GENERALIZED LORENTZ TRANSFORMATIONS

A. Blato

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Argentina

This article presents the generalized Lorentz transformations of time, space, velocity and acceleration which can be applied in any inertial or non-inertial (non-rotating) frame.

Introduction

If we consider a (non-rotating) frame S relative to another inertial frame Σ then the time (t), the position ($\mathbf{r}$), the velocity ($\mathbf{v}$) and the acceleration ($\mathbf{a}$) of a (massive or non-massive) particle relative to the frame Σ are given by:

$$t \;=\; \int_{0}^{\mathtt{t}} \gamma \; \mathtt{dt} \;-\; \gamma \; \frac{\vec{r} \cdot \mathbf{V}}{c^2} \;+\; \mathtt{k}$$

$$\mathbf{r} \;=\; \vec{r} \;+\; \frac{\gamma^2}{\gamma+1} \, \frac{(\, \vec{r} \cdot \mathbf{V} \,)\, \mathbf{V}}{c^2} \;-\; \mathbf{R} \;-\; \frac{\gamma^2}{\gamma+1} \, \frac{(\, \mathbf{R} \cdot \mathbf{V} \,)\, \mathbf{V}}{c^2}$$

$$\mathbf{v} \;\doteq\; \frac{d\mathbf{r}}{dt}$$

$$\mathbf{a} \;\doteq\; \frac{d\mathbf{v}}{dt}$$

where ($\mathtt{t}$, $\vec{r}$) are the time and the position of the particle relative to the frame S ($\mathbf{R}, \mathbf{V}, \mathbf{A}$) are the position, the velocity and the acceleration of the origin of the frame Σ relative to the frame S, ($\mathtt{k}$) is a particular constant between the frames Σ & S, (c) is the speed of light in vacuum, and $\gamma \;\doteq\; (1 - \mathbf{V} \cdot \mathbf{V}/c^2)^{-1/2}$

- $\dfrac{\gamma^2}{\gamma+1}\,\dfrac{1}{c^2} \;=\; \dfrac{\gamma-1}{\mathbf{V}^2} \qquad (\, \mathbf{V}^2 = \mathbf{V}\cdot\mathbf{V}\,)$

- $\vec{r} \,+\, \dfrac{\gamma^2}{\gamma+1}\,\dfrac{(\,\vec{r}\cdot\mathbf{V}\,)\,\mathbf{V}}{c^2} \;=\; \gamma\,\vec{r} \,+\, \dfrac{\gamma^2}{\gamma+1}\,\dfrac{(\,\vec{r}\times\mathbf{V}\,)\times\mathbf{V}}{c^2}$

- $\mathbf{R} \,+\, \dfrac{\gamma^2}{\gamma+1}\,\dfrac{(\,\mathbf{R}\cdot\mathbf{V}\,)\,\mathbf{V}}{c^2} \;=\; \gamma\,\mathbf{R} \,+\, \dfrac{\gamma^2}{\gamma+1}\,\dfrac{(\,\mathbf{R}\times\mathbf{V}\,)\times\mathbf{V}}{c^2}$

The frame S is inertial when $(\,\mathbf{A} \;=\; 0\,)$

The frame S is non-inertial (rectilinear accelerated motion) when $(\,\mathbf{A} \;\neq\; 0\,)$ and $(\,\mathbf{A}\times\mathbf{V} \;=\; 0\,)$

The frame S is non-inertial (uniform circular motion) when $(\,\mathbf{A} \;\neq\; 0\,)$ and $(\,\mathbf{A}\cdot\mathbf{V} \;=\; 0\,)$

If the frame S is inertial then the observer S must use a fixed origin O such that $(\,\mathbf{R}\times\mathbf{V} \;=\; 0\,)$

If the frame S is non-inertial (rectilinear accelerated motion) then the observer S must use a fixed origin O such that $(\,\mathbf{R}\times\mathbf{V} \;=\; 0\,)$

If the frame S is non-inertial (uniform circular motion) then the observer S must use a fixed origin O such that $(\,\mathbf{R}\cdot\mathbf{V} \;=\; 0\,)$

