

Refutation of Metamath proof explorer axiom ax-garth for Tarski–Grothendieck set theory

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From: en.wikipedia.org/wiki/Tarski–Grothendieck_set_theory

Axiom ax-garth:

$$\vdash \exists y(x \in y \wedge \forall z \in y (\forall w(w \subseteq z \rightarrow w \in y) \wedge \exists w \in y \forall v(v \subseteq z \rightarrow v \in w)) \wedge \forall z(z \subseteq y \rightarrow (z \approx y \vee z \in y))) \quad (2.1)$$

We assume the Meth8/VL4 apparatus and method.

LET v w x y z: v w x y z.

necessity, for all; % possibility, for one or some;

> Imply, greater than; < Not Imply, less than, ≡ = Equivalent to, ≈.

$$\%y \& (((x < y) \& ((\#z < y) \& (\#w \& (\sim(w > z) > (w < y)))))) \& (((\%w < y) \& (\#v \& (\sim(v > z) > (v < w)))))) \& (\#z \& (\sim(z > y) > ((z = y) + (z < y)))))) ; \text{repeated fragment FFFF FFFF FFFF FFFF} \quad (2.2)$$

Eq. 2.2 as rendered is *not* tautologous. This means the axiom ax-garth in the Metamath proof explorer for rendering the axiom of Tarski-Grothendieck set theory in Eq. 2.1 is refuted.