A New Concept in Spherical Geometry for Physical and Cosmological Applications

S. Kalimuthu

Received: 29 May 2013/Accepted: 25 July 2013
© The National Academy of Sciences, India 2013

Abstract After establishing the fundamental physics prizes, Yuri Milin said: "Unlike the Nobel in physics, the Fundamental Physics Prize can be awarded to scientists whose ideas have not yet been verified by experiments, which often occurs decades later. Sometimes a radical new idea "really deserves recognition right away because it expands our understanding of at least what is possible.". Keeping this mind the author formulated two spherical geometrical theorems which may applied for the studies and probes of fundamental particles, quantum gravity, gravitational waves, dark matter and dark energy and also for engineering sciences.

Keywords Non-Euclidean math - New geometry - Physical and engineering applications

Introduction

The Pauli exclusion principle was postulated in an attempt to explain some of the properties of electrons in an atom. This principle says that in a closed system, no two electrons can occupy the same state. Heisenberg’s uncertainty principle states that the position and momentum of a particle cannot be simultaneously measured with arbitrarily high precision. Special relativity applies only to cases in which objects are moving at a uniform velocity. General relativity, however, is applicable to all forms of accelerated motion. This theory of general relativity arose from Einstein’s principle of equivalence. Einstein formulated this principle by examining a given mass in two different states.

Einstein’s equivalence principle is any of several related concepts dealing with the equivalence of gravitational and inertial mass, and to Albert Einstein’s observation that the gravitational "force" as experienced locally while standing on a massive body (such as the Earth) is actually the same as the pseudo-force experienced by an observer in a non-inertial (accelerated) frame of reference. Like these easy and brief principles, the author proposes the following spherical geometrical theorems.

Theorem 1 There exists a spherical triangle whose interior angle sum adds to 360°.

Theorem 2 There exists a spherical triangle whose interior angle sum adds to 540°.

Construction

First proof for Theorem 1

In spherical Fig. 1 consider NB, WE and EN as the three sides of triangle NEW. WE is the equator of spherical Fig. 1 and both EN and WN are perpendicular to WE. Since the angle WNE is a straight angle, we get that the sum of the interior angles of spherical triangle WNE is equal to 360°. And hence the proof.

Second proof for Theorem 2

In spherical Fig. 2, consider WN, NE and EW as the three sides of spherical triangle WNE. Since the angles WNE, NEW and EWN are straight angles, we obtain that the sum of the interior angles of spherical triangle WNE is 540°. And hence the proof.
Let \( y, z \) and \( m \) are three distinct spherical triangles.

Since the interior angle sum of a spherical triangle is more than 180°,

Let us assume, \( y + z = 360° + a \)

\( m + z = 360° + b \)

\( y - m = a - b \)

Squaring (3),

\[ y^2 + m^2 - 2my = a^2 + b^2 - 2ab \]  

Multiplying (1) and (2),

\[ y(m + z) + z(m + z) = 360°^2 + 360°b + 360°a + ab \]  

Adding (3a) and (4) we get that,

\[ y(m + z + y - 2m) + m(m + z) + (z + b)(z - b) = a(a - b + 360°) + 360°(b + 360°) \]

Applying (1) in the first factor, and (2) in the second and third factors of LHS we have,

\[ y(360° + a - m) + m(m + z) + (z + b)(360° - m) = a(a - b + 360°) + 360°(m + z) \]

i.e. \( y(360° + a - m) + (m - 360°)(m + z) = a(a - b + 360°) \)

Putting (3) in RHS, \( y(360° + a - m) + (m - 360°) \)

\( (m + z) = a(y - m + 360°) \)

\( z + a = 360°(a + m + z) \)

i.e. \( y(360° - m) + (m + z + a)(m - 360°) = 0 \)

i.e. \( (m - 360°)(m + z + a - y) = 0 \)

Assuming (3) in the second factor, \( \phi(m - 360°) \)

\( (z + a + b = a) = 0 \)

i.e. \( (m - 360°)(z + b) = 0 \)

i.e. \( a = 360° \)  

(5)

So, (5) establishes our first theorem.

Third proof for Theorem 1. (See Figure 3)

Since angles WAB, ABC, and BCE are straight angles they are all equal to 180°.

Let \( v \) be the value of this 180°.