If the frame S is inertial then $(\,\mathbf{A} \;=\; 0\,)$, $(\,\mathbf{V} \;=\; \text{constant}\,)$, $(\,\gamma \;=\; \text{constant}\,)$ $(\,\int_0^{\mathtt{t}}\gamma\,\mathtt{dt} = \gamma\,\mathtt{t}\,)$, $(\,\mathbf{R} \;=\; \mathbf{V}\,\mathtt{t} + \text{constant}\,)$ and $(\,\mathbf{R}\times\mathbf{V} \;=\; 0\,)$

If the frame S is non-inertial (rectilinear accelerated motion) then $(\,\mathbf{A} \;\neq\; 0\,)$ $(\,\mathbf{A}\times\mathbf{V} \;=\; 0\,)$ and $(\,\mathbf{R}\times\mathbf{V} \;=\; 0\,)$

If the frame S is non-inertial (uniform circular motion) then $(\,\mathbf{A} \;\neq\; 0\,)$ $(\,\mathbf{A}\cdot\mathbf{V} \;=\; 0\,)$, $(\,\gamma \;=\; \text{constant}\,)$, $(\,\int_0^{\mathtt{t}}\gamma\,\mathtt{dt} = \gamma\,\mathtt{t}\,)$ and $(\,\mathbf{R}\cdot\mathbf{V} \;=\; 0\,)$

If the frame S is inertial or non-inertial (non-rotating) then the observer S can use test particles such that $(\,\vec{r}\times\mathbf{V} \;=\; 0 \;\text{ or }\; \vec{r}\cdot\mathbf{V} \;=\; 0\,)$

According to this article, the relative velocities of the origins between the frames Σ & S are reciprocal.

General Observations

It is known that in inertial frames the local geometry is Euclidean and that in non-inertial frames the local geometry is in general non-Euclidean.

According to this article, the local line element in the frame S must be obtained from the local line element of the frame Σ.

The local line element in the frame Σ (in rectilinear coordinates) is given by:

- $ds^2 \;=\; c^2 dt^2 - d\mathbf{r}^2$

The kinematic quantities $(\, t, \mathbf{r}, \mathbf{v}, \mathbf{a}\,)$ are the proper kinematic quantities of the frame Σ.

Therefore, the kinematic quantity $(\, t \,)$ is a tensor of rank 0 and the kinematic quantities $(\, \mathbf{r}, \mathbf{v}, \mathbf{a}\,)$ are tensors of rank 1.

According to this article, if the frame S is inertial or non-inertial (rectilinear accelerated motion) or non-inertial (uniform circular motion) then:

- $\dfrac{d\mathbf{r}}{dt} \;=\; \left(\dfrac{d\mathbf{r}}{\mathtt{dt}} \;+\; w\,\mathbf{r} \;+\; \Omega \times \mathbf{r}\right)\left(\dfrac{1}{dt\,/\,\mathtt{dt}}\right) \quad , \quad w \;\doteq\; \dfrac{\gamma^2}{\gamma+1}\,\dfrac{(\,\mathbf{A}\,\cdot\,\mathbf{V}\,)}{c^2}$

- $\dfrac{d\mathbf{v}}{dt} \;=\; \left(\dfrac{d\mathbf{v}}{\mathtt{dt}} \;+\; w\,\mathbf{v} \;+\; \Omega \times \mathbf{v}\right)\left(\dfrac{1}{dt\,/\,\mathtt{dt}}\right) \quad , \quad \Omega \;\doteq\; \dfrac{\gamma^2}{\gamma+1}\,\dfrac{(\,\mathbf{A} \times \mathbf{V}\,)}{c^2}$

Finally, the velocity of light in vacuum is $(\, \mathbf{c} \,)$ in the frame Σ and $(\, \vec{c}\,)$ in the frame S and $(\, \mathbf{c}\cdot\mathbf{c}\,)$ & $(\, \vec{c}\cdot\vec{c}\,)$ are constant in the frames Σ & S, respectively.

Bibliography

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