Goldbach's Conjectures

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Abstract:The prime numbers set is the three primes addition closed; each prime is the sum of three not necessarily distinct primes. All natural numbers are created on the set of all prime numbers according to the laws of the weak and strong Goldbach's conjectures. Thus all natural numbers are the Goldbach's numbers.

Key words: Goldbach weak and strong conjectures, residual sets, prime partition matrix, prime vector, prime invariant transformation.

Introduction

The purpose of this article is to show that the prime numbers are not only multiplication but addition operation fundamental blocks of the natural numbers. The property of the prime numbers to be the Goldbach's numbers, the sum of three primes, is fundamental and extends naturally to the all even and odd natural numbers. We are showing that:

1. The prime number set is closed under 3-primes addition.
2. Every even number greater then 2 is the sum of two primes.
2. Every odd number greater then 2 is the sum of two primes.

The first part of the article shows that prime numbers by themselves are the Goldbach's numbers. For this the concept of the primality and the prime partitions, the prime residual sets, and their descriptive characteristics are considered The prime 3-partition matrix is naturally suited to preserve the properties of the primes, and finally the prime invariant transformation, essential to prove that the primes are sums of three primes, is introduced.
Finally weak and strong and weak conjecture follow as the mere consequence of the fundamental nature of the prime numbers.
We will use a, b, c and x, y, z for the natural integers and their variables and ξ, η, ζ for the prime numbers. The set of natural numbers is $\mathbf{N}$ and for the convenience the collection of all primes Π includes the number one.

Primality and Partition

The primality of an integer is based on the multiplication operation, which is nothing more than a sophisticated addition operation. This means that the primality may be characterized in the terms of the number addition operation, or equivalently by the properties of its addition partition components. Here, we will look only at partitions of the primes in two parts, 2-partitions, and three parts, 3-partitions.

1. Clearly, any prime can be split into two parts in which case one part must be an even and another part an odd number. The case of the prime 31

$$31 \quad = \quad 24+7 \quad = \quad 6+25 \quad = \quad 18+13 \quad = \quad 26+5$$

shows that the integer x in its $p = x + 2y \sim (x, 2y)$ decomposition is either prime or composite. While the odd integer 33 has common factor partition $x+2y = 24+9 = 3(8+3)$ it is impossible to find a pair $(x, 2y)$ with common factor for the prime 31. In general

Corollary: Natural integer n is an odd prime if and only if there is no integer apart for the unit to be factored out from its any 2-partition $(x, 2y)$.

□ Let $(x, 2y)$ be an arbitrary 2-partition of an odd integer n with $x = \alpha a$ and $y = 2\beta b$ factorization of its parts. Clearly n is an odd prime if and only if $\alpha = \beta = 1$ and

$$n \quad = (\alpha a, 2\beta b) = (1 \cdot x + 2(1 \cdot y)) = 1 \cdot (x, 2y) = \ (x, 2y).$$

□

Corollary: Any odd prime greater or equal to 3 has necessarily a 1-prime 2- partition $(\xi, 2y)$.

□ By our definition, the integer 1 is included into the prime set and at least $n = (1, a)$ for any natural number n greater or equal to 3 □

2. Any odd prime greater or equal to 3 can be split into three parts. The example of the prime

$$31 = 6 + 15 + 10 = 16 + 8 + 7 = 15 + 9 + 7 = 19 + 7 + 5.$$

shows that the partition triples are two even and one odd, two even and one prime, all three odd and finally three prime components. Since any odd prime greater or equal to 3 can be split into a prime and an even integer greater or equal to 2, and since an odd prime may be selected from its odd part an odd prime greater or equal to 3 has the following not necessarily the prime partitions

$$n = (x, y, z) = (\xi, 2m) = (\xi, \eta, C).$$

Corollary: Natural integer n is an odd prime if and only if its any 3-partition (x, y, z) allows only the trivial factor.

□ Let (x, y, z) be an arbitrary 3-partition of an odd integer n with composite components $x = \alpha a$, $y = \beta b$ and $z = \gamma c$. If $k = \alpha = \beta = \gamma \neq 1$ is a common factor the integer is not prime. Hence n is an odd prime if and only if $\alpha = \beta = \gamma = 1$ and

$$n \quad = (\alpha a, \beta b, \gamma c) = (1 \cdot x + 1 \cdot y + 1 \cdot y) = 1 \cdot (x, y, z) = \ (x, y, z).$$

□

The primality test is very much simplified by the following obvious corollary.

