

# Dark Matter is Negative Mass

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The fundamental properties of negative mass can explain important characteristics of dark matter. 1) additional centripetal effects, 2) explanations derived from fundamental principles about the reason why dark matter does not have electromagnetic interaction, 3) repulsive gravity ensuring almost even distribution and lower interaction of dark matter, 4) accelerating expansion of the universe can be explained with negative mass. Therefore, we should seriously examine the negative mass model.

## I. Introduction

The dark matter had been reported by Fritz Zwicky, 1933, [1] [2] and then was rediscovered by Vera Rubin and Kent Ford, 1979. [3] Since then, it has emerged as one of important issues in the academic world.

There have been a lot of articles on the dark matter, a very important issue regarding the formation of galaxy and evolution of the universe. [4] [5] Therefore, this paper does not explain such things, but will focus on the negative mass model.

### 1. Brief current conditions of dark matter candidates

1) WIMP succeeds in rough explanations by combining with Cosmological Constant, [4] [5] but fails in the direct detection experiment. [6] [7] [8] Moreover, it also fails in production with LHC, a completely different approach. [9] [10]

2) Until now, supersymmetry particles have not been detected from LHC [9] [10] and the evidence that 4th neutrino exists has not been found by observation of the Plank satellite. [11]

3) The MOND theory usually requires fine tuning of each galaxy, [12] has difficulty explaining Bullet Cluster [13] and collides with many observational evidences of the dark matter. [4] [5]

## II. Main Characteristics of Negative Mass

**1. Positive mass is stable at the low energy state, while the negative mass is stable at the high energy state.** [14] [15]

One of physics' fundamental principles, 'lower energy state is associated with stability' can be only applied to positive mass. However, both negative mass and negative energy level have been denied, as it has been wrongly applied to negative mass.

$$\begin{aligned} \vec{F} &= -m_- \vec{a} \\ (m_- > 0) \end{aligned} \tag{1}$$

$$\vec{a} = -\frac{\vec{F}}{m_-} \tag{2}$$

When negative mass exists within potential with maximal and minimal points, different directions of force and acceleration should be considered for negative mass.

**The acceleration of negative mass is opposite to the direction of force. Therefore, the negative mass has harmonic oscillation at the maximum point and it is also stable at the maximum point.**

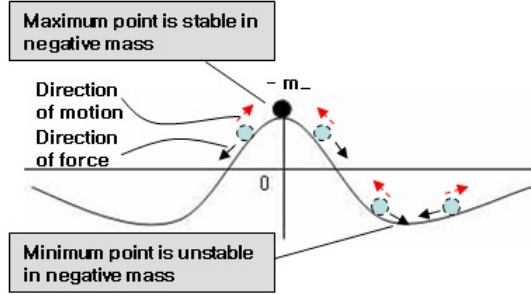


Figure 1: When there is negative mass in potential which has a point of maximum value and a point of minimum value.

In the case of positive mass, it was stable at the minimum point at which energy is the low. However, in case of negative mass, stable equilibrium is a point of maximum value, not a point of minimum value.

It is stable at a low energy state in the case of positive mass. However, it is stable at a high energy state in the case of negative mass. Duetto this, “the problem of transition to minus infinite energy level” does not occur, therefore negative mass(energy) and positive mass(energy) can exist stably in our universe.

**2. Negative mass has both negative inertial and gravitational mass.**

There are repulsive gravitational effects between negative masses.

One of problems regarding movement of negative mass, which researchers are likely to misunderstand is that if there are negative mass particles around the earth (or galaxy), large positive mass, they may not fly to the universe, but freely fall to the earth (or galaxy). [15]

**3. See the following video produced to help understand movement of negative mass. [16]**

<http://www.youtube.com/watch?v=MZtS7cBMlc4>

### III. Dark Matter is Negative Mass

**1. Negative mass is an object whose existence is required by the law of the conservation of energy. [15] [16]**

If the conservation of energy that “energy will be always conserved” is applied to the initial state of the universe, this gives rise to the question of “where the energy of our universe did come from?”. The most natural answer to this question is the assumption that the energy itself was not something created and that zero-energy state went through phase transition keeping conservation of energy and generating negative and positive energy. Therefore, in order to offset the known positive energy of matter, negative energy is needed.

