A DIRECT, SIMPLE DERIVATION OF FLT V5

PHILIP AARON BLOOM; ELLENB2357@GMAIL.COM

Abstract. For $n \in \mathbb{N}$: We design an algebraic identity $r^n + s^n = t^n$ holding for $r, s, t > 0 \in \mathbb{R}$, to relate to $x^n + y^n = z^n$ holding for co-prime $x, y, z \geq 1$. We show for $n > 2$ that there exists no co-prime $(r, s, t) \in \mathbb{N} \subset \mathbb{R}$. We can infer that such {r,s,t} equals such {x,y,z} since our identity has an unrestricted variable. Hence, for $n > 2$, there exists no co-prime nor integral (x, y, z).

1. Introduction

Fermat's last theorem (FLT) states, for integral $n \geq 3$, that no positive integral x, y, z satisfy $x^n + y^n = z^n$. No simple proof of FLT is established for every $n \geq 3$.

2. The Direct Argument, Defined As Not By Way Of Contradiction

We start a deductive chain of reasoning with a detailed *algebraic identity* that we have designed to be sufficient for implying FLT, namely, our equation (1) :

$$\left(\left(2^{p+1}q^n\right)^{\frac{1}{n}}\right)^n + \left(\left(m - 2^p q^n\right)^{\frac{1}{n}}\right)^n = \left(\left(m + 2^p q^n\right)^{\frac{1}{n}}\right)^n. \tag{1}$$

For all integral $n \geq 1$: We restrict q to all positive rational values, and restrict p to positive odd values, with m as all positive real values such that $m > 2^p q^n$.

Use $r, s, t \geq 1 \in \mathbb{N}$, respectively, to *denote* $(2^{p+1}q^n)^{\frac{1}{n}}$; $(m-2^p q^n)^{\frac{1}{n}}$; $(m+2^p q^{n_1})^{\frac{1}{n}}$.
With $r^n, s^n, t^n \geq 1$, existing values of $r, s, t \in \mathbb{N}$ each is a unique n-th root.

Solely rational q is *legitimate*, *sufficient* for our argument, per Prop. 2.1, below.
For $n = 2$ with even $p \geq 0$, (1) does not hold for $r, s, t \in \mathbb{N}$: By inspection, for $n = 2$, even $p \geq 0$ yields solely irrational r, e.g., $p = 2$ yields $r = \sqrt{8}q$.

For $n = 1, 2$, set $\{r, s, t \in \mathbb{N}\}$ is non-empty, as clearly is $\{x, y, z \in \mathbb{N}\}$).
Example : For $n = 1$, values $m = \frac{3}{4}$, $p = 1$, and $q = \frac{11}{2}$ result in $3 + 4 = 7$.
Example : For $n = 2$, values $m = \frac{3}{2}$, $p = 1$ and $q = \frac{41}{2}$ result in $3^2 + 4^2 = 5^2$.
For $n > 2$, no necessary relation exists between $(r, s, t) \in \mathbb{N}$ and $(x, y, z) \in \mathbb{N}$.

In section 3 we determine, for $n > 2$, that $\{r, s, t \in \mathbb{N}\} = \varnothing$.
But, the argument stays valid by maintaining the generality of n : We achieve this goal using our proof of proposition 2.1, in which, for any given value of n, we relate nonempty $\{(r, s, t) \in \mathbb{R}\}$ with $\{(x, y, z) \in \mathbb{N}\}$ which we take as nonempty because, for $n > 2$, we act as if the "fact", $\{(x, y, z) \in \mathbb{N}\} = \varnothing$, is not yet established.

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For any given n : *Let* A be $\{(r,s,t)|r,s,t$ are coprime $\geq 1\}$ for which (1) holds.
For any given n : *Let* B denote $\{(x,y,z)|x,y,z$ are coprime $\in \mathbb{N}\}$ for which $x^n + y^n = z^n$ holds.

We intend to infer values of n for $B = \varnothing$, a *hypothetical example* being $n = 3$.

For any given n : *Let* C be $\{(r,s,t) \in \mathbb{R} \supseteq \mathbb{N}|r,s,t > 0\}$ for which (1) holds.

