

A DIRECT, SIMPLE DERIVATION OF FLT V6

PHILIP AARON BLOOM; ELLENB2357@GMAIL.COM

ABSTRACT. For $n \in \mathbb{N}$: We design an algebraic identity $r^n + s^n = t^n$ holding for $r, s, t > 0 \in \mathbb{R}$, to relate to $x^n + y^n = z^n$ holding for co-prime $x, y, z \geq 1$. We show for $n > 2$ that there exists no co-prime $(r, s, t) \in \mathbb{N} \subset \mathbb{R}$. We can infer that such $\{r, s, t\}$ equals such $\{x, y, z\}$ since our identity has an unrestricted variable. Hence, for $n > 2$, there exists no co-prime nor integral (x, y, z) .

1. INTRODUCTION

Fermat's last theorem (FLT) states, for integral $n > 2$, that no positive integral x, y, z satisfy $x^n + y^n = z^n$. No simple proof of FLT is established for every $n > 2$.

2. THE DIRECT ARGUMENT, DEFINED AS NOT BY WAY OF CONTRADICTION

We start a deductive chain of reasoning with a detailed *algebraic identity* that we have designed to be sufficient for implying FLT, namely, our equation (1) :

$$(1) \quad \left((2^{p+1}q^n)^{\frac{1}{n}} \right)^n + \left((m - 2^p q^n)^{\frac{1}{n}} \right)^n = \left((m + 2^p q^n)^{\frac{1}{n}} \right)^n.$$

For all integral $n \geq 1$: We restrict q to all positive rational values, and restrict p to all positive odd values, with m as all positive real values such that $m > 2^p q^n$.

Use $r, s, t \geq 1 \in \mathbb{N}$, respectively, to denote $(2^{p+1}q^n)^{\frac{1}{n}}$; $(m - 2^p q^n)^{\frac{1}{n}}$; $(m + 2^p q^n)^{\frac{1}{n}}$. With $r^n, s^n, t^n \geq 1$, existing values of $r, s, t \in \mathbb{N}$ each is a unique n -th root.

Rational q is *legitimate*, being *sufficient* for our argument, per Prop. 2.1, below.

For $n = 2$ with even $p \geq 0$, (1) does not hold for $r, s, t \in \mathbb{N}$: By inspection, for $n = 2$, even $p \geq 0$ yields solely irrational r , e.g., $p = 2$ yields $r = \sqrt{8}q$.

For $n = 1, 2$, set $\{r, s, t \in \mathbb{N}\}$ is non-empty, as clearly is $\{x, y, z \in \mathbb{N}\}$.

Example : For $n = 1$, values $m = \frac{3}{4}$, $p = 1$, and $q = \frac{11}{2}$ result in $3 + 4 = 7$.

Example : For $n = 2$, values $m = \frac{3}{2}$, $p = 1$ and $q = \frac{41}{2}$ result in $3^2 + 4^2 = 5^2$.

For $n > 2$, no necessary relation exists between $(r, s, t) \in \mathbb{N}$ and $(x, y, z) \in \mathbb{N}$.

In section 3 we determine, for $n > 2$, that $\{(r, s, t) \in \mathbb{N}\} = \emptyset$.

But, the argument is valid since we maintain the generality of n : We achieve this goal using our proof of proposition 2.1, in which, for any given value of n , we relate nonempty $\{(r, s, t) \in \mathbb{R}\}$ with $\{(x, y, z) \in \mathbb{N}\}$ that we take as nonempty because, for $n > 2$, we act as if the "fact", $\{(x, y, z) \in \mathbb{N}\} = \emptyset$, is not yet established.

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For any given n : Let A be $\{(r, s, t) | r, s, t \text{ are coprime } \geq 1\}$ for which (1) holds.

For any given n : Let B denote $\{(x, y, z) | x, y, z \text{ are coprime } \in \mathbb{N}\}$ for which $x^n + y^n = z^n$ holds.

We intend to infer values of n for $B = \emptyset$, a *hypothetical example* being $n = 3$.