The negative energy (mass) is the demand and outcome of the law of conservation of energy.

$$\begin{array}{c}
 \text{Energy conservation} \\
 \overbrace{E_T = 0 = (+E) + (-E) = \sum +m_+ c^2 + \sum -m_- c^2 + \sum +K_{+m} + \sum -K_{-m} + \sum U = 0} \\
 \underbrace{\hspace{1cm}} \quad \underbrace{\hspace{1cm}} \quad \underbrace{\hspace{1cm}} \quad \underbrace{\hspace{1cm}} \quad \underbrace{\hspace{1cm}} \quad \underbrace{\hspace{1cm}} \\
 \text{Nothing} \quad \text{Pair Creation} \quad \text{Positive mass} \quad \text{Negative mass} \quad \text{Kinetic Energy} \quad \text{Potential Energy} \\
 \text{Something} \quad \text{Something}
 \end{array}$$

Figure 2: Zero Energy Universe model-2. [17] Model that considers negative mass as negative energy. Simply put, negative mass indicates that energy can be locally distributed and has characteristics as particle.

**2. Negative mass does not have charges, so is unlikely to have electromagnetic interaction.**

Think about whether negative mass has charges.

It is possible that the model designer assumes two models of negative mass with and without charges. However, let's infer what a state is more proper, based on the existing physical laws.

We have observed pair production due to the collision of photon with atomic nucleus. Such phenomena suggest that the process of pair production exists where the charge  $Q$  does not exist. Now, let's examine whether when negative and positive energy exist, each of them can have charges.

1) Electrostatic self-energy  $U_{es}$

Since the charge  $Q$  is the set of infinitesimal charge  $dQ$ , electric force is operated between infinitesimal charges and therefore, electrostatic self-energy exists due to the presence of charge  $Q$  itself.

When spherical symmetry and even distribution of charges are assumed,  $U_{es}$ , the electrostatic self-energy (or electrostatic binding energy ( $-U_{es}$ )) is as follows:

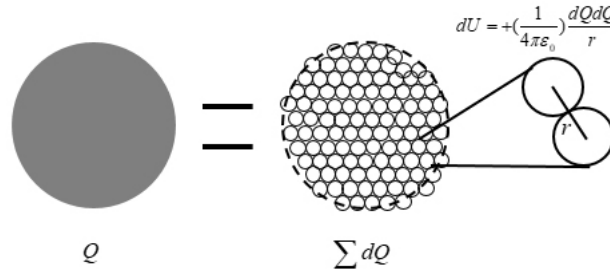


Figure 3: Since all charge  $Q$  is a set of infinitesimal charge  $dQ$ s and each  $dQ$  is electromagnetic source, too, there exists electrostatic potential energy among each of  $dQ$ s.

$$U_{es} = +\frac{3}{5} \left( \frac{1}{4\pi\epsilon_0} \frac{Q^2}{R_{es}} \right) \quad (3)$$

However, **electrostatic self-energy has positive values for both positive and negative charges.**

$$U_{es} = +\frac{3}{5} \left( \frac{1}{4\pi\epsilon_0} \frac{Q_+^2}{R_{es}} \right) = +\frac{3}{5} \left( \frac{1}{4\pi\epsilon_0} \frac{Q_-^2}{R_{es}} \right) > 0 \quad (4)$$

2) Positive energy can have charges, since a part of its positive energy or mass is able to be transformed into electrostatic self-energy of charges.

$$E_T = +E_0 = +E_0 + (-U_{es}) + (+U_{es}) = (+E_0 - U_{es}) + \frac{3}{5} \left( \frac{1}{4\pi\epsilon_0} \frac{Q^2}{R_{es}} \right) \quad (5)$$

However, **negative energy does not have positive energy to produce charges, so it is very unlikely to have charges.**

3) Limit of spontaneous process by the principle of negative mass

In positive mass, the transformation for  $E_0$  to  $E_0 - U_{es}$  is the process in which energy is lowered, which corresponds to the fundamental principle that positive mass is stable at the low energy state.

$$E_T = -E_0 = -E_0 + (-U_{es}) + (+U_{es}) = -(E_0 + U_{es}) + \frac{3}{5} \left( \frac{1}{4\pi\epsilon_0} \frac{Q^2}{R_{es}} \right) \quad (6)$$

Meanwhile, in negative mass, the transformation for  $-E_0$  to  $-E_0 - U_{es}$  is the process in which energy is lowered, which does not correspond to the fundamental principle that the negative mass is stable at the high energy state.