Let angle WNB = s, ANC = t and WBE = u

Assuming (1) and (2) and adding we get that,

\[ x + y = 2v + s \]

\[ y + z = 2v + t \]

\[ z + m = 2v + u \]

(3a)  

(3)  

(4)  

(5)  

(6)  

(7)  

Squaring (3)

\[ x^2 + y^2 + 2xy = 4v^2 + s^2 + 4vs \]  

Squaring (7)

\[ m^2 + t^2 + 2mt = y^2 + u^2 + 2uy \]  

Adding (3a) and (7a) we get that,

\[ (x + u)(x - u) + (m + 2v)(m - 2v) + (t + s)(t - s) + 2y(x - u) + 2mt - 4vs = 0 \]

i.e. \( (x - u)[x + u + 2y] + (m + 2v)(m - 2v) + (t + s)(t - s) + 2mt - 4vs = 0 \)

Replacing \( x + y \) by \( 2v + s, y + u \) by \( m + t, m - 2v \) by \( u - z \) and \( t - s \) by \( z - x \) we have,

...
A New Concept in Spherical Geometry

[See Eqns. (3), (5), (6) and (7)]

\[(x - u)[m + v + x + u + z] + (m + 2v)(u - z) + (t + s)(t - s) + 2vt - 4vs = 0\]

123 **Rearranging**, \((m + 2v)x - u + u - z + (t + s)[t - s - x - u] + 2vt - 4vs = 0\)

i.e. \((m + 2v)x + t = (t + s)[t - s - x - u] + 2vt - 4vs = 0\)

126 **Substituting** \(s = t\) for \(x - z\) and \(s + t\) for \(x + t\) [See Eq. (6)] we have,

\[(m + 2v)[s - t] + (t + s)[x - u] + 2vt - 4vs = 0\]

129 i.e. \(t(x - u + 2m - m - 2v) + s(x - u - 4v - m - 2v) = 0\)

131 **Replacing** \(m + x\) by \(2v + u\) [see Eq. (5)] we obtain,

\[2m + 4v = 0\]

i.e. \(m + 2v = 0\), i.e. \(m = -2v\) \((8)\)

134 It is well known that in geometry minus theta represents

the vertically opposite angles. Since vertically opposite
angles are equal it implies from \((8)\) that

\(m = 2v\).

138 Combining \((9)\) and \((2)\) we get that the sum of the interior
angles of spherical triangle NCE is equal to four right
angles \((10)\)

141 **Equation** \((10)\) proves our first theorem.

142 **Fourth proof for Theorem 1**

143 In the above spherical figure, the angles at \(W, A, B, C, A\) and

\(BNE\) respectively

146 Considering straight angles \(WAB, ABC, BCE\) and

147 adding,

\[x + y = 2v + s\]

\[x + z = 2v + t\]

\[m + x = 2v + u\]

\[(2) - (3)\) gives,

\[m + x = y + u\]

**Squaring** \((1)\),

\[x^2 + y^2 + 2xy = 4v^2 + s^2 + 4vs\]

**Squaring** \((4)\),

\[m^2 + s^2 + 2mt = y^2 + u^2 + 2yu\]

159 **Adding** \((5)\) and \((6)\),

**x(s + y) + xy + (m + 2v)(m - 2v) + (t + s)(t - s) + 2mt = y(u + y) + 2mt - 4vs = 0\]

160 i.e. \(x(x + y) + y(x - u) + 2mt - 4vs + (t + s)(t - s) + = 0\]

162 Applying \((3)\) in the first factor, and \((6)\) in the third factor

we have, \(x(2v + s) + y(x - u) + 2mt - um - ut - 4vs +\)

\((t + s)(t - s) + = 0\)

164 i.e. \(2v(x + s) + s(x - 2v) + y(x - u) + m +

\((t - u) + t(m - u) + t(s + s) + = 0\)

166 From \((3)\) we have, \(x - s = 2v - y\) and \(x - 2v = s - y\).

167 Assuming these values in the above relations,

\[2v(2v - y) + s(s - y)y(x - u) + m(t - u) + t(m - u)\]

\[+ (t + s)(t - s) + = 0\]

170 i.e. \(y(x - u - 2v) + 4v^2 + s^2 + m(t - u) + t(m - u)\]

\[+ (t + s)(t - s) + = 0\]