Corollary: For an odd prime to have a 3-partition it is necessary to have at least one 3-partition with one and two prime components. Moreover, any odd prime greater or equal to 3 may always be partitioned into two odd primes and a reminder, not necessarily a prime.

□ Clearly, the prime does not have 3-primes partition if and only if its all 3-partitions do not allow one and two prime partitions. Further, the prime 3 has partition $((1, 1); 1)$. For any odd prime greater than

3 the $p = (1, 3, z)$ is a good partition. Hence, any prime equal or greater than 3 may be partitioned into two primes and an odd remainder, not necessarily a prime, Explicitly

$$p = (\xi, \eta, c), \quad \xi, \eta \in \Pi,\ c \in 2\mathbf{N} + 1.$$

□

The fact is that all practical prime numbers are decomposable into three primes. To prove that all prime numbers are 3-primes non decomposable may be easier than to prove that they are. Hence, a criterion of a prime 3-primes non-decomposability is as good as the criterion of its 3-primes decomposability.

Corollary**: The prime numbers are not 3-primes decomposable if and only if there is at least one prime such that its all possible 3-partitions have at least one composite partition component.

□ Explicitly, the prime numbers are decomposable into three primes if

$$\forall p \in \Pi,\ \exists (\xi, \eta \zeta) \in \Pi\ \therefore\ p = (\xi, \eta, \zeta).$$

The negative statement of the three primes decomposability is

$$\begin{aligned} & \exists p \in \Pi,\ \forall (\xi, \eta, \zeta) \in \Pi\ \therefore\ p \neq (\xi, \eta, \zeta) \\ \Leftrightarrow \quad & \exists p \in \Pi,\ \forall (\xi, \eta) \in \Pi \therefore\ p = (\xi, \eta, c), \\ & c \text{ - is a composite integer.} \end{aligned}$$

The second line is justified by the particular choice of two prime components ξ and η allowed by the presence of the quantifier $\forall$. □

The Prime Partition and its Residual Sets

The complete residual set $\{0.1, 2, 3, 4, \cdots, p-1\}$ is the natural companion of any prime p. Furthermore, instead of the complete we will need its reduced residual set $\mathsf{R}_o = \{1, 2, 3, 4, \cdots, p-1\}$, which by itself is the union of the mutually complementary reduced residual, shortly r-residual, sets of the odd $\mathsf{R} = \{1, 3, 5, \cdots, p-2\}$ and the even $2\mathsf{R} = \{2, 4, 6, \cdots, p-1\}$ natural integers.

The collection $\mathrm{P} = \{1, 3, 5, \cdots, p\}$ of all odd primes smaller or equal to the prime p is its prime family. We refer by the Π_p to the set P without the prime p. Clearly

$$\mathsf{R}_o = \mathsf{R} \cup 2\mathsf{R}, \quad \mathrm{P} \supset \mathsf{R} \cup \{p\}.$$

Each odd prime number greater than 1 is represented by the sum of a three integers from its r-residual set R. The collection of all such triples is the prime 3-partition set

$$3\mathcal{P} = \{(x, y, z) : x + y + z = p\}.$$

Furthermore, we identify each prime p by the collection $\{p, \mathsf{R}, 2\mathsf{R}, 3\mathcal{P}\}$.

Remark: We notice that the function $\hat{w} : z \to p - z,\ \ z \in \mathsf{R}$ is the one to one and onto mapping of the r-residual set R of the odd integers to the r-residual set 2R of the even integers. The

prime p is the restriction of the three variable function $f(x, y, z) = x + y + z$ to the p so that the prime 3-partition set is the collection $3\mathcal{P} = \{((x, y), z) : \ z = p - (x + y)\}$.
The p restriction of the linear function makes the component $z = z(x, y) = z_{xy}$, explicitly dependent on the other two integer variables, which lets as the following definition.

Definition: *The function* $R \times R \to R$

$$\star : (x, y) \to x \star y = p - (x + y).$$

is the star operation mapping the odd r-residual set of the prime p *onto itself.*

The star operation partitions the collection of all triples into the set $2\mathcal{P}$ of the pairs (x, y) on the integers from the r-residual set R, and the set $\mathcal{Z} = \{z = x \star y\}$. Hencet $3\mathcal{P} = (2\mathcal{P}, \mathcal{Z})$.