We intend to relate C to B and, subsequently, to relate A to B.

For any given n : *Let* D be $\{\frac{rs}{t}|r,s,t \in C$ with real m, rational q, and odd p$\}$.
For any given n : *Let* E be $\{\frac{rs}{t} \subset D|r,s,t \in A\}$.
For any given n : *Let* F be $\{\frac{xy}{z}|x,y,z \in B\}$.

Proposition 2.1. *For any given* $n \in \mathbb{N}$ *with nonempty A, B, it is true that A=B.*

Proof. For any given n with nonempty B : Due *solely* to varying unrestricted m with arbitrary p,q, real $\frac{rs}{t} \in D$ or $\frac{(2^{p+1}q^n)^{\frac{1}{n}}(m-2^pq^n)^{\frac{1}{n}}}{(m+2^pq^n)^{\frac{1}{n}}}$ takes every value of $\frac{xy}{z} \in F$.
Let us keep rational $\frac{xy}{z} \in F$ in simplest terms.
Let us keep *real* $\frac{rs}{t} \in D$ in the form $\frac{rs}{t}$ such that, for the following reason, $rs = xy$ and $t = z$: Since $\frac{rs}{t} = \frac{xy}{z}$, or sets $E = F$, evidently, $rs = xy$ and $t = z$.
Unrestricted real values m vary to allow real values (rs), with both r,s simultaneously rational and r,s simultaneously irrational, to take every value of integral (xy). The same arbitrary values m, varying the same *at the same time*, allow real values of s or $(m - 2^pq^n)^{\frac{1}{n}}$ to take every value of integral y. Thus, values of real r must take every value of integral x. So, it is true that $r = x$, $s = y$, and $t = z$. □

Rational q, which is legitimate, is sufficient for Prop. 2.1 to be true, as follows :
Irrational values of q are irrelevant because values taken by m,p,q, with p,q independent of determining proposition 2.1, are *sufficient* for proving Prop. 2.1.

Allowing q to vary as described, above, results in set C *being restricted to* set A.
We choose *now* to restrict odd p to $p = 1$ since, per remark 3.1, below, with $p = 1$, equation (1) yields an empty A for the largest $\{n|n \in \mathbb{N}, n \geq 3\}$.
Hence, for (1), the value of $(r,s,t) \in A$ is $((4q^n)^{\frac{1}{n}};\ (m-2q^n)^{\frac{1}{n}};\ (m+2q^n)^{\frac{1}{n}}$.

3. Results and Conclusion

Remark 3.1. *By inspection, with* $r = (2^{p+1}q^n)^{\frac{1}{n}}$*, which reduces to* $2^{\frac{p+1}{n}}q$ *:*
For odd $p = 1, ..., 19...$ *we get, respectively, integral* $r = 2^{\frac{2}{n}}m, ...2^{\frac{20}{n}}m...$*, that have progressively smaller sets of n for which r can not be integral.*
For example, with $p = 19$*, the values of odd n for which r can not be integral show as* $n = 3, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19$*, plus* $\{n|n \in \mathbb{N}, n > 20\}$.

We show for $n > 2$ with $p = 1$ that $\{(r,s,t)|r,s,t$ are coprime $\geq 1\}$ is empty, or, alternately, no values exist for $((4q^n)^{\frac{1}{n}}, (m-2q^n)^{\frac{1}{n}}, (m+2q^n)^{\frac{1}{n}})$, as follows :
By inspection, $(4q^n)^{\frac{1}{n}} = 2^{\frac{2}{n}}q$. Hence, for $n > 2$ with $p = 1$ and rational q (which is *legitimate*, per proof of Prop. 2.1), $\{2^{\frac{2}{n}}q \in \mathbb{Q}\} = \varnothing$, thus, $\{2^{\frac{2}{n}}q \in Z \subset \mathbb{Q}\} = \varnothing$.
So, for $n > 2$, no co-prime nor integral $(r,s,t) \geq 1$ exists for which (1) holds.
So, for $n > 2$, equation $x^n + y^n = z^n$ does not hold for $((x,y,z) \in \mathbb{N})$. QED