Therefore, negative mass is unlikely to have charges, according to this principle.

4) **Electrostatic self-energy has a potential to explain substantial parts of elementary particle mass.** [18]

Most parts of elementary particles' rest mass are likely to be originated from the electrostatic(or electromagnetic) self-energy, due to the presence of charges. Although there are many points to be additionally considered, such a model's suggestions for the magnitude of quark should be currently examined with electrostatic self-energy equivalent to mass energy.

$$E = mc^2 = \frac{3}{5} \left( \frac{1}{4\pi\epsilon_0} \frac{Q^2}{R_{es}} \right) \quad (7)$$

$$R_{es} = \frac{3}{5} \left( \frac{1}{4\pi\epsilon_0} \frac{Q^2}{mc^2} \right) \quad (8)$$

For convenience of calculation, it is defined as follows  $Q = ke$ ,  $mc^2 = \alpha(eV)$

$$R_{es} = \frac{3}{5} \left( \frac{1}{4\pi\epsilon_0} \frac{Q^2}{mc^2} \right) = \frac{3}{5} \left( \frac{1}{4\pi\epsilon_0} \right) \left( \frac{k^2 e^2}{mc^2} \right) = \frac{k^2}{\alpha} (8.64 \times 10^{-10}) [m] \quad (9)$$

Since mass is  $2.4MeV/c^2$  and quantity of electrical charge is  $(+2/3)e$ , for up quark, if  $k=2/3$  and  $\alpha = 2.4 \times 10^6$  are substituted,

$$R_{es-U} = \frac{k^2}{\alpha} (8.64 \times 10^{-10}) [m] = \frac{\left(\frac{2}{3}\right)^2}{2.4 \times 10^6} (8.64 \times 10^{-10}) [m] = 1.6 \times 10^{-16} m \quad (10)$$

By interpreting this, it can be inferred that the mass of up quark is derived from electrostatic self-energy, if the radius of up quark is approximately  $R_{es-U} = 1.6 \times 10^{-16} m$  and here, charges are evenly distributed.

If inferences of other quarks can be acquired,

Down quark's mass:  $4.8MeV/c^2$ , charge:  $(-1/3)e$

$$R_{es-D} = 2.0 \times 10^{-17} m \quad (11)$$

Strange quark's mass:  $95MeV/c^2$ , charge:  $(-1/3)e$

$$R_{es-S} = 1.01 \times 10^{-18} m \quad (12)$$

Charm quark's mass:  $1.275GeV/c^2$ , charge:  $(2/3)e$

$$R_{es-C} = 3.01 \times 10^{-19} m \quad (13)$$

Bottom quark's mass:  $4.18GeV/c^2$ , charge:  $(-1/3)e$

$$R_{es-B} = 2.29 \times 10^{-20} m \quad (14)$$

Top quark's mass:  $172.44GeV/c^2$ , charge:  $(2/3)e$

$$R_{es-T} = 2.22 \times 10^{-21} m \quad (15)$$

It is desirable to think that the magnitude of quark from such a simple model is smaller than that of an atomic nucleus.

**The fact that the six quark radii vary by roughly  $10^1$  orders of magnitude, suggests that there is some principle or rule. We considered only the electrostatic self-energy value. Nonetheless, some rules seem to suggest that the self-energy of the charge may be the most important term.**

If there is a fourth-generation quark, we can predict the mass of the fourth-generation quark by using this reasoning.

The calculation above strongly suggests the possibility that electromagnetic self-energy due to the presence of charges can explain some parts of elementary particle mass. In order to make a precise model, the model related with kinetic and other energy, relativistic effects and orbital model(in case of electron) related with charge distribution can be also considered

This inference also suggests that negative energy is unlikely have charges, because electrostatic self-energy is positive mass (energy), which provides an evidence about the fact that electromagnetic interaction does not occur.

Hence, negative mass does not have any charges and is very unlikely to have electromagnetic interaction. In addition, even if other fundamental forces are applied to the negative mass, the fundamental charges underlying the forces may be supposed to have positive self-energy.