Definition: The prime 3-partition matrix is an array $\mathtt{Z}$ of the matrix entries z from the set $\mathcal{Z}$ placed in the coordinate frame of the pair of the axes x and y of the integer from the odd r-residual set R. Corresponding matrix table is the star multiplication matrix on the odd r-residual set.

The **Table 1**. shows the 3-partition matrix for the prime $p = 31$. The even row and the column adjacent to the axes x and y are the complements $\mathsf{x} = p - x$ and $\mathsf{y} = p - y$ in the p.

Table 1. The odd residual set multiplication matrix

31	X	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	0
Y	$y \backslash x \quad \to$	1	3	**5**	7	9	11	13	15	17	19	21	23	25	27	29	31
30	1	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	
28	3	27	25	23	21	19	17	15	13	11	9	7	5	3	1		
36	5	25	23	21	19	17	15	13	11	9	7	5	3	1			
24	7	23	21	19	17	15	13	11	9	7	5	3	1				
22	9	21	19	17	15	13	11	9	7	5	3	1					
20	11	19	17	15	13	11	9	7	5	3	1						
18	13	17	15	13	11	9	7	5	3	1							
16	**15**	15	13	**11**	9	7	5	3	1								
14	17	13	11	9	7	5	3	1									
12	19	11	9	7	5	3	1										
10	21	9	7	5	3	1											
8	23	7	5	3	1												
6	25	5	3	1													
4	27	3	1														
2	29	1															
0	31																

After the 3-partition matrix is constructed the following operation may be introduced.

Definition: The wedge operation is the function $\hat{\wedge} : 2\mathcal{P} \to 2\mathsf{R}$

$$\hat{\wedge}(x, y) = x + y = w \in 2\mathsf{R}, \ \ \exists z \in \mathtt{Z} \ \therefore \ w = \hat{w}z = p - z.$$

Corollary**: The function $\hat{w}$ maps all matrix entries z from the the matrix Z onto the set of the even r-residual numbers 2R, and

$$\hat{w}z = \hat{\wedge}(x,y), \quad z = x \star y.$$

$\square$ The array Z contains all odd r-residual numbers by its construction. Hence the function $\hat{w} : z \to p - z$ is the mapping of the set Z to its complementary set 2R of the even r-residual set integers and must be onto, Since $z = x \star y = p - \hat{\wedge}(x,y)$

$$\hat{w}z = p - z = p - (p - \hat{\wedge}(x,y)) = \hat{\wedge}(x,y).$$

Definition: *The explicit function $z = x \star y$ is the position of the prime point p or the prime vector $\mathbf{p}(x,y)$ on the set Z in the coordinate frame (x,y).*

<u>Descriptive Characteristic of the Matrix Set Z</u>

The coordinate integer axes are the odd r-residual sets R. The inspection shows that the first matrix row is the odd residual set $\mathsf{R}_1 = \mathsf{R}$ of the prime $\mathrm{z}_1 = p$, the second row is the odd residual set R_2 of the integer $\mathrm{z}_2 = p - 2 \cdot 1$, the third row is the odd residual set R_3 of the integer $\mathrm{z}_3 = p - 2 \cdot 2$, ..., and the k^{th} row is the residual set R_k of the integer $\mathrm{z}_k = p - 2(k-1)$. The residual sets are subsets of the r-residual set R, and can be related to the column variable y as well as to their ordering indexes k. However, depending on what is wanted to be stresed we may use any from the following notation $\mathsf{R}(y) \equiv \mathsf{R}(y_k) \equiv \mathsf{R}_k \equiv \mathsf{R}(z_{1k}) \equiv \mathsf{R}(\mathrm{z}_y)$.

Definition: *The odd r-residual sets of the prime p are all sub-collections*

$$R(y) = \{z_{xy} : x = 1, 3, \cdots, p - 2(y-1)\} = |z_{y1}, z_{y2}, \cdots, 3, 2, 1\rangle,$$

of the multiplication matrix entries in the either rows of the array Z

In the same way are defined the $\mathsf{R}(x)$ residual sets. Clearly the $\mathsf{R}(y)$ and $\mathsf{R}(x)$ are subsets of the odd r-residual set of the prime p. Because of the (x,y) symmetry we may refer only to the row y r-residual sets.

