a speed v, was equal to $\Delta t = 2v\Delta l/c2$, whether the segment was moving uniformly in rectilinear or circular motion. The existence of a Sagnac effect in rectilinear uniform motion is at odds with the prediction of SRT, and with the Lorentz invariance principle and, thus, should qualify as a strong refutation of both theories. However, despite the fact that Wang and his colleagues published their findings in well-respected mainstream journals, their falsification of SRT's second axiom, and the Lorentz transformations, has been completely ignored. To the best of my knowledge, no effort was done by SRT experimentalists to replicate Wang at al.'s falsifying test of SRT.

In this short note, we provide strong theoretical support to the aforementioned findings regarding the identity between the rectilinear and circular Sagnac effects, by underscoring the fact that, in disagreement with the acceptable Newton's definition of inertial motion, the dynamics of rectilinear and circular types of motion are completely equivalent, and that this equivalence holds true for both non-accelerated and accelerated motion. We elucidate this fact in the following section and draw conclusions regarding the contradiction between the rectilinear and circular Sagnac effects, and the predictions of special relativity theory.

II. On the equivalence between circular and rectilinear kinematics

The common view in physics is that the above-mentioned two types of motion are, in general, qualitatively different. Linear motion with constant velocity is considered inertial, while circular motion, even with constant radial velocity, is considered an accelerated (non-inertial) motion. The above view is not restricted to the Sagnac effect, or to relativistic motion, but it is believed to be a general distinction in classical mechanics as well, and is repeated in all books on physics. This common view maintains that the centrifugal force acting on a rigid rotating mass causes continual change in its velocity vector, reflected in change in its direction (keeping it in a tangential direction to the circular path).

Here, we challenge this convention by claiming that there is a one-to-one correspondence between the linear and circular types of motion. In the language of systems analysis, the two types of motion are completely *equivalent* systems[22,23]. The proof for our claim is trivial. To verify that, consider a dynamical system of any type (physical, biological, social, etc.), which could be completely defined by a set of dynamical parameters p_i (i = 1, 2, .. , n), and a set of equations *R* defined in (1):

$$R = \{p_2 = \dot{p_1},\ \ p_3 = \ddot{p_1}, p_5 = p_3\, p_4, p_6 = \int p_5 dp_1\,, p_7 = \tfrac{1}{2}\, p_4\, {p_2}^2\}. \qquad (1)$$

If we think of p_1, p_2, p_3, as representing rectilinear position *x*, velocity *v*, and acceleration *a*, respectively, and of p_4, p_5, p_6, p_7, as mass *m*, rectilinear force *F*, work *W*, and kinetic energy *E*, respectively, then the dynamical system defined by *R* gives a full description of a classical *rectilinear motion* (see Table 1, Appendix A). Alternatively, if we think of p_1, p_2, p_3, as representing angular position *θ*, velocity *w*, and acceleration α, respectively, and of p_4, p_5, p_6, p_7, as radial inertia *I*, torque *τ*, work *W*, and kinetic energy *E*, respectively (see Table 1, Appendix A), then the dynamical system defined by *R* gives a full description of a classical *circular motion* (Q.E.D.).

It is worth noting that the equivalence between rectilinear and circular dynamical systems is not restricted to the special case of rotation with constant angular velocity or even with constant acceleration.

We note here that the equivalence demonstrated above between the dynamics of uniform rectilinear and uniform circular types of motion is inconsistent with Newton's first law, which states that, *unless acted upon by a net unbalanced force, an object will remain at rest, or move uniformly forward in a straight line*[24]. According to this definition of inertial motion, which was adopted by Einstein, a circular motion with uniform radial velocity, is considered an accelerated motion. However, the above demonstrated equivalence is at odds with Newton and Einstein's views of inertial systems. In fact, based on Newton's mechanics, the first law for circular motion could be derived simply by replacing, in the original statement of the law, the words "straight line" by the word "circle," thus yielding the following law:

"*A body in circular motion will continue its rotation in the same direction at a constant angular velocity unless disturbed.*"

Quite interestingly, our view of what defines an inertial system is in complete agreement with Galileo's interpretation of inertia. In Galileo's words: "All external impediments removed, a heavy body on a spherical surface concentric with the earth will maintain itself in that state in which it has been; if placed in movement toward the west (for example), it will maintain itself in that movement"[25]. This notion, which is termed "circular inertia" or "horizontal circular inertia" by historians of science, is a precursor to Newton's notion of rectilinear inertia[26,27].

In fact, a rectilinear inertial motion, according to Newton's definition, cannot exist in reality because there are always forces acting on a body with mass. The closest approximation of a rectilinear inertial motion is the motion of a body on a perfectly horizontal and frictionless surface, like a billiard ball on a pool table. However, in such cases, the net force on the body is not zero. What maintains the inertial motion is the fact that, at all times during the body's motion, the gravitational force is orthogonal to the velocity vector. But this is also the case of circular motion with fixed radial velocity, in which the centripetal force, which supports the circular motion, is always orthogonal to the tangential velocity vector.