173 **Assuming** \((3)\) in the first factor, \(- y(y + u) + 4v^2 + s^2\)

\[+ m(t - u) + t(m - u) + (t + s)(t - s) = 0\]

176 **Rearranging**, \(- y(y + u) + 4v^2 + m(t - u) + t(m - u + t + s) + s(x - s) = 0\)

178 Putting \((4)\) in the first factor,

\[- y(y + u) + 4v^2 + m(t - u) + t(m - u + t + s) + s(x - s) = 0\]

176 i.e. \(4v^2 + m(t - u) + t(m - u + t + s) + s(x - s) = 0\)

178 **Once again** assuming \((4)\), \(4v^2 - m^2 = 0\)

\[(7)\]

180 i.e. \(m = 2v\)

182 **And** \((7)\) is the fourth proof of our first theorem.

**Proof of Theorem 2**

In spherical Fig. 2, consider WN, NE and EW as the three
sides of spherical triangle WNE. Since the angles WNE,
NEW and EWN are straight angles, we obtain that the sum
of the interior angles of spherical triangle WNE is 540
degrees. And hence the proof.

**Discussion**

Let us recall that after the publication of non-Euclidean
math. work Riemann concluded: “Here after it is up to
physicists to apply my findings.” Similarly I request the
research community to apply my results to theoretical
physics and cosmology. There are many burning problems
in physics such like quantum gravity, dark matter, dark
energy and pre big bang phenomena. When Lobachevsky
published his first non Euclidean math in 1824, the whole
research community did NOT approve it. They have
remarked non-Euclidean math, only a mathematical trick.
But this concept was widely applied in Einstein’s special
relativity after 1905 and the formulae of hyperbolic
geometry are being applied to study the atomic properties in quantum physics. Riemann’s second non-Euclidean math. was published in 1854. It was assumed to formulate general relativity in 1915. The hyperbolic math. had to wait for 81 years and the elliptic had to wait for 61 years for recognition and application. Similarly my new findings will be very useful in theoretical physics and cosmology. There was a trouble with Maxwell’s equations. A deep analysis of his equations predicted that light is an electromagnetic wave. And PAM Dirac also encountered such a physical phenomenon. Then Dirac’s equations revealed that there exists anti-particles. Similarly the author’s finding that the sum of the interior of spherical triangle NCE is equal to four straight angles will unlock some of the mysteries of cosmological problems such like gravitational waves, dark energy and dark matter. The applications of Eq. (7) to spherical trigonometry and differential equations will predict new cosmological phenomena.

Algebra is the extension of number theory. It occupies almost all the areas of science, technology, and administrative. The famous French mathematician used to tell time and again that As long as algebra and geometry have been separated, their progress have been slow and their uses limited; but when these two sciences have been united, they have lent each mutual forces, and have marched together towards perfection. And once Einstein proposed to the scientific community to put all the equations of physics in algebra. These two says are the foundational guide lines for the author for the preparation of this paper. Future probes and studies will surely create a new field of spherical geometry & trigonometry.

Kepler’s law of planetary motions, Galileo’s inventions, Einstein’s special and general relativity theories, De Broglie’s matter waves hypothesis, Paul’s exclusion principle, Heisenberg’s uncertainty principle, Fractals geometric idea, Lobachevsky’s non-Euclidean geometric concept, Riemann’s non-Euclidean geometric theory, Peter Higgs’ Bosons papers and many other ground breaking inventions were initially NOT accepted/approved by the research community. Later on these findings were gradually agreeable to the scientists. In quasicrystals, we find the fascinating mosaics of the Arabic world reproduced at the level of atoms: regular patterns that never repeat themselves. However, the configuration found in quasicrystals was considered impossible, and Dan Shechtman had to fight a fierce battle against established science. The Nobel Prize in Chemistry 2011 has fundamentally altered how chemists conceive of solid matter. The research community, Editors and referees of professional journals, research institutes and chemistry departments completely ignored, insulted and avoided and Dan Shechtman. But he did not bother about this and continued his research and finally won the Nobel prize. In this work, by applying algebra to geometry, the author has found challenging results.

References
7. www.slibborhall.org/pdfs/spherical_trigonometry2.pdf