1. The matrix set Z is partitioned into the collection of the row r-residual sets $\mathsf{R}(y)$ and the column residual sets $\mathsf{R}(x)$. The y residual sets are descending throughout the array Z, down the axes y; each y step $h = 2$ reduces the size of the residual set row vector $|\cdots\rangle$ for the matrix entry z_{1y} and

$$\mathsf{R} = \mathsf{R}_1 \supset \mathsf{R}_3 \supset \mathsf{R}_5 \supset \cdots \supset \mathsf{R}_{p-2} = |z_{1,p-2}\rangle.$$

For, the r-reduced sets at y and x are identical to the y row and the x column vectors, Hence, the matrix set Z is partitioned into collection of the disjoint r-residual sets. Further

$$\begin{aligned} \mathsf{R}_y &= |z_{1y}, z_{2,y}, \cdots, 9, 7, 5, 3, 2, 1\rangle \\ &= |z_{1y}, 0, \cdots, 0, 0, 0, 0, 0\rangle + |0, z_{2,y}, \cdots, 9, 7, 5, 3, 2, 1\rangle \\ &= |z_{1y}, \mathsf{R}_{y+2}\rangle. \end{aligned}$$

Hence $\{z_{1,y+2}\} = \mathsf{R}_y \backslash \mathsf{R}_{y+2}$ so that $\mathsf{R}_y \supset \mathsf{R}_{y+2}$ for all y, and rest follows.

2. The variables x and y are either the odd prime integers from the subset Π_p in the odd r-residual set R, or the composite integers from the set $\Pi_p^c = \mathsf{R} \setminus \Pi_p$. Hence, in the terms of the star product the array

$$\mathrm{Z} = Z_\Pi \cup Z_\Pi^c, \quad Z_\Pi = \Pi_p * \Pi_p, \quad Z_\Pi^c = \Pi_p * \Pi_p^c \cup \Pi_p^c * \Pi_p.$$

The collection Z_{Π} spanned by primes are the prime sectors, and the collection Z^c_{Π}, spanned by the mixed integer bass are the net sets of the matrix Z.

3. We identify the points $z = x \star y$ by the intersections $\mathsf{R}_x \cap \mathsf{R}_y$ of the odd r-residual sets. Hence, the prime p integer partition structure is completely characterized by the point

$$z = x \star y = \mathsf{R}_x \cap \mathsf{R}_y \in \begin{cases} Z_{\Pi}, & \text{if } x \in \Pi_p, \quad y \in \Pi_p \\ Z^c_{\Pi}, & \text{if } x \notin \Pi_p, \quad \text{or} \quad y \notin \Pi_p. \end{cases}$$

of its vector $\mathbf{p}$. on the matrix array set Z.

Finally we summarize the conditions of the 3-primes partition and non-partition in the terms of the property of the prime matrix of the entries Z.

Corollary**: • The prime p 3-partition $((x, y), z)$ is a 3-prime partition if and only if the prime vector $\mathbf{p}$ is at a prime number prime point z in a prime sector of the set Z_{Π}, or equivalently, if and only if the matrix entry z is a prime number at the intersection of two r-residual sets R_ξ and R_η of two primes ξ and η.

• The prime p does not have a 3-prime partition if and only if either its entry or at least one coordinate axes component is composite.

☐ The partition (x, y, z) is the 3-prime partition of the p if

$$x = \xi, \ y = \eta, \ z = \zeta \in \mathsf{R}_x \cap \mathsf{R}_y.$$

Neither the prime with prime matrix entry z or with at least one axes composed component is prime nor the prime may have composts in its 3-partitions. ☐

The Prime Transformations

All triples $(x, y, z) \in 3\mathcal{P}$ are all equivalent representations of the prime p, what are also all the entries z points or the prime vector $\mathbf{p}$ positions on the matrix set Z. A transition from one triple point to another triple is a transformation of the prime representation into itself, and such a transformation reflects as the motion of the prime vector $\mathbf{p}$ or matrix entry z on the matrix entry set Z.

Definition: *The function $\hat{t}$ mapping the prime triple into a prime triple,*

$$\hat{t} \, : (x, y, z) \to (x', y', z') = p,$$

preserving the prime p is the prime transformation.

An example of such transformation is the creation of an r-residual set R_y from its initial entry z_{1y} point.