**This corresponds to characteristics required for the dark matter, and also gives some explanations to ‘why does the dark matter not have electromagnetic interaction?’**

**3. Centripetal force effects that negative mass outside the galaxy creates inside it [15] [16]**

If negative mass(energy) and positive mass(energy) were came into being together at the beginning of universe, since positive masses have attractive gravitational effects with each other, so it forms stars and galaxy. However, negative masses have repulsive gravitational effects towards each other, so it cannot form any giant structure and may spread out almost uniformly across the whole area of universe.

Owing to the effect of negative mass and positive mass, negative mass disappears near massive positive mass structures (such as the galaxy and galaxy clusters, etc.) after meeting positive mass. However, negative mass, which came into existence at the beginning of universe, can still exist in a vacuum state outside of general galaxy.

The current structure of the galaxy is a structure that survived in the pair-annihilation of positive mass and negative mass and, since negative mass existed outside of this galaxy structure, therefore it has not been observed at the Earth.

Negative mass has repulsive gravitational effect towards each other so it cannot make massive mass structures like stars or galaxies.

We observed only the effect of matter in the Earth. However, it is presumed that the observation of the gravitational effect of dark matter and dark energy at the galactic scale or above, is due to the existence of negative energy and negative mass outside the galactic structure.

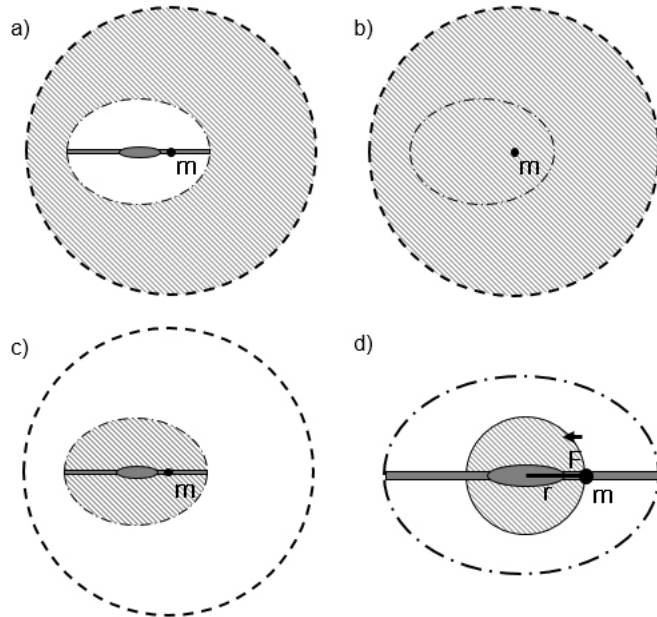


Figure 4: The structure of the galaxy surrounded by negative mass that is distributed equally.(a) The white area is the area where negative mass almost does not exist. The oblique line area is the area where negative mass is distributed.

Let’s examine the effect of the centripetal force of negative mass that is outside the galaxy on mass m, which is located within the galaxy.

1) If we assume that the empty space is filled with both negative mass and positive mass of the same density then,

$$\text{Total energy of the white empty space} = 0 = (+mc^2) + (-mc^2) = 0$$

2) Negative mass is now uniformly distributed over the whole area so the effect of negative mass on mass m becomes 0.

3) The remaining positive mass is distributed over the white area at the density of negative mass. Gravity that uniformly distributes positive mass works on positive mass m located on radius r is worked upon only by the distribution of mass within radius r. -Shell Theorem.

Therefore, the effect of negative mass that remains outside of the galaxy is compressive to the distribution of positive mass within the radius r in the galaxy.

This means that the dark matter, consisting of negative mass outside galaxy, has additional effect of centripetal force on stars within the galaxy.

$$\frac{mv^2}{r} = \frac{G(M_{matter} + M_{dark})m}{r^2} \tag{16}$$

$$v = \sqrt{\frac{G(M_{matter} + M_{dark})}{r}} = \sqrt{\frac{G(M_{matter} + \frac{4\pi r^3 \rho_{dark}}{3})m}{r}} \approx \sqrt{\frac{4\pi G \rho_{dark}}{3}} r = \sqrt{\frac{4\pi G \rho_-}{3}} r \tag{17}$$

This effect suggests that the further from the center of the galaxy, the more mass effect exists and agrees with the current situation where the further from the center of the galaxy, the more dark matter exists.