A deeper inquiry of the different opinions of the notion of "inertia" throughout the history of physics is beyond the scope and aims of the present paper. Nonetheless, we dare to put forward the following definition of an inertial motion, which agrees well with Galileo's conception. According to the proposed definition, *a rigid body is said to be in a state of inertial motion if and only if the scalar product between the sum of all the forces acting on the body, and its velocity vector is always equal to zero,* or

$$\left(\sum \overrightarrow{\boldsymbol{F}_i}(t)\right).\ \vec{\boldsymbol{v}}(t) = 0 \quad \text{for all } t. \tag{2}$$

Note that the condition in (2) is satisfied (under ideal conditions) only by two types of motion: the rectilinear and the circular types of motion.

III. Conclusions and general remarks

Although it is not the subject of the present paper, our demonstration of the complete equivalence between the circular and the rectilinear dynamics, based on Newtonian kinematics, calls for a reformulation of Newton's first law, which is in line with Galileo's view of inertial motion. Such reformulation is far from being semantic. By accepting the fact that the circular and rectilinear dynamics are completely equivalent, it becomes inevitable but to conclude that the dynamics of the Sagnac effect in uniform circular motion is completely equivalent to the dynamics of the Sagnac effect in uniform rectilinear motion, and that both effects contradict special relativity theory.

Moreover, the claim that the circular Sagnac effect should be treated in the framework of GRT simply does not make sense. In most Sagnac experiments, the experimental apparatus is of small physical dimensions, allowing us to assume that the gravitational field in the apparatus is uniform, thus excluding any GRT effects.

Another erroneous justification for the coexistence between special relativity theory and the Sagnac effect is that the observed effect could be derived from SRT[28,29], e.g., by using Lorentz transformations expressed in coordinates of a rotating frame. This claim is based on fact that the difference between the detected effect, and the one predicted by SRT, amounts to $\frac{1}{2}\left(\frac{v}{c}\right)^2$, which is claimed to be negligible for all practical cases and applications. We argue that this line of reasoning is erroneous in more than one aspect: 1) The directionality of the Sagnac effect is dependent on the direction of light travel with respect to the rotating object, whereas the time dilation effect is independent of the direction of motion; 2) Special relativity is founded on the axiom postulating that the motion of the source of light, relative to the detector, has no effect on the measured velocity of light, whereas in the Sagnac effect, the Galilean kinematic composition of velocities ($c+v$, $c-v$) is the reason behind its appearance; 3) At relativistic velocities, for which SRT predictions become practically relevant, the second order of v/c can amount to values approaching infinity; and (4) The aforementioned difference, even if infinitesimally small, as in the case of GPS, could not be overlooked because it is a systematic deviation between the model's prediction and reality, and not some kind of statistical or system's error.

Finally, we note that the abundance of experimental findings in support of SRTs, mainly its prediction of time dilation[30-33], is no more than what Carl Popper calls "confirmation tests" of the theory. What is needed is to subject SRT to stringent tests, i.e., to what Carl

Popper has termed a “risky” or “severe” falsification test[34,35]. Evidently, the Sagnac effect, arising in rectilinear and in circular motion, qualifies as a “severe” test of SRT. But such experiments have already been performed in linear and circular motion by Wang and his colleagues[18-20], and we have shown here that the two types of motion are completely equivalent.

We have no other way but to conclude that the physics community is acting irrationally and unscientifically. The logic behind the second axiom of SRT is shaky, and Herbert Dingle’s argument[36-38] that it leads to contradiction, has never been answered without violating the principle of relativity itself. On the experimental side, the Sagnac effect detected in linear motion is a clear falsification of the theory, and we have closed the loophole by showing here that what applies to rectilinear motion applies to circular motion.

In science, it takes one well-designed and replicated experiment to falsify a theory. As put most succinctly by Einstein himself: “If an experiment agrees with a theory it means ‘perhaps’ for the latter… but If it does not agree, it means ‘no.’”[39] (p. 203). Meanwhile, an experiment falsifying SRT is flying above our heads in the GPS and similar systems, but there are no good and brave experimentalists to observe them and register their results.

We are not aware of a similar case in the history of modern science, where a theory, which defies reason, and contradicts with the findings of crucial tests, holds firm. We believe that it is due time for a serious reconsideration of SRT and the Lorentz transformations.

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Appendix A

Table 1
Dynamical equations of rectilinear and circular systems

Variable	Rectilinear	Circular	General
Position	x	θ	p_1
Velocity	$v = \frac{dx}{dt}$	$\omega = \frac{d\theta}{dt}$	$p_2 = \frac{dp_1}{dt}$
Acceleration	$a = \frac{dv}{dt}$	$\alpha = \frac{d\omega}{dt}$	$p_3 = \frac{dp_2}{dt}$
Mass/Inertia	M	I	p_4
Newton's second law	$F = ma$	$\tau = I\,\alpha$	$p_5 = p_4\,p_3$
Work	$W = \int F dx$	$W = \int \tau\ d\theta$	$p_6 = \int p_5 dp_1$
Kinetic energy	$E = \frac{1}{2} mv^2$	$E = \frac{1}{2} I\,\omega^2$	$p_7 = \frac{1}{2} p_4\,{p_2}^2$
	…….	…….	…….