Corollary: Invariant transformation of the prime p is the motion of the prime vector point $\mathbf{p}$ throughout the matrix set Z. Corresponding collection of the points $z = z_{xy} \in \mathrm{Z}$ is the prime trajectory throughout the set Z.

☐ The function $\hat{t}$ maps triple $((x, y), z)$ into $(\hat{t}\,(x, y); \hat{t}\,z)$ preserving the prime p. Hence

$$\hat{t}\,z = p - \hat{t}\,(x + y) = \hat{t}x \star \hat{t}y = z'.$$

The integer variable z' is an explicit function of the integer coordinate variables (x,y) and will be identified as the trajectory of the prime vector $\mathbf{p}$ on the matrix set Z. $\square$

Among all invariant prime transformation, the most particular place takes the transformation preserving the matrix entry z.

Corollary**: The prime invariant transformation preserving its matrix entry z is the left to right-up diagonal translation of the prime vector $\mathbf{p}$ point throughout the matrix set Z.

$\square$ Let the initial prime point be $\mathbf{p}_o = ((x_o, y_o), z_o)$. The function preserving the matrix entry z requires $z = z_o$ which translates into

$$x\star y = \hat{t}\,x\star\hat{t}\,y \quad\Leftrightarrow\quad \hat{t}\,x_o + \hat{t}\,y_o = x + y \equiv x_o + y_o = const,$$

The linear function, continuous over the integers, is exactly the left to right-up diagonal translation of the prime point z throughout its set Z. The **Table 1** shows the translation of the prime entry 11. $\square$

Remark: The translation transformation may be achieved by the local discreet integer step function. For, the translation transformation defines the argument integer step $\mathbf{h} = (h_x, h_y, 0)$

$$h_y = \hat{h}\,y - y_o \;=\; h \;=\; -(\hat{h}\,x - x_o) = -h_x$$

Hence there is the discreet integer steps function

$$\begin{aligned} &\hat{h}: \;\; (x,y,z) \to (\hat{h}\,x - x_o, \hat{h}\,y - y_o, 0) = (h, -h, 0) = \mathbf{h}. \\ \therefore \;\; &\hat{h}(x,y,z) \to (x_o + h, y_o - h, z) = (x_0, y_0, z) + (h, -h, 0) = \mathbf{p}_o + \mathbf{h}.. \end{aligned}$$

The function with the step $h = 2$ translates continuously over the odd r-residual set.

Now we will look at the r-residual set properties of the matrix set Z.

Corollary**: The matrix Z is the collection of either rows or columns odd r-residual sets arranged by the set order of the integer variables y and x. If the set R_{y-h} is before the set R_y then R_y is the proper subset of the set R_{y-h} and in the row vector notation

$$\mathsf{R}_{y-h} \;\;=\;\; |\mathsf{R}^{y-h}_{y+1}\;, \mathsf{R}_y\rangle. = |\mathsf{R}^{y-h}_{y+1}\;, 0\rangle. + \;|\,0\;, \mathsf{R}_y\rangle.$$

$\square$ The first part follows directly from the descriptive characteristics of the matrix Z.

For the second part, each row vector R_y is the collection of the matrix entries $z = x\star y = p - \hat{\wedge}(x,y)$ starting at $z_{1,y}$ with x taking values from 1 to $\mathrm{N}_y = p - (y+1)$. Written explicitly in the vector form

$$\begin{aligned} &\mathsf{R}_y = |\, z_{1,y}, z_{3,y}, \cdots, 9, 7, 5, 3, 1\,\rangle. \\ &\mathsf{R}_{y-h} = |\, z_{1,y-h}, z_{3,y-h}, \cdots, z_{h,y-h},\; z_{h+1y-h}, \cdots, 9, 7, 5, 3, 1\,\rangle. \end{aligned}$$

However the explicit calculation gives

$$\begin{aligned} &z_{1,y} = p - \hat{\wedge}(1,y), \;\; z_{1,y-h} = p - \hat{\wedge}(1, y-) = z_{1,y} + h > z_{1,y}, \\ &z_{h+1,y-h} = p - \hat{\wedge}(h+1, y-h) = p - \hat{\wedge}(1,y) = \; z_{1,y} \end{aligned}$$

Hence the matrix entries
$, z_{h+1y-h}, \cdots, 9, 7, 5, 3, 1\,\rangle$ belong to the r-residual set R_y so that

$$\mathsf{R}_{y-h} = |\, z_{1,y-h}, z_{3,y-h}, \cdots, z_{h,y-h},\;, \mathsf{R}_y\rangle = \; |\mathsf{R}^{y-h}_{y+1}\;, \mathsf{R}_y\rangle.$$

$\square$

Instead of the matrix Z spanned by the odd r-residual sets we would need the matrix Z_Π spanned only by the odd primes from the r-residual sets. The matrix reduction is achieved by the elimination of all composite integers from the coordinate r-residual sets. The reduction operation eliminates all the net sets of the matrix Z. Hence, the reduced matrix Z_Π is the collection of all prime sectors.