If dark matter is exist in the galaxy, dark matter will be affected by the distribution of stars and interstellar gas in the galaxy. That is, the density of dark matter around the solar system will be higher than the average of the galaxy. The existing CDM model suggests that there is a local distribution of dark matter around the local gravitational source (such as solar system). Some models offer a theoretical background on how dark matter is not detected in the solar system by lowering the total amount of dark matter around the solar system by assuming that dark matter is completely evenly distributed in the galaxy.

This model explains the presence of additional gravitational effects in the galaxy and the reason why the dark matter is not found on the earth or in the vicinity of the solar system. **In this model, dark matter (negative mass) exists outside the galaxy, and there is only a gravitational effect in the galaxy** that is not affected by the local gravitational source (such as solar system) in the galaxy.

4) Computer simulation [16]

<https://youtu.be/MZtS7cBMic4?t=364>

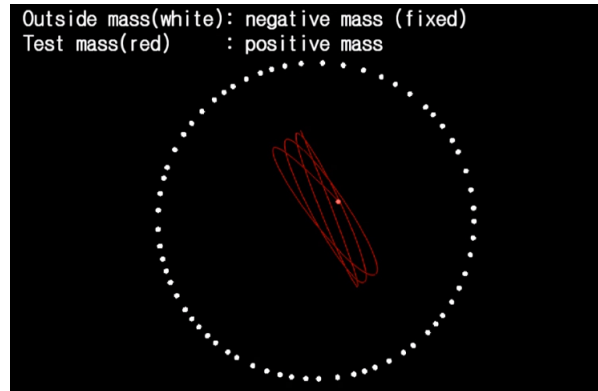


Figure 5: White : negative mass(fixed), Red : positive mass(free). The positive mass vibrates.

If the negative mass is disposed at the outline, the positive mass vibrates. Therefore a kind of centripetal force exists. This corresponds to the “centripetal force” when considering rotation of the galaxy.

5) The model above is a principle explanation. Moreover, axisymmetric distribution due to rotation of the galaxy should be also considered. It is also possible to consider repulsive force affected by negative mass as well as negative mass’s free fall onto the center of the galaxy.

#### **4. Low interaction between dark matters: occurring from repulsive gravitational effects between dark matters**

The existing CDM models argue that dark matters are almost evenly distributed as they only have gravitational interaction, except for electromagnetic one, even though dark matters are five times or hundreds times more than other masses. [19] However, is this process true?

There are more certain explanations and rules in the model of negative mass.

Positive masses have attractive gravitational effects with each other, so it forms stars and galaxy. However, negative masses have repulsive gravitational effects towards each other, so it cannot form any giant structure and may spread out almost uniformly across the whole area of universe. Also, negative mass exists outside the galaxy, and there is only a gravitational effect inside the galaxy.

This provides strong explanations to the phenomenon that current dark matters do not strongly interact with each other, but are almost evenly distributed.

#### **5. Negative mass can also explain dark energy. [15] [16] [20]**

Negative mass engenders antigravity which is consistent with effects of dark energy that allows for accelerating expansion of the universe.

This model can prove the energy composition(Matter : Dark matter : Dark energy) ratio of the universe and CCC (Cosmological Constant Coincident) Problem. [16] [21]

Roughly calculation :

Average of WMAP and Planck - Matter : Dark Matter : Dark Energy = 4.75% : 25.05% : 70.20%

Average of Pair Creation Model - Matter : Dark Matter : Dark Energy = 4.75% : 25.00% : 70.25%

#### **6. Gravitational lens effect by negative mass [15]**

Generally, it is possible to examine the existence of dark matter with the existence of additional mass with the effect of gravitational lensing. If negative mass is dark matter, so if we try to examine the effect of gravitational lensing, previously the gravity between positive mass is attraction, so it has the shape of convex lens to collect within the form, whereas the force between negative mass and positive mass is repulsion, so a set of massive negative mass can make the effect that distorts observation target in the form of concave lens.

In order to observe gravitational lens effects due to negative mass in the universe, the negative mass should independently constitute large mass. However, it is not possible, since the negative masses have repulsive gravitational interaction with each other. Therefore, it is difficult for us to observe gravitational lens effects independently caused by negative mass. However, there are spaces in which negative masses are concentrated, in the universe. They are in the vicinity of the galaxy or the cluster of galaxies, which consist of positive masses, because of attractive gravitational effects of large positive mass on negative mass.