For any given n : Let C be $\{(r, s, t) \in \mathbb{R} \supseteq \mathbb{N} | r, s, t > 0\}$ for which (1) holds.

We intend to relate C to B and, subsequently, to relate A to B .

For any given n : Let D be $\{\frac{rs}{t} | r, s, t \in C \text{ with real } m, \text{ rational } q, \text{ and odd } p\}$.

For any given n : Let E be $\{\frac{rs}{t} \in D | r, s, t \in A\}$.

For any given n : Let F be $\{\frac{xy}{z} | x, y, z \in B\}$.

Proposition 2.1. *For any given $n \in \mathbb{N}$ with nonempty A, B , it is true that $A=B$.*

Proof. For any given n , with nonempty B : Due *solely* to varying unrestricted real m , real $\frac{(2^{p+1}q^n)^{\frac{1}{n}}(m-2^p q^n)^{\frac{1}{n}}}{(m+2^p q^n)^{\frac{1}{n}}}$ or $\frac{rs}{t} \in D$ takes every value of $\frac{xy}{z} \in F$. So, sets $E=F$.

The same arbitrary values m , varying the same, *at the same time*, allow real values of $(2^{p+1}q^n)^{\frac{1}{n}}(m-2^p q^n)^{\frac{1}{n}}$ or rs (with both r, s simultaneously irrational and r, s simultaneously rational, including simultaneous co-prime r, s) to take every value of xy , allow real values of $(m-2^p q^n)^{\frac{1}{n}}$ or s to take every value of integral y , and allow real values of $(m+2^p q^n)^{\frac{1}{n}}$ or t to take every value of integral z . Hence, $rs = xy$, $s = y$, and $t = z$. Therefore, values of real r must take every value of integral x .

It logically follows that $r = x$, $s = y$, and $t = z$. \square

Rational q is legitimate, being sufficient for Prop. 2.1 to be true, as follows :

Irrational values of q are irrelevant because values taken by m, p, q , with p, q independent of determining proposition 2.1, are *sufficient* in our proof of Prop. 2.1.

Allowing q to vary as described, above, results in set C *being restricted* to set A .

We choose *now* to restrict odd p to $p = 1$ since, per remark 3.1, below, with $p = 1$, equation (1) yields an empty set A for the largest $\{n | n \in \mathbb{N}, n > 2\}$.

Hence, for (1), the value of $(r, s, t) \in A$ is $((4q^n)^{\frac{1}{n}}; (m-2q^n)^{\frac{1}{n}}; (m+2q^n)^{\frac{1}{n}})$.

3. RESULTS AND CONCLUSION

Remark 3.1. *By inspection, with $r = (2^{p+1}q^n)^{\frac{1}{n}}$, which reduces to $2^{\frac{p+1}{n}}q$:*

For odd $p = 1, \dots, 19 \dots$ we get, respectively, integral $r = 2^{\frac{2}{n}}m, \dots, 2^{\frac{20}{n}}m \dots$, that have progressively smaller sets of n for which r can not be integral.

For example, with $p = 19$, the values of odd n for which r can not be integral show as $n = 3, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19$, plus $\{n | n \in \mathbb{N}, n > 20\}$.

We show for $n > 2$ with $p = 1$ that set B is empty, or, alternately, no co-prime numbers exist for $((4q^n)^{\frac{1}{n}}, (m-2q^n)^{\frac{1}{n}}, (m+2q^n)^{\frac{1}{n}})$, as follows :

By inspection, $(4q^n)^{\frac{1}{n}} = 2^{\frac{2}{n}}q$. Consequently, for $n > 2$ with $p = 1$ and rational q , $\{2^{\frac{2}{n}}q \in \mathbb{Q}\} = \emptyset$, hence, $\{2^{\frac{2}{n}}q \in Z \subset \mathbb{Q}\} = \emptyset$.

Thus, for $n > 2$, no co-prime nor integral $(r, s, t) \geq 1$ exists for which (1) holds.

So, for $n > 2$, equation $x^n + y^n = z^n$ does not hold for $((x, y, z) \in \mathbb{N})$.

QED