Corollary**: Reduced matrix Z_Π preserves all the matrix entries from the odd r-residual set.

$\square$ We know already that all matrix entries z are contained in the matrix set Z. Since the translation operation is the left to right-up trajectory of each matrix entry z must intersect both horizontal and vertical strips spanned by the primes $1, 3, 5, 7$. Hence, each matrix entry is "recreated" in at least one prime sector, and reduced matrix Z_Π preserves all the matrix entries from the odd r-residual set. $\square$

Finally, we will introduce the functions mapping the matrix set Z onto itself by mapping one r-residual set to another one. The transformation keeps the matrix structure unchanged and serves only to recognize the one r-residual structure as the part of the other one.

Definition: *The r-residual set transformation of the matrix Z is an operation*

$$\hat{\cup}_h : \ R_y \to \hat{\cup}_h R_y \ = \ R_{y-h} = |\, R_{y+1}^{y-h} \ , R_y \,\rangle,$$

embedding the r-residual set R_y in the residual set R_{y-h} at the place of the matrix entries of the R_y. Transformation $\hat{\cap}_h$ is the r-residual set restriction transformation if

$$\hat{\cap}_h : \ R_y \to \hat{\cap}_h R_y \ = \ R_{y+h} = |R_{y+h}\,\rangle.$$

Corollary**: The translation function $\hat{h}$ maps the prime vector $\mathbf{p}$ into $(\hat{\cup}_h \mathsf{R}_y) \cap (\hat{\cap}_h \mathsf{R}_x)$ set

$\square$ We chose an initial prime vector $\mathbf{p}_o = ((x_o, y_o), z_o)$ to be the intersection point of the r-residual sets R_{y_o} and R_{x_o}. The translation transformation $\hat{h} = (h, -h)$ is the left to the right-up diagonal translation for the h in the x and the $-h$ in the y, and

$$\hat{h}\mathbf{p}_o = (x_0 + h, y_0 - h, z) = \mathbf{p}_o + (h, -h, 0).$$

The embedding transformation $\hat{\cup}_h$ deposits the residual set R_y into residual set R_{y-h} at the point (h, y). Hence the point x_o is translated to the column $x_o + h$. The reduction transformation $\hat{\cap}_h$ reduces the r-residual set R_x to the r-residual set R_{x+h} placed in the column at $x + h$, which implies that the vector point $\mathbf{p}_o$ is in the intersection of the r-residual sets $\hat{\cup}_h \mathsf{R}_y$ and $\hat{\cap}_h \mathsf{R}_x$. $\square$

The Primes are Goldbach's Numbers

Fundamental nature of the prime numbers manifests not only in the structure of all natural numbers but in their own structure.

Statement: All odd primes greater than one are the sums of a three, not necessarily distinct, primes.

$\square$ The proof will be given by the contradiction. The statement will not be true if there exist at least one prime number for which the assertion is not true. This means that there is a prime P such that its every 3- partition (x, y, z) has at least one composite partition part.

If it is true for every partition than it must be true for a particular prime entry ζ, partition $\mathrm{P}(\mathrm{X}, \mathrm{Y}, \zeta)$ submersed into matrix Z net set.

The prime vector of the partition is at the X point in the r-residual set R_{Y}, there are $\pi(\mathrm{X})$ primes in the r-residual subset $\{\pi(\mathrm{X}), \cdots, 7, 5, 3, 1\}$ so that such partition must exist.

To give the visual presentation of what has been said we use the 3-partition matrix Z of the prime $p = 31$, see the **Table 2**. We may imagine that all matrix entries are general, that their names are their integer values, and that the prime vector $\mathbf{p} = (\mathrm{X}, \mathrm{Y}, \zeta)$ is identified with the vector $(((5, 15), 11)$ i in the matrix net set.