But in the hypothesis of dark matter with negative mass in this study, negative mass is distributed out of galaxy, not within galaxy. Therefore, the effect of concave gravitational lensing by negative mass out of galaxy will be presented to an observer on the earth in the same form as the effect of convex gravitational lensing which galaxy works.

For the situation presented on the left when observing the light on the earth which passed through galaxy surrounded with negative mass, this study suggested that there is invisible dark matter in galaxy, and owing to this gravity it can be happened owing to the effect of convex lens, it means, if there is any matter on the left and right side of galaxy that plays a role of concave lens, it is difficult to classify convex lens and its form.

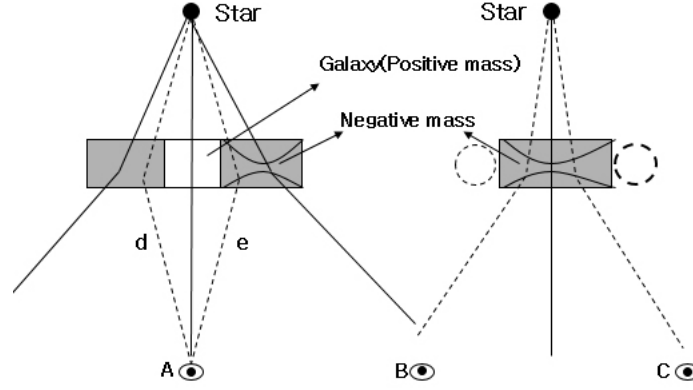


Figure 6: Concave Gravitational lensing effect. The left side is the Gravitational lensing effect that occurs when the galaxy is surrounded by negative mass. The right side is Gravitational lensing effect that occurs when negative mass exists alone

If we consider the situation that we can observe purely the effect of concave gravitational lensing, that situation is just the same as the situation when there is single concave lens on the right in Figure above.

First, an observer on the earth must observe it on Position B and C after moving by considerably great distance as compared with the size of galaxy, but the earth has never changed the position of observation from the target for distant observation.

second, real observation on the earth is the single B or C situation in Figure on the right, and if an observer on the earth performed observation previously on C, generally an observer of C would describe the effect of gravitational lensing is generated because there is invisible dark matter on the right of black and white area(circle drawn with dotted line on the right).

Anyway, it is possible that gravitational lens effects by negative mass are similar with those by positive mass. However, according to this model, the formula for gravitational lens based on the existing positive mass may be accurate for the scale of each fixed star, but inaccurate for the scale more than the galaxy.

### 7. Other characteristics related with dark matter

1) Dark matter is required by rotation curve of the galaxy. However, why is it not detected on the earth or the solar system?

According to negative mass model, negative mass exists outside the galaxy, while additionally exerting centripetal force effects inside the galaxy, so it cannot be observed in the vicinity of the earth. Dark matter (negative mass) exists outside the galaxy, and there is only a gravitational effect in the galaxy.

2) Why is negative mass not detected in LHC?

Broken particles also only produce particles with positive energy, since the experiment is conducted only with positive energy particle.

3) Negative mass satisfies the condition that requires dark matter be particles outside the Standard Model.

4) Lower interaction between dark matters and almost even distribution: occurring from repulsive gravitational effects between negative masses.

5) Peculiar phenomenon where dark matter is observed to be separated with the galaxy, galaxies with little dark matter [22]

Negative mass can be separated with the galaxy, in this model. For example, this phenomenon can occur, if nearby galaxies attract much of negative mass.

6) Although negative mass may undergo pair-annihilation by colliding with positive mass, this process can be limited by other physical laws.



For example, consider the charge conservation law. When a charge with positive energy collides with negative mass, they may undergo pair annihilation, from the perspective of energy. However, if an electron (or positive energy) disappears, where should charge go? In that case, the conservation law of electrical charge may be invalid. If the conservation law of electrical charge should be held, such a pair-annihilation may be prohibited. In other words, **the pair-annihilation is unlikely to occur even when negative mass meets positive mass.**

Negative mass essentially satisfies many characteristics of dark matter, without introducing a separate assumption. It is necessary to seriously examine the negative mass model just because negative mass is required by the law of conservation of energy and can explain accelerating expansion of the universe.

**Way to go!**

**Newton and Einstein already died, but we live now!, though sometimes we feel sorrow.**

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