Table 2. The **31** prime multiplication matrix

31	X	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	0
Y	$y\backslash x$ $\rightarrow$	1	3	**5**	7	9	11	13	15	17	19	21	23	25	27	29	31
30	1		27	25		21			15			9		5	3		
28	3	27	25		21	19		15	13		9	7		3	1		
26	5	25		21		17	15		11	9		5		1			
24	7		21			15			9			3					
22	9	21	19	17	15	13	11	9	7	5	3	1					
20	11			15		11	9		5								
18	13		15			9			3								
16	**15**	15	13	**11**	9	7	5	3	1								
14	17			9		5	3	1									
12	19		9			3	1										
10	21	9	7	5	3	1											
8	23																
6	25	5	3	1													
4	27	3	1														
2	29																
0	31																

To make contradiction we construct the translation trajectory of the prime vector $\mathbf{p}$ under the sequential function

$$\hat{h} = \{\hat{\mathbf{2}}^0, \hat{\mathbf{2}}^1, \hat{\mathbf{2}}^2, \cdots, \hat{\mathbf{2}}^m\}.$$

of the integer step $h = 2$. The transformation is the left to right-up diagonal and continuous -it takes a value on each row and column of odd r-residual sets. Hence the sequence of the points on the set Z

$$\hat{h}(\zeta) = \{\, \zeta, \hat{\mathbf{2}}\zeta, \hat{\mathbf{2}}^2\zeta, \cdots, \hat{\mathbf{2}}^m\zeta \,\}$$

is the trajectory of the prime vector $\mathbf{p}$ continuously descending across the rows, and continuously ascending across the columns of the r-residual sets.The trajectory may be extended to the prime vector points prior to the $\mathbf{p}$. Consequently, the prime vector $\mathbf{p}$ under translation transformation must cross all corresponding prime sectors of the matrix Z, and finally it must intersect the horizontal and vertical prime strips $7, 5, 3, 1$, see the **Table 2**, Since translation transformation preserves the matrix entity ζ the prime vector must receive another two prime components at each point of a prime sectors and $p = \xi + \eta + \zeta$. This contradicts to our assumption that all the prime 3-partitions must have at least one composed partition component. Hence, each prime p must have at least one 3-primes partition and the prime numbers are the Goldbach's numbers by their very nature. $\square$

The Goldbach's Conjectures

The property of the prime numbers to be the Goldbach's numbers has direct structural consequences of all natural numbers.

Statement: All even numbers are the sums of two, not necessarily distinct, primes.

$\square$ We found already that the function $\hat{w}$ maps the odd r-residual set onto the even r-residual set. Moreover the set Z contains all odd r-residual integers and the function maps all Z set onto even r-residual set and

$$2\mathsf{R} = \{\hat{w}z = p - z = \hat{\wedge}(x, y), \quad z = x \star y \in \mathrm{Z}\}$$

However, we found that the reduced matrix Z_Π of Z preserves all odd r-residual set elements so that x and y take values on the set of the primes Π. Hence

$$2\mathsf{R} = \{\hat{\wedge}(\xi, \eta) = \xi + \eta, \quad \xi, \eta \in \Pi_p.\}$$

The conclusion is elevated from r-residual set $2\mathsf{R}$ of a prime p to the all set of the even integers by the induction. All practical even integers are the sums of two not necessarily distinct primes. Let the same be true for an arbitrary prime p and let q be the first next prime after p. Since q is the prime all even integers in its residual set are the sums of two, not necessarily distinct primes. Hence all even natural integers are the Goldbach's numbers. $\square$

Statement: Every odd natural integer greater or equal to 3 is the sum of three, not necessarily distinct odd primes.

$\square$ The sets of the even and odd natural integers are in one to one correspondence related by the function $\hat{\mathbf{1}} : 2\mathbf{N} \to 2\mathbf{N} + 1$. Hence every odd integer is created by $2n + 1$ from natural integer n. However, every even integer by itself is sum of two not necessarily distinct primes so that

$$m = \xi + \eta' + 1 = \xi + (\eta' + 1) = \xi + \eta + \zeta.$$

$\square$

Conclusion

All even natural numbers greater or equal to 3 are the sums of three, not necessarily distinct primes and all odd natural numbers are the sums of two, not necessarily distinct primes. Thus, all natural numbers are the Goldbach's numbers, the integer 1 being included by the